Statisztikus tanulás az idegrendszerben

ORBÁN GERGŐ

golab.wigner.mta.hu

WIGNER RESEARCH CENTRE FOR PHYSICS

Neurális válaszok

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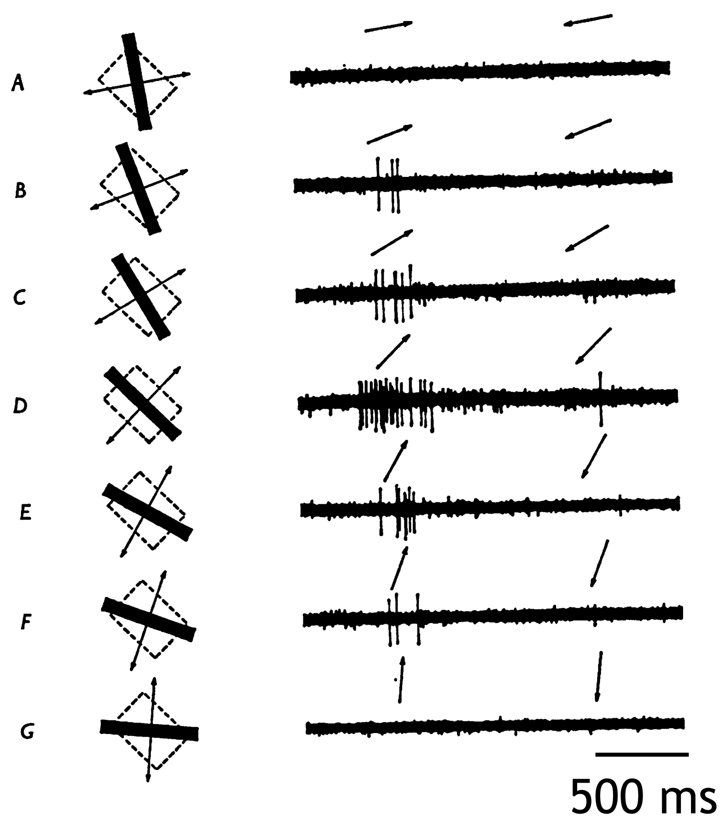
V1 characteristic response



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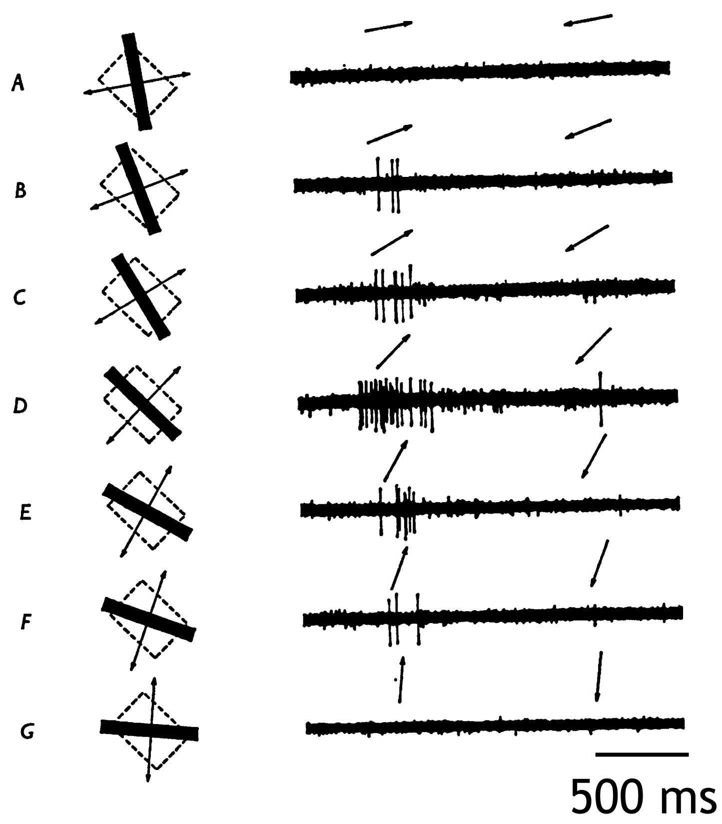

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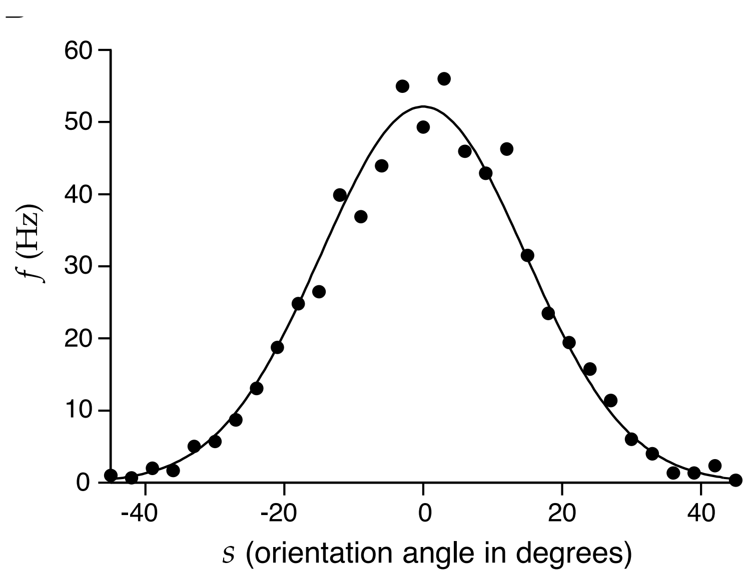

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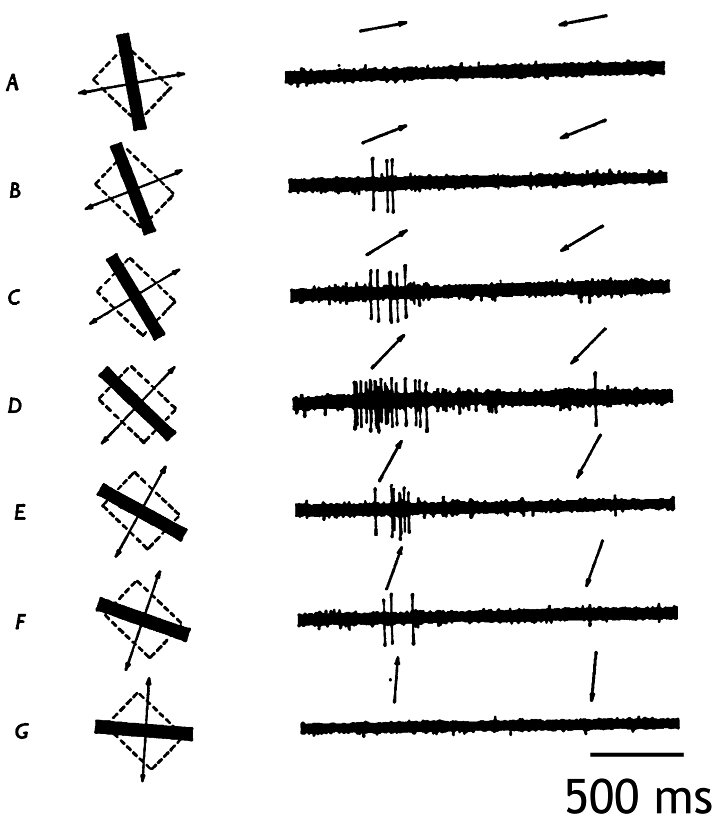

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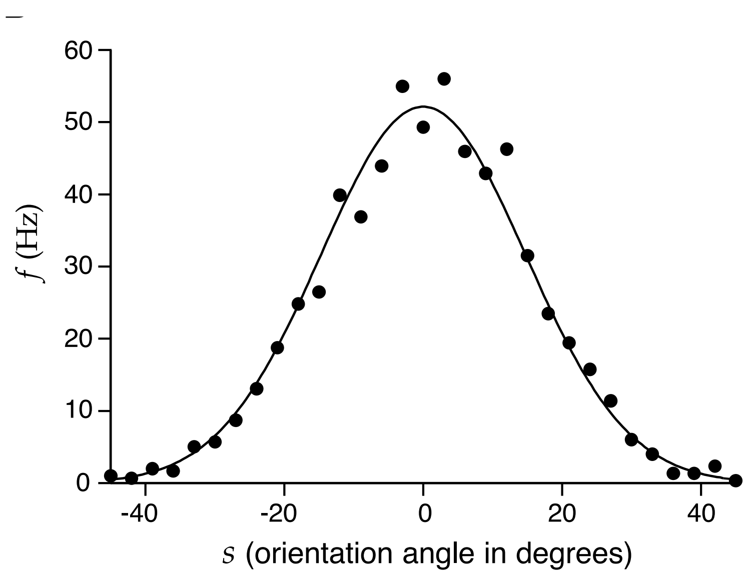
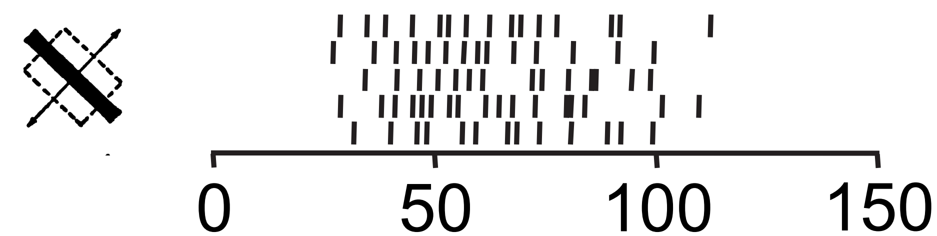


V1 spike train variability



Gur & Snodderly, Cereb Cortex 2006

Neurális válaszok

$$s \sim r$$

Encoding:

$$P\left[r \mid s\right]$$

Decoding:

$$P\left[s \mid r\right] = \frac{P\left[r \mid s\right] P\left[s\right]}{P\left[r\right]}$$

For binary discrimination:

$$P\left[s_1 \mid r\right] = \frac{P\left[r \mid s_1\right] P\left[s_1\right]}{P\left[r\right]} =$$

$$\frac{P\left[r \mid s_1\right] P\left[s_1\right]}{P\left[r \mid s_1\right] P\left[s_1\right] + P\left[r \mid s_2\right] P\left[s_2\right]}$$

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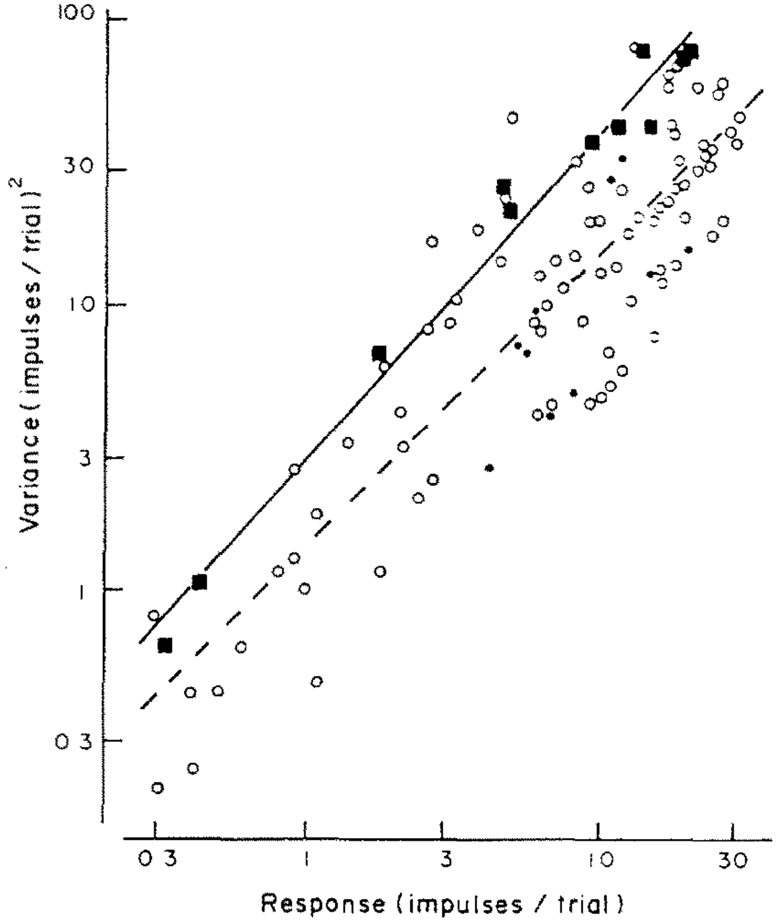
Discrimination is linear:

$$P[s_1 \mid r] = \sigma(\mathbf{w}^\mathsf{T} r + w_0)$$

$$\mathbf{w} = \Sigma^{-1}(\mu_1 - \mu_2)$$

$$w_0 = \frac{1}{2}\mu_1^\mathsf{T} \Sigma^{-1} \mu_1 + \frac{1}{2}\mu_2^\mathsf{T} \Sigma^{-1} \mu_2$$

Neurális zaj



Tolhurst et al (1983) Vision Res

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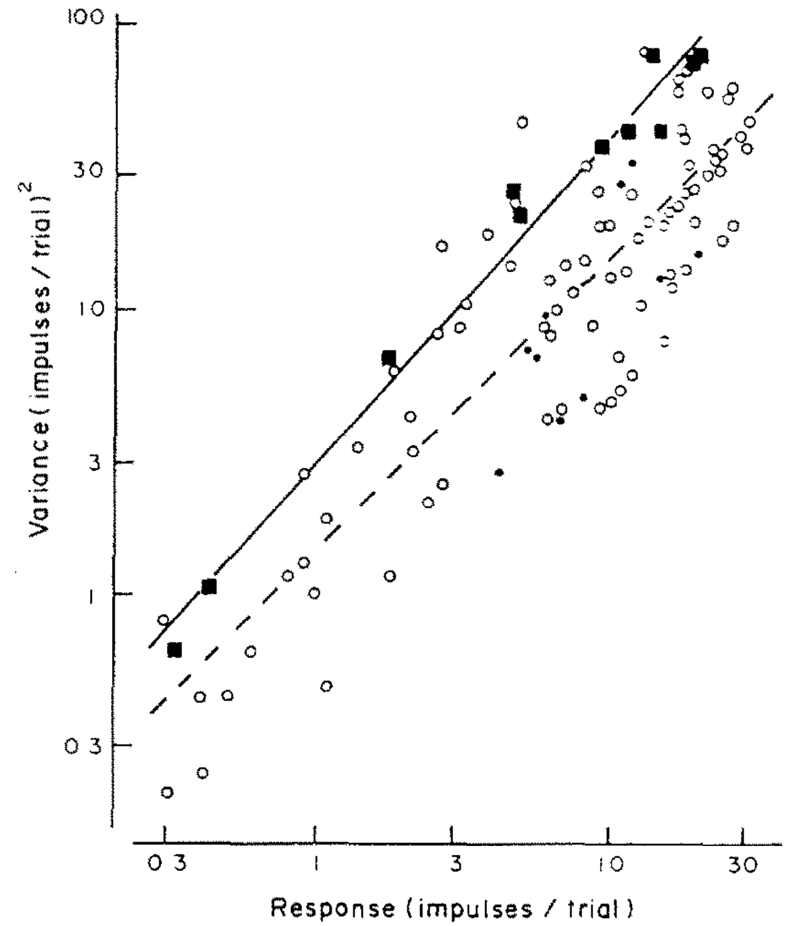

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Poisson process: In any given time window λ the probability of firing is determined by the firing rate

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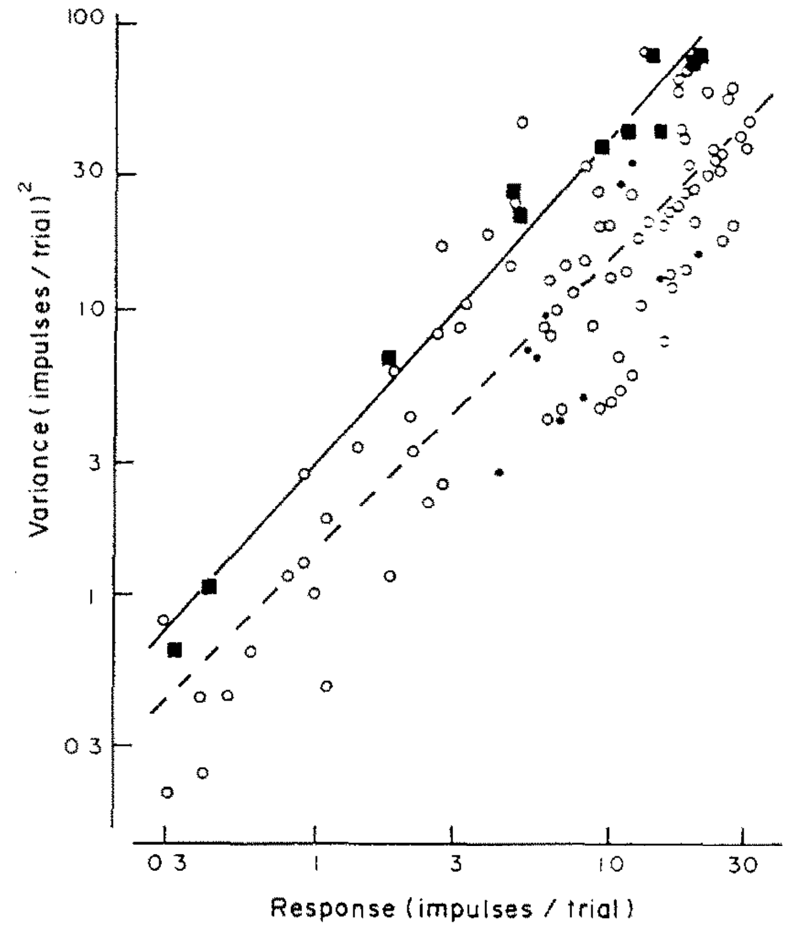

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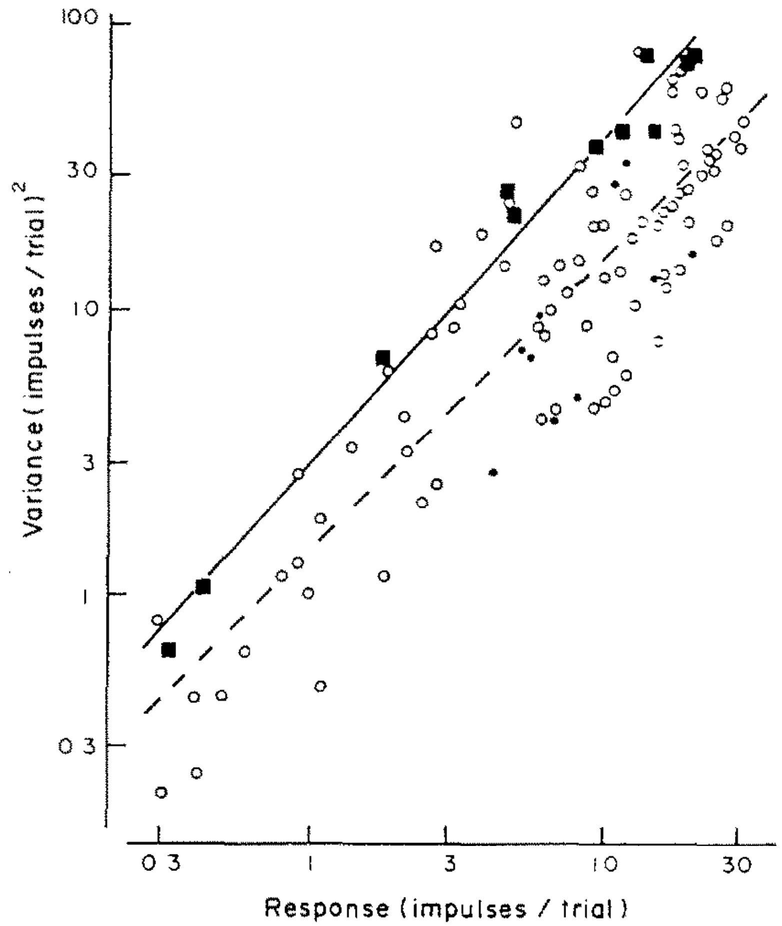

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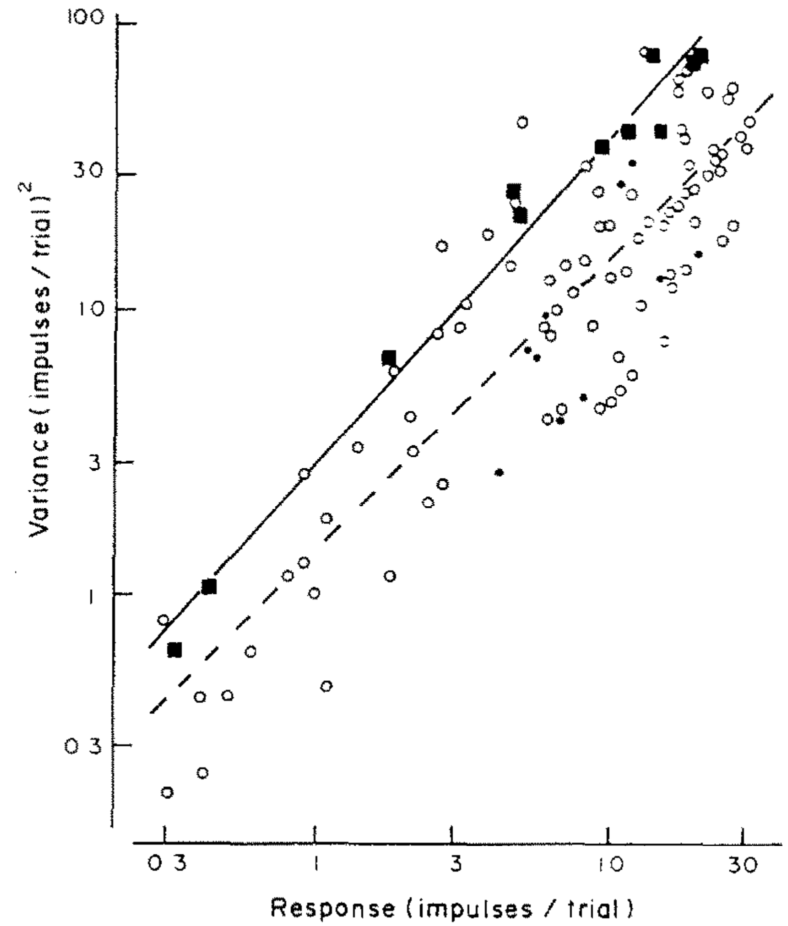

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Homework: prove that we will obtain a linear decoder if the response noise is Poisson

Bayes inferencia neuronhálózatokkal: PPC

V1 orientáció-szelektív neuronok

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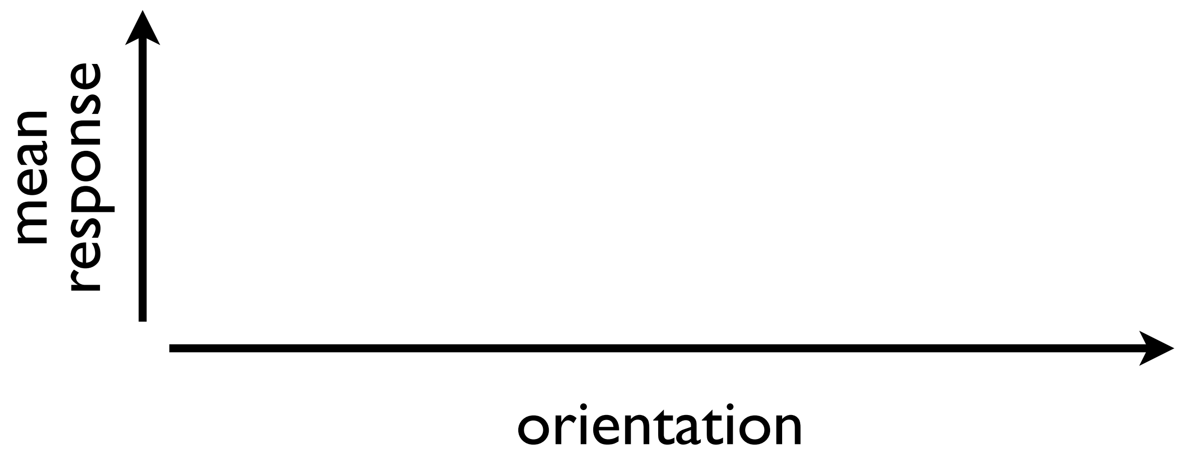

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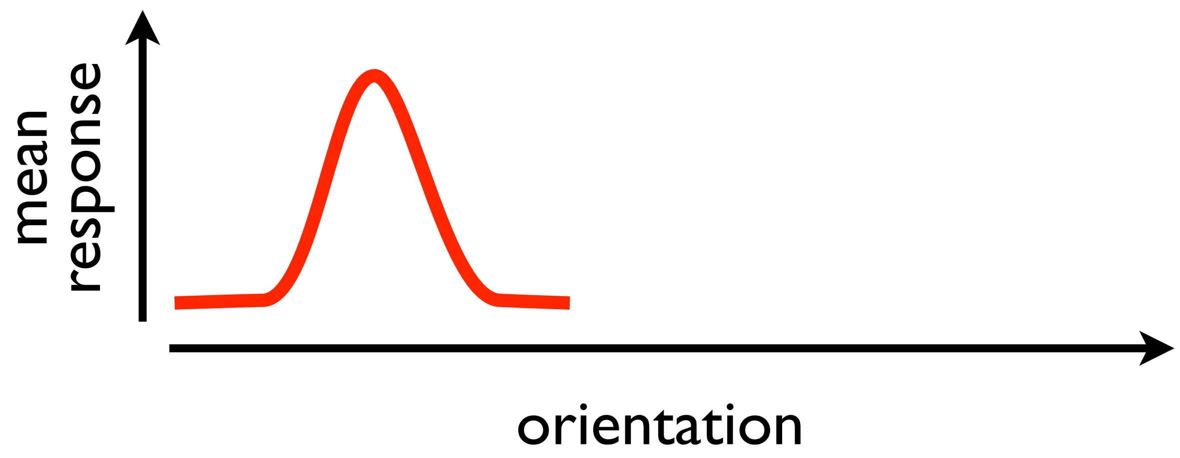

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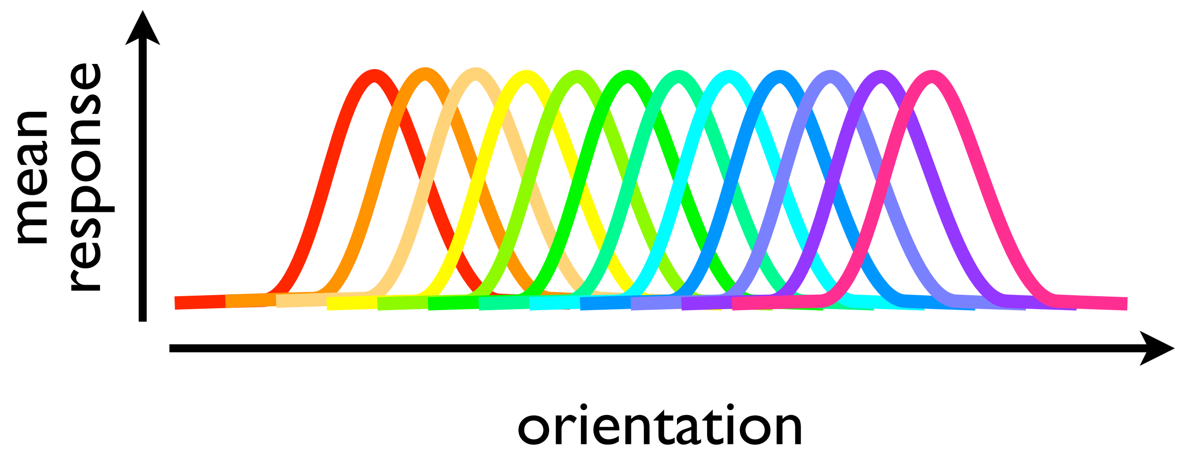

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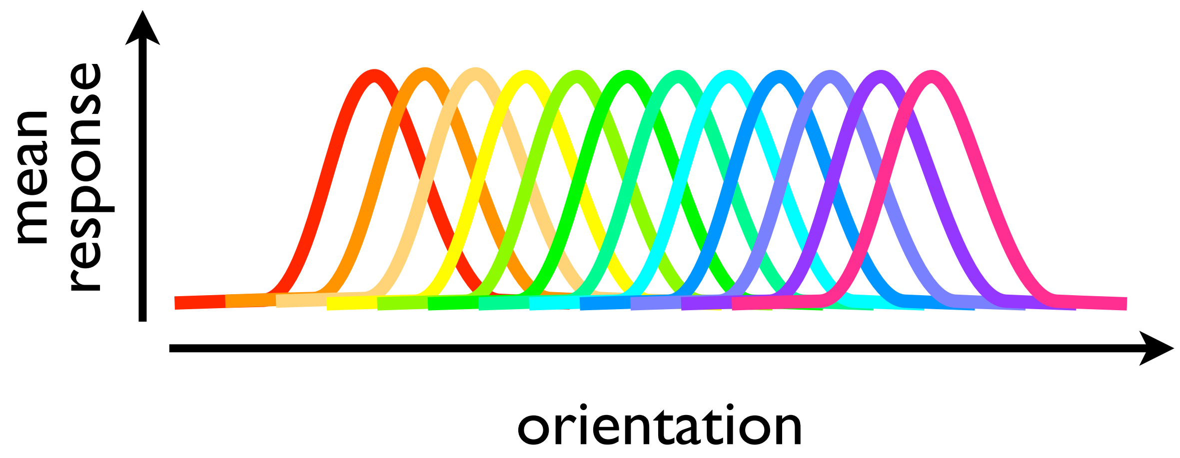

a neuronok azonban zajosak:
az átlag körül az átlaggal
arányos variabilitás van jelen

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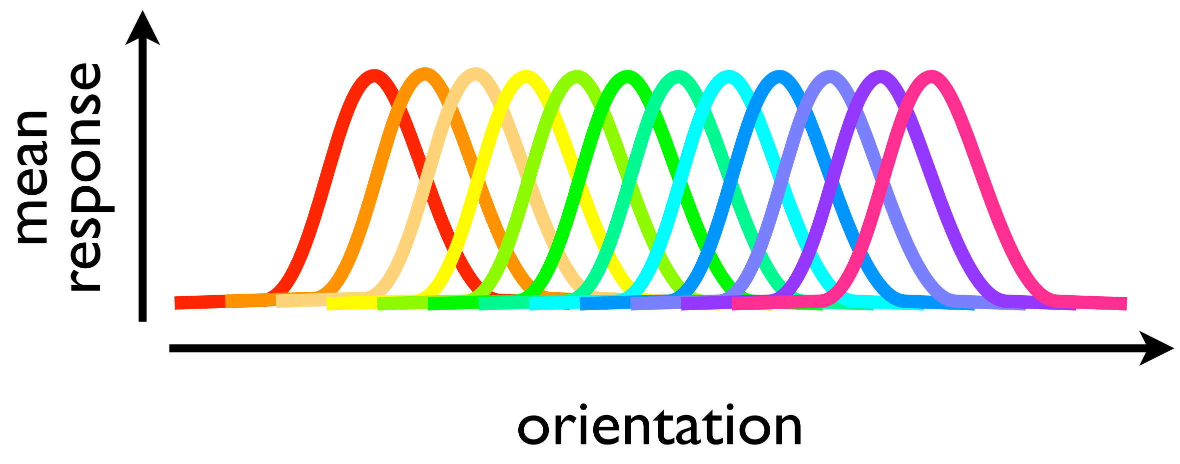

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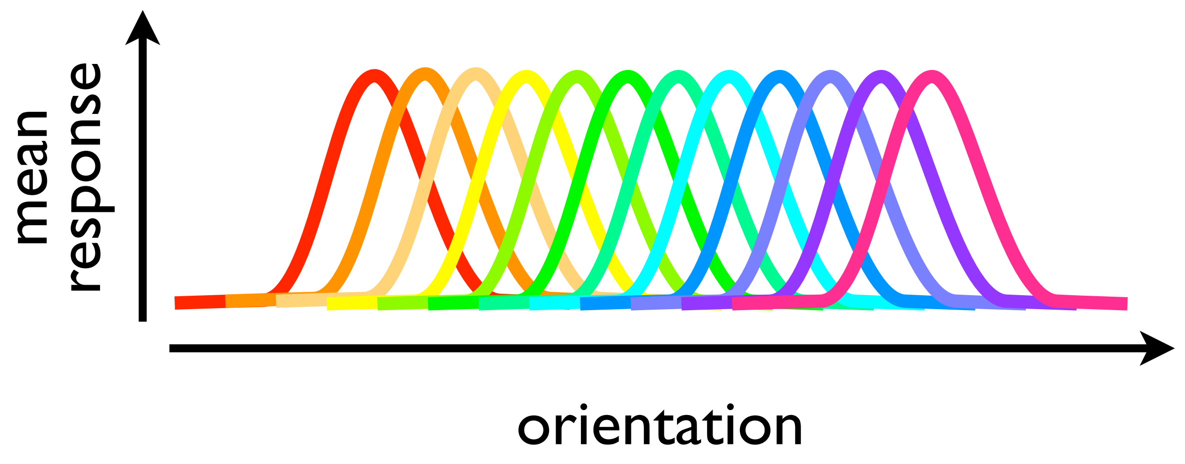

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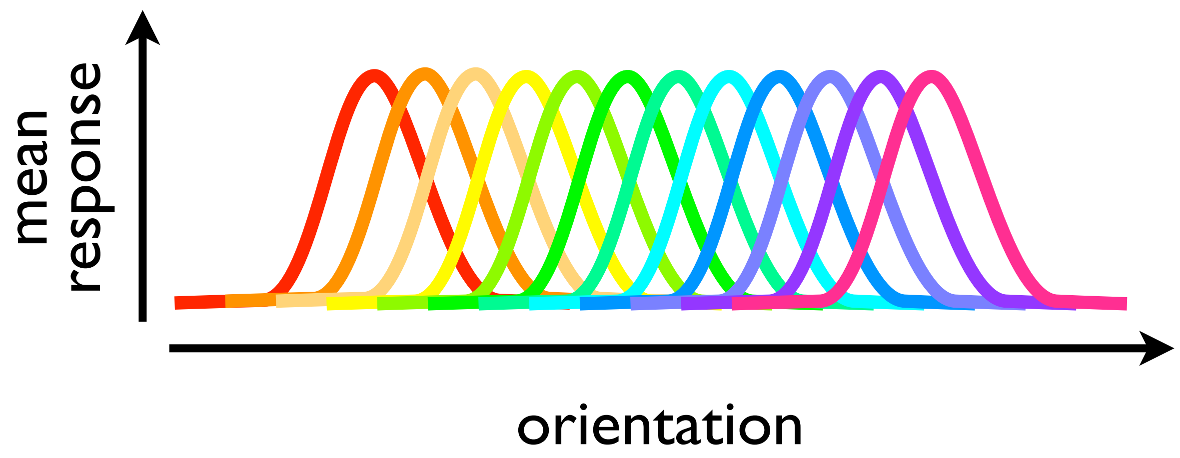

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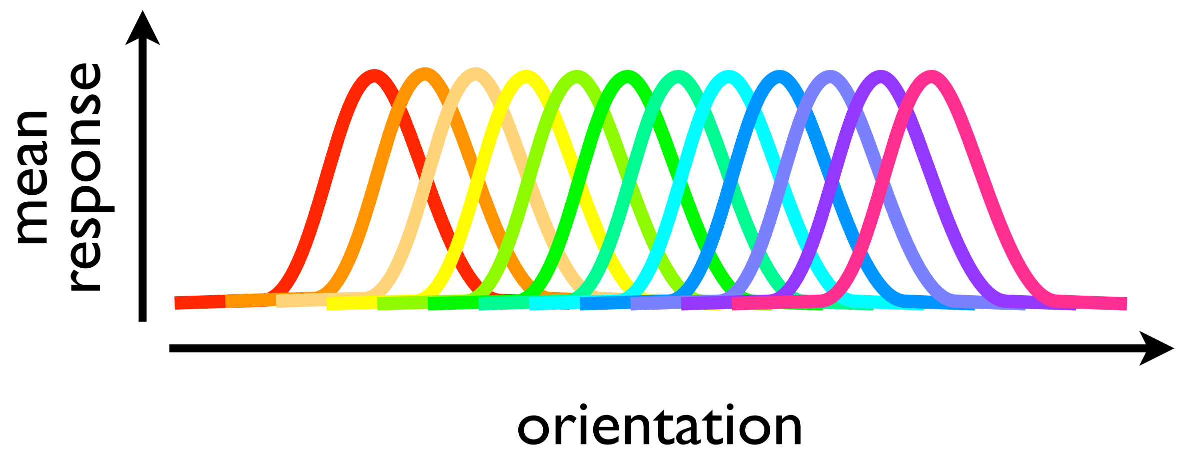

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Bayes: $P(s \,|\, \mathbf{r}) \propto P(\mathbf{r} \,|\, s)\, P(s)$

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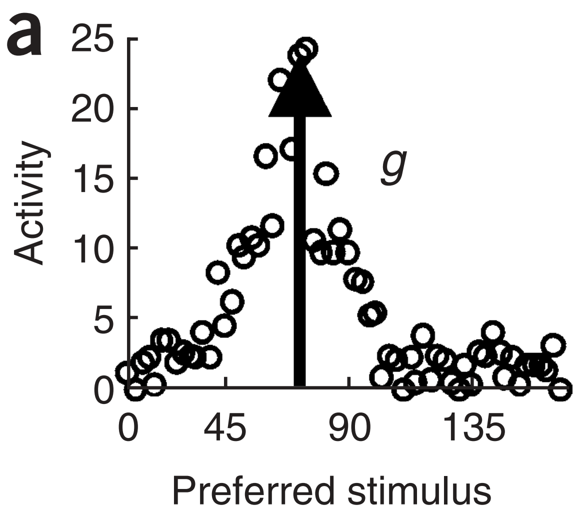

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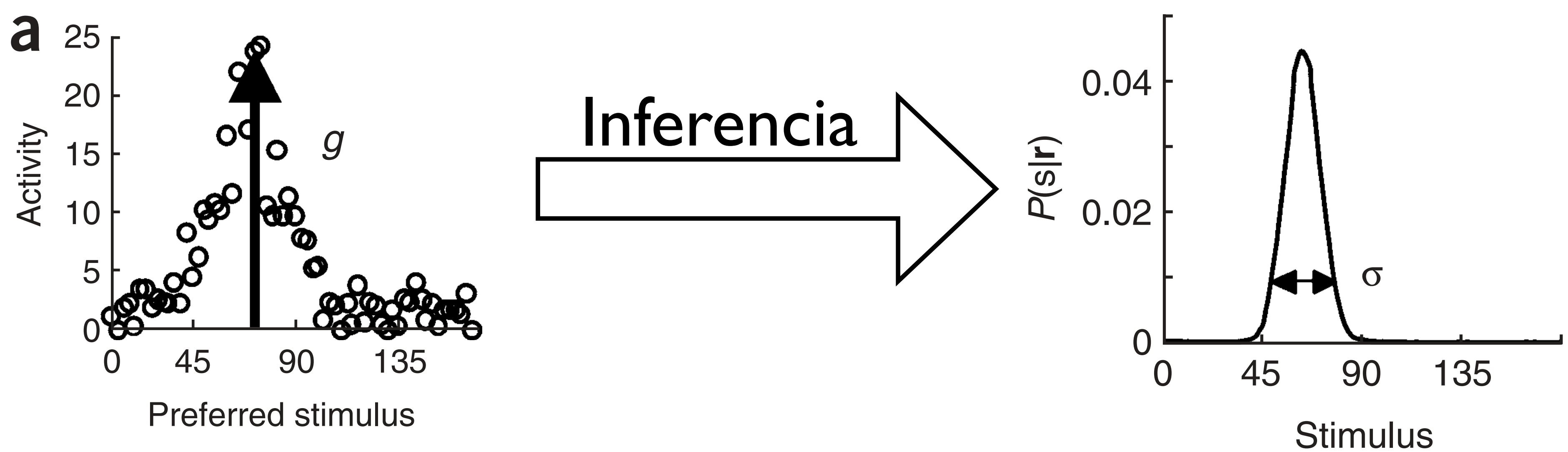

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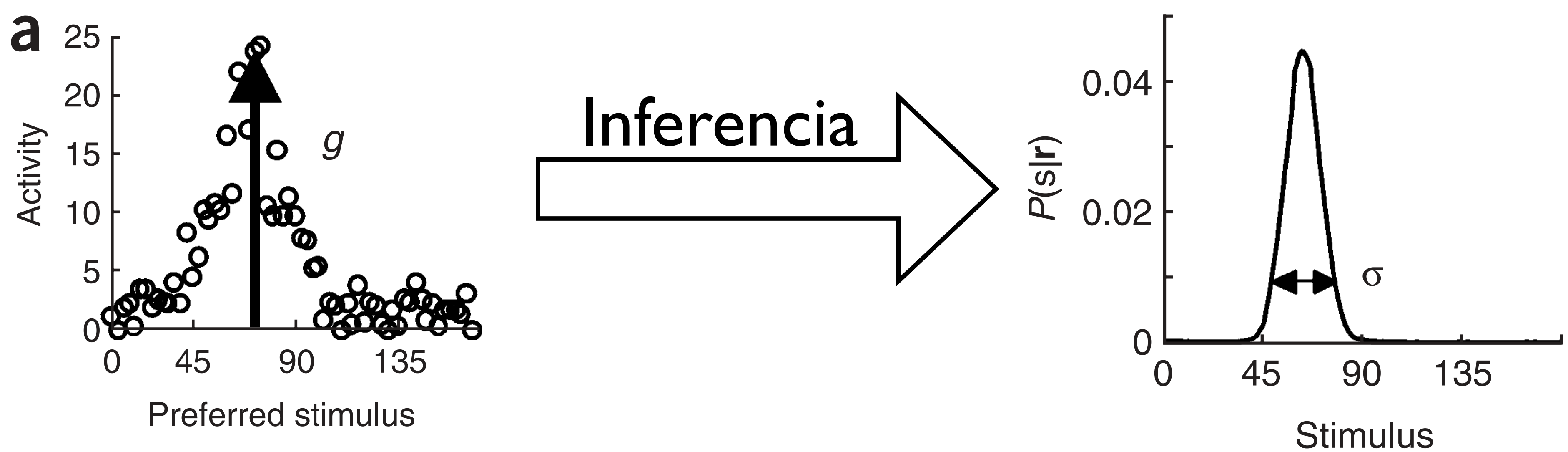

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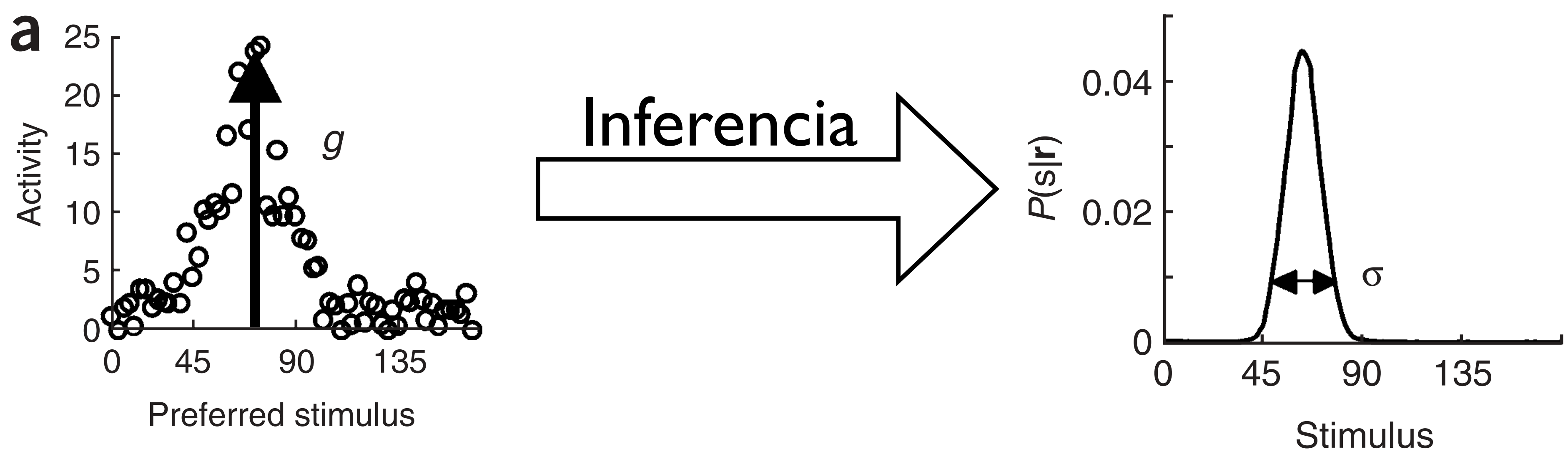

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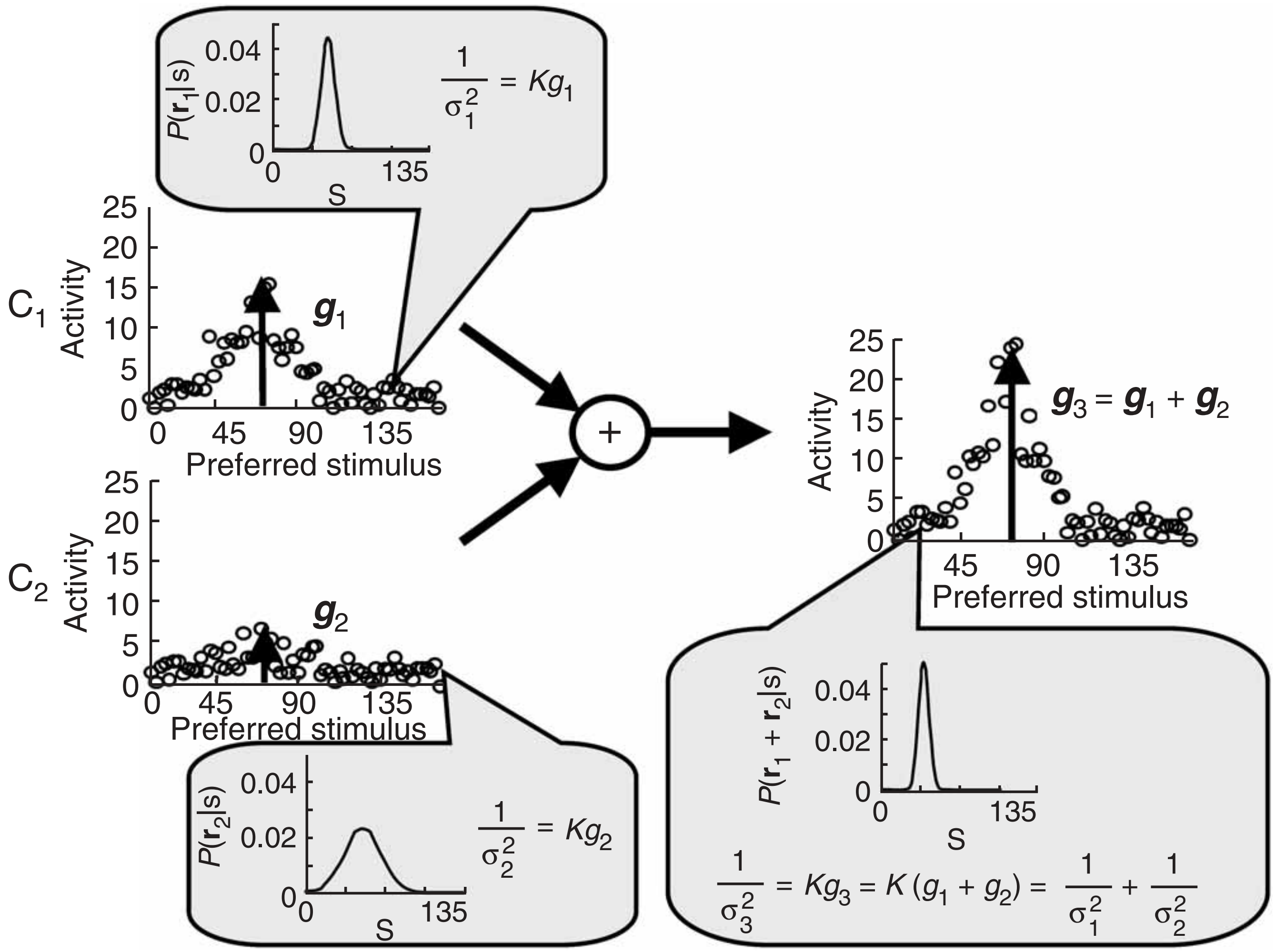
$$g \propto \frac{1}{\sigma^2}$$

PPC: Multiszenzoros integráció



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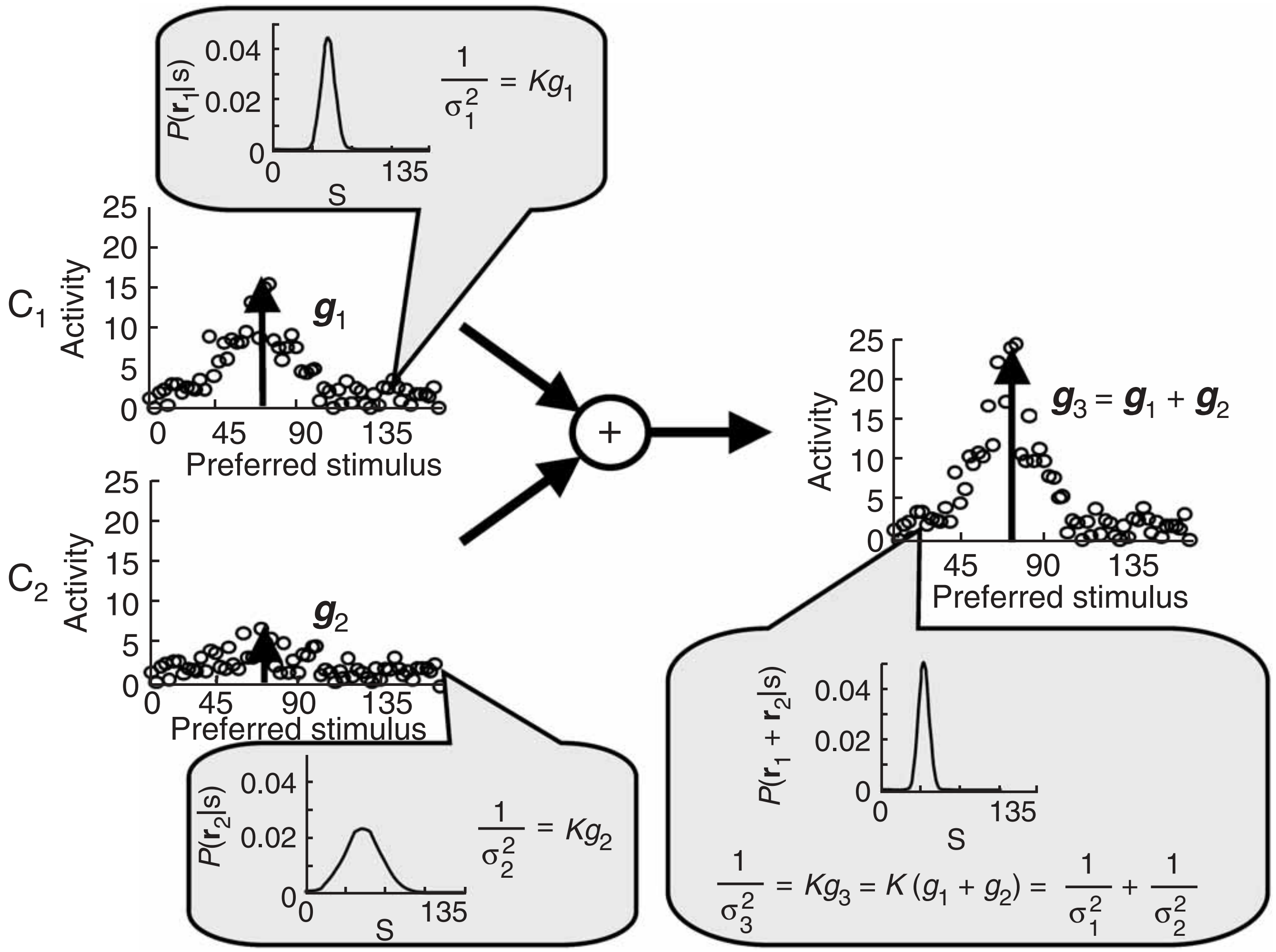

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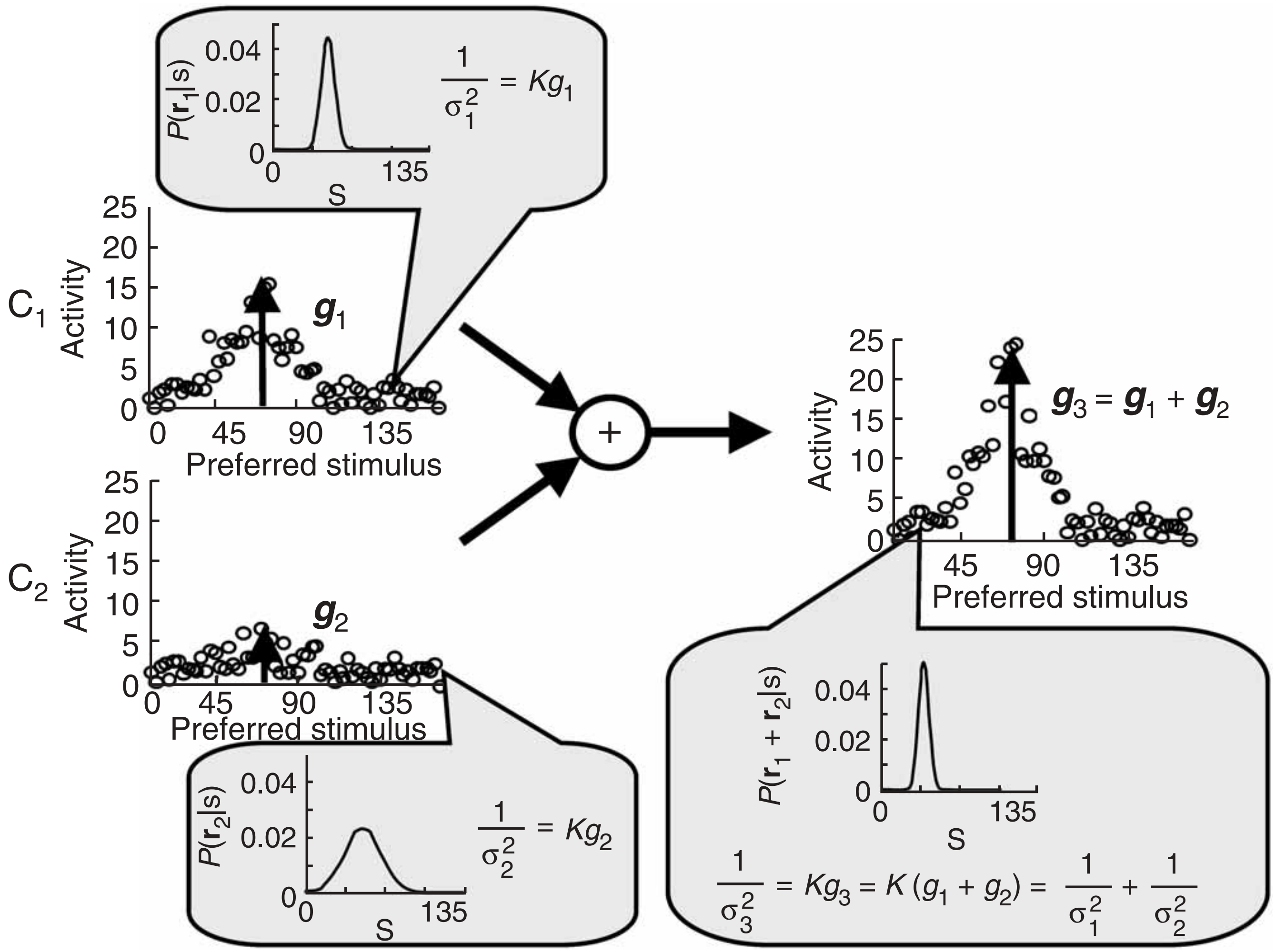

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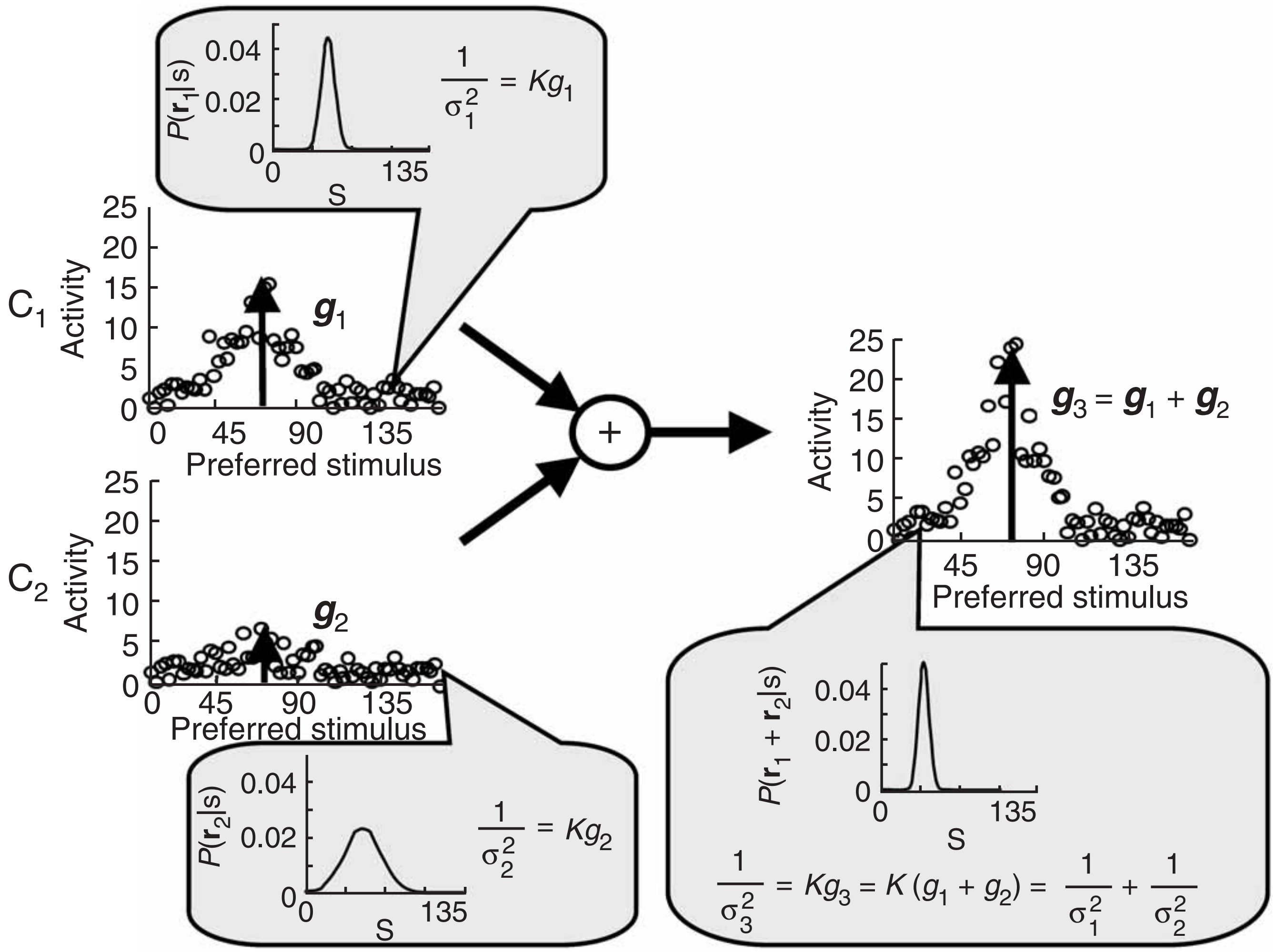

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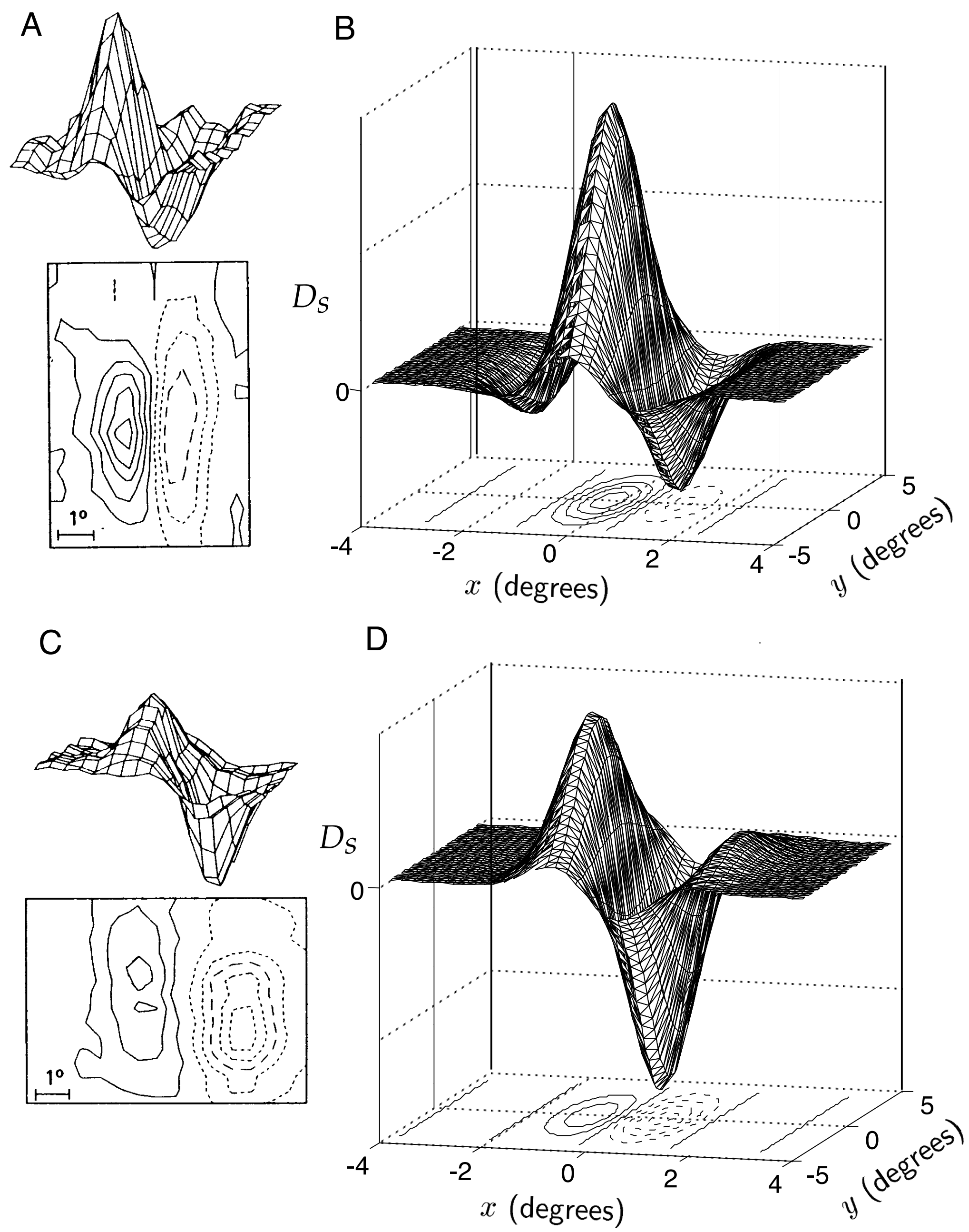
$$\frac{1}{\sigma_3^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$

$$\mu_3 = \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\mu_1 + \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}\mu_2$$

Receptív mezők

Stimulus is not orientation or any other simple stimulus feature , rather a 2D image (sequence)



Dayan & Abbott (2000) Theoretical

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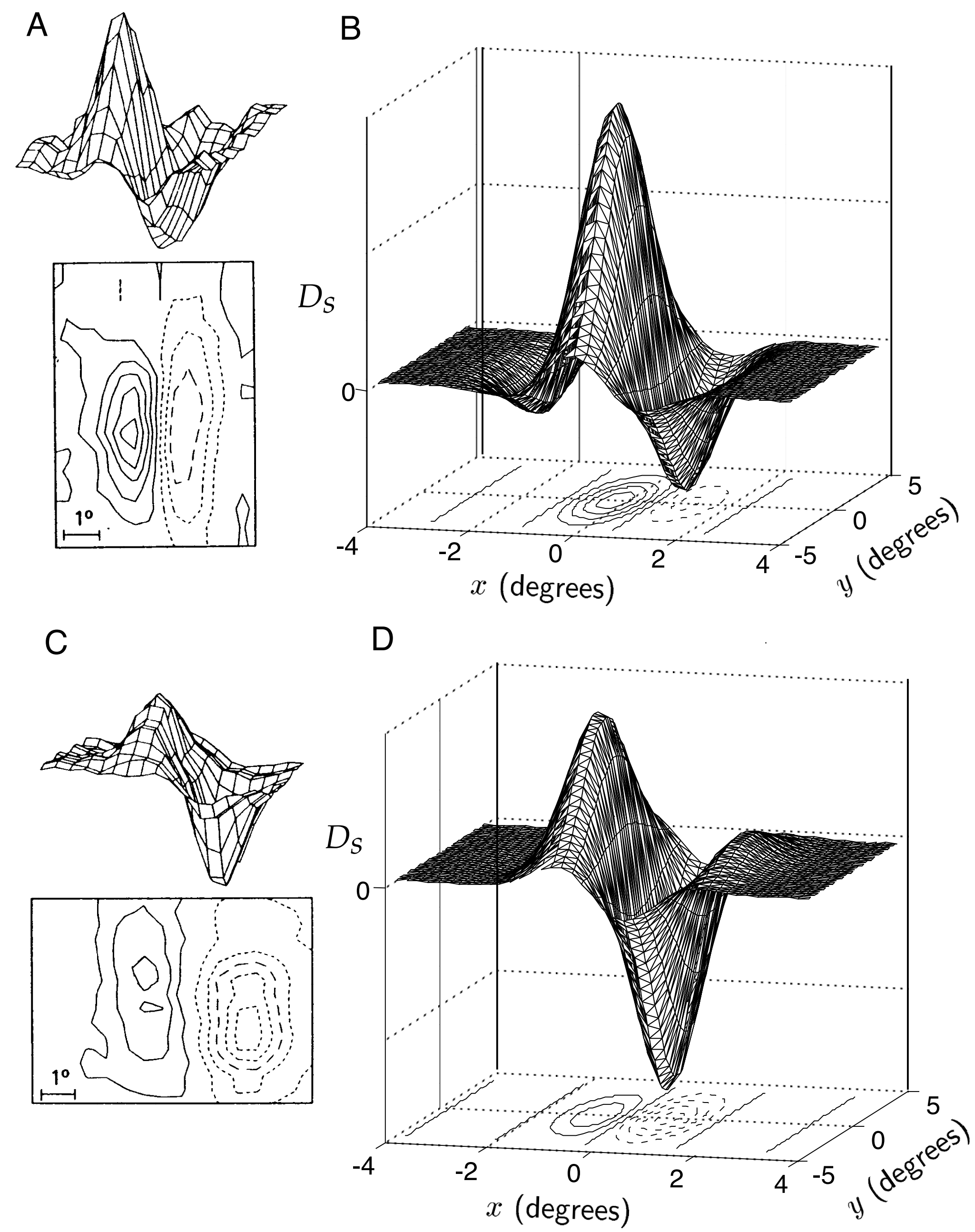

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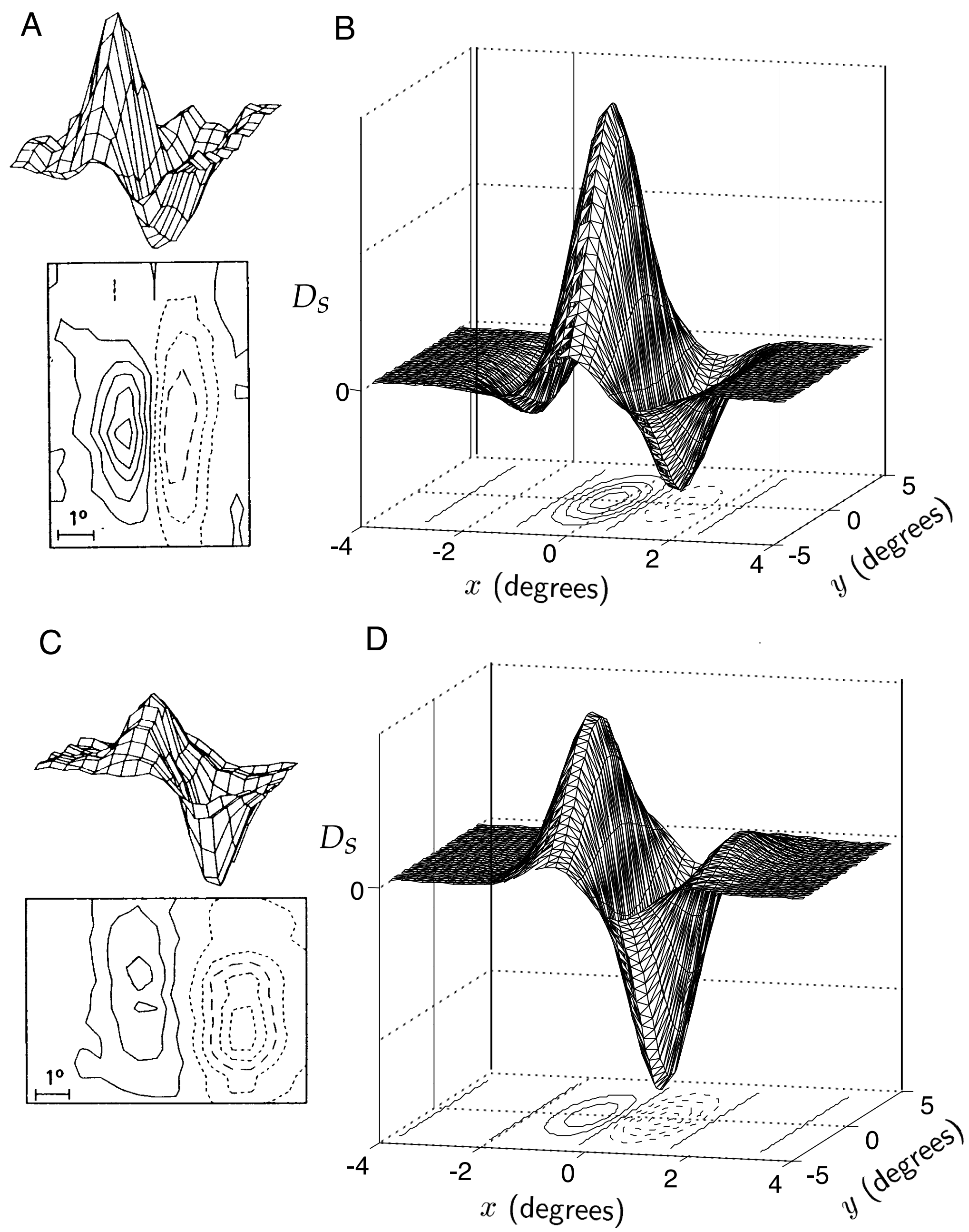

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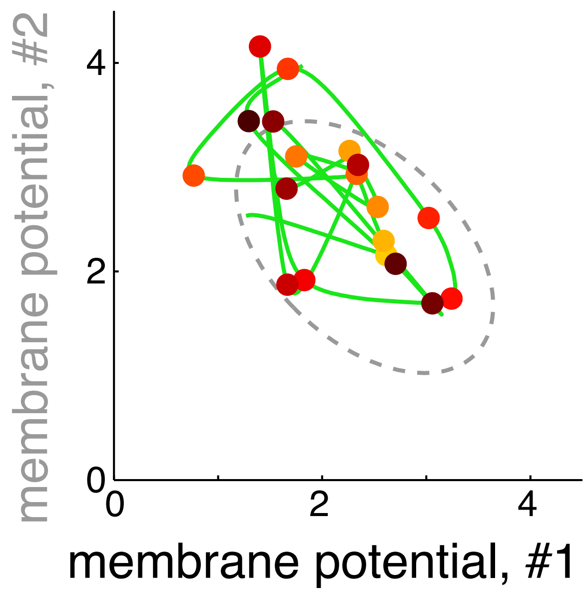
maximum likelihood fitting?

stochastic sampling



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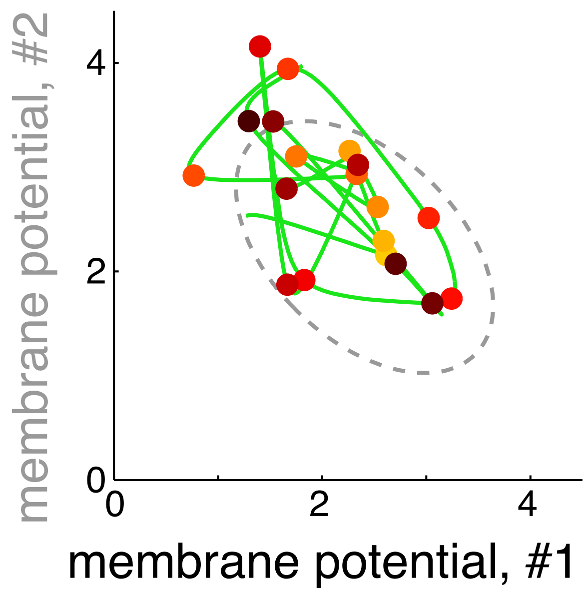

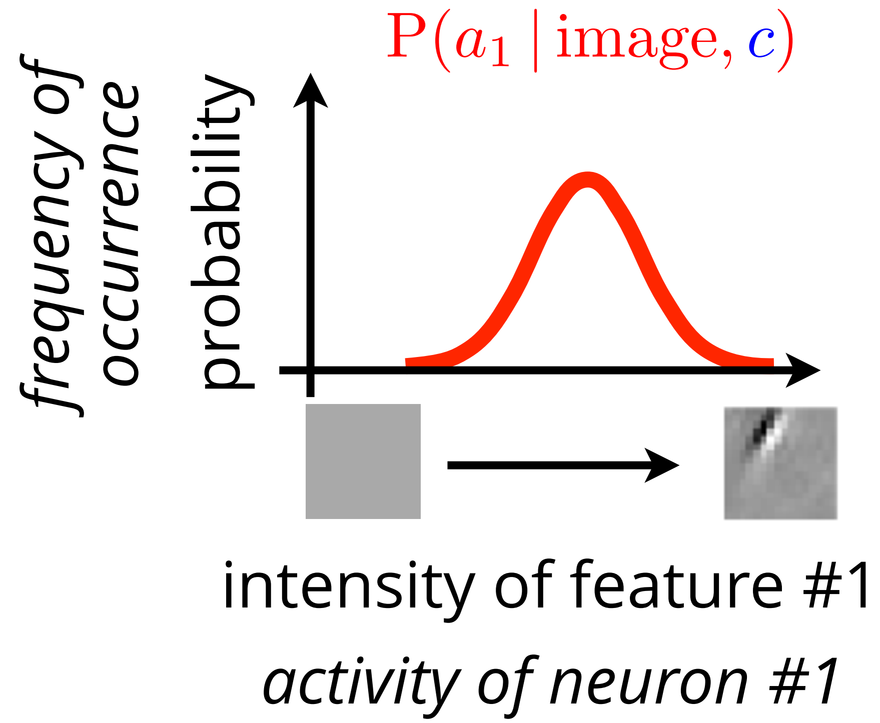

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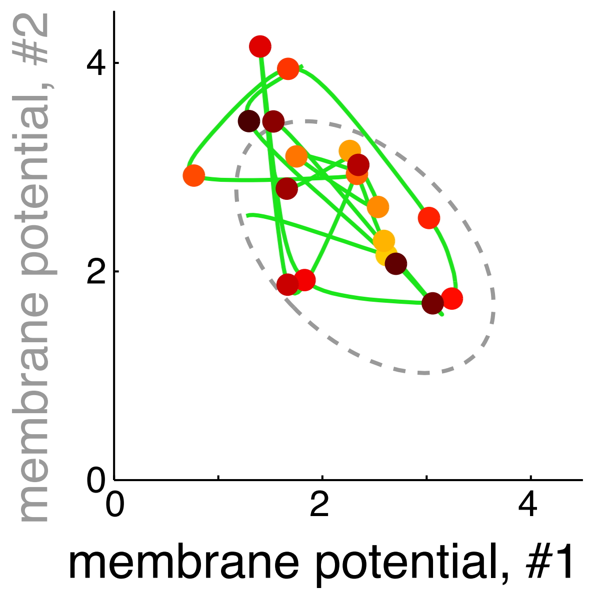

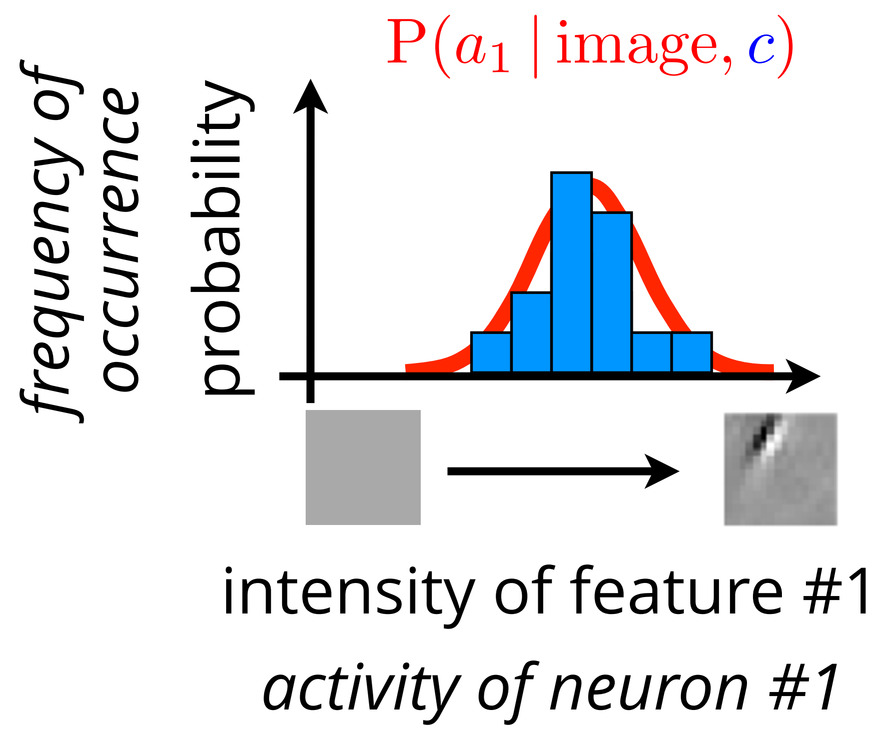

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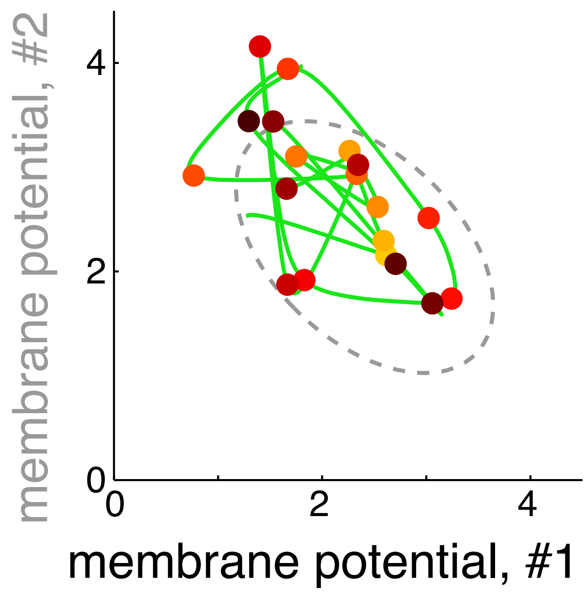

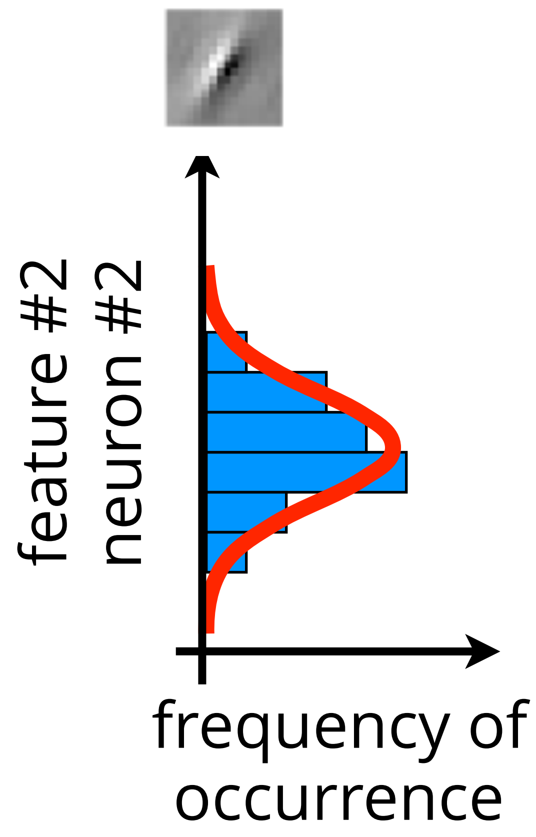

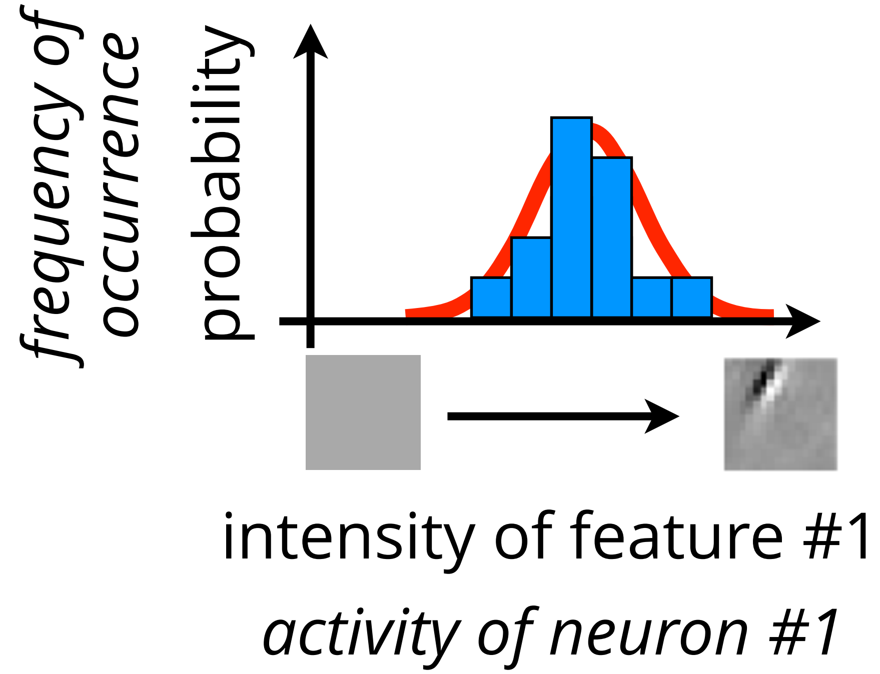

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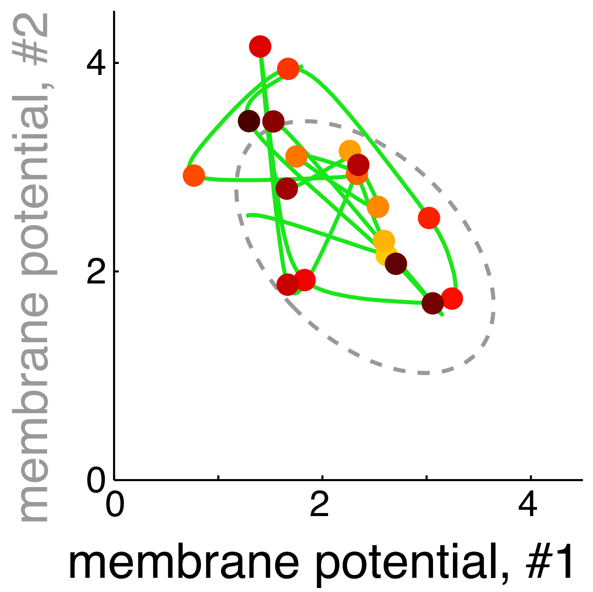

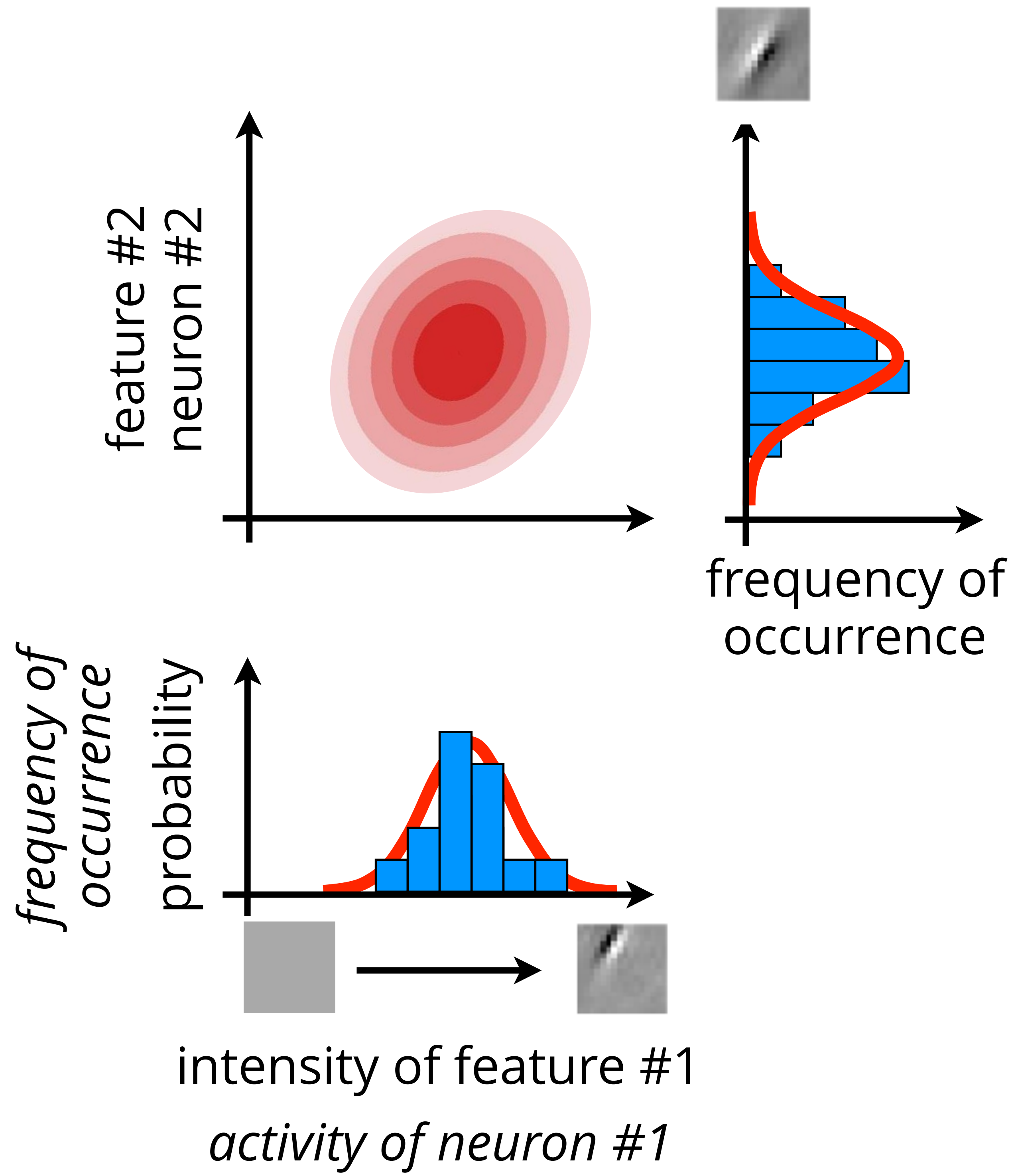

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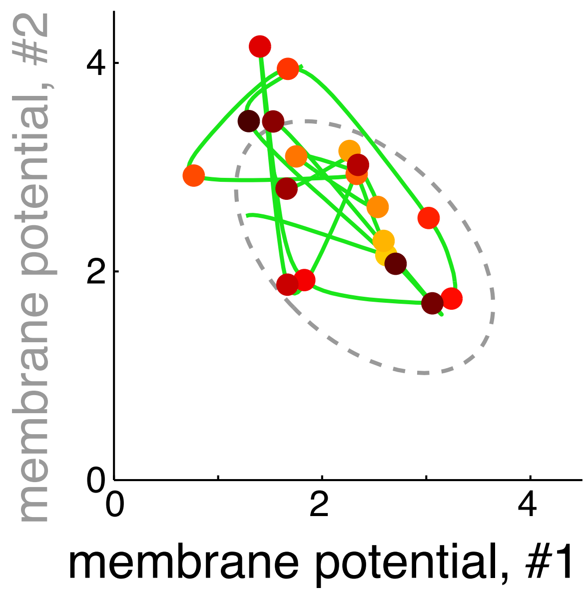
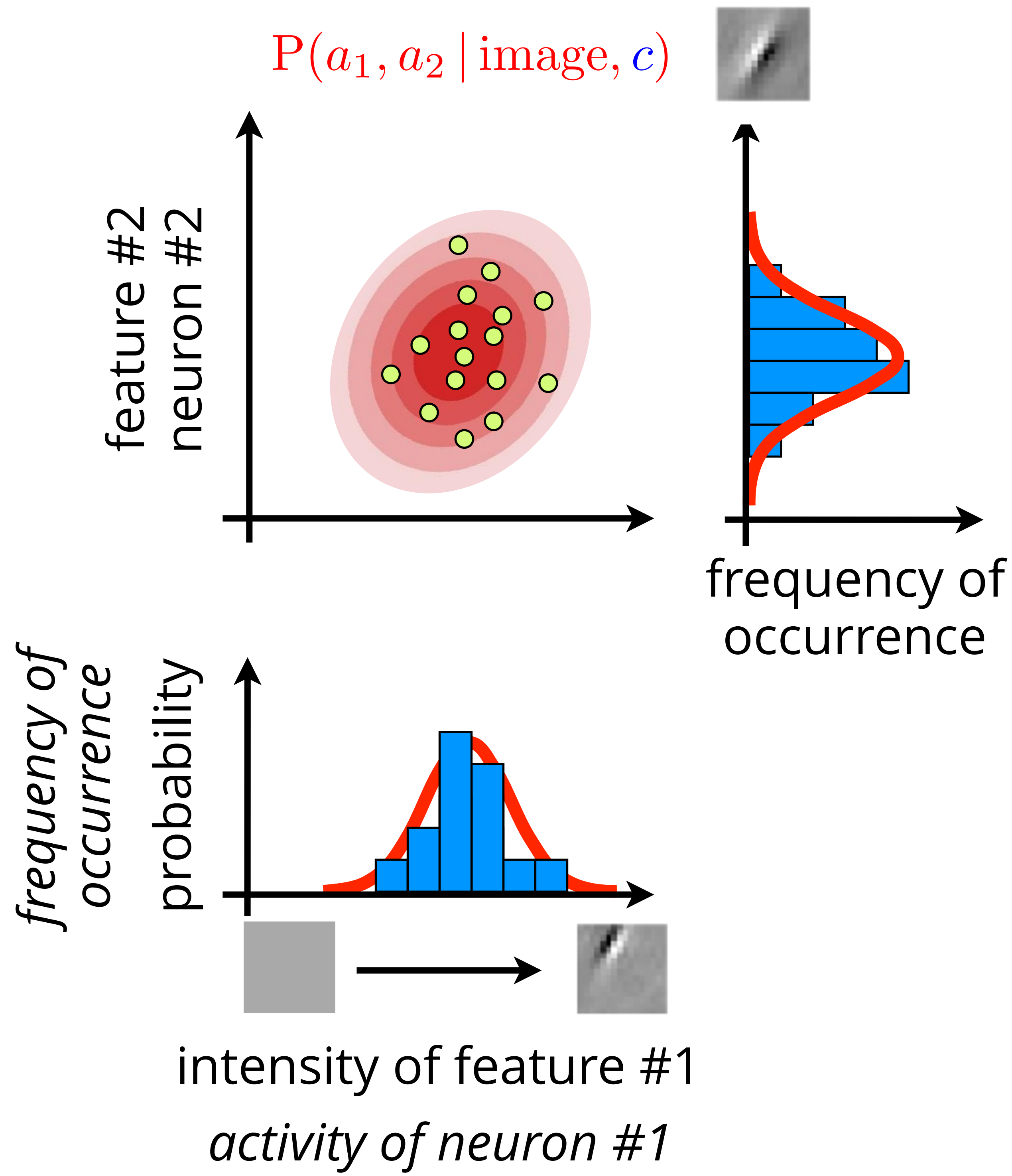
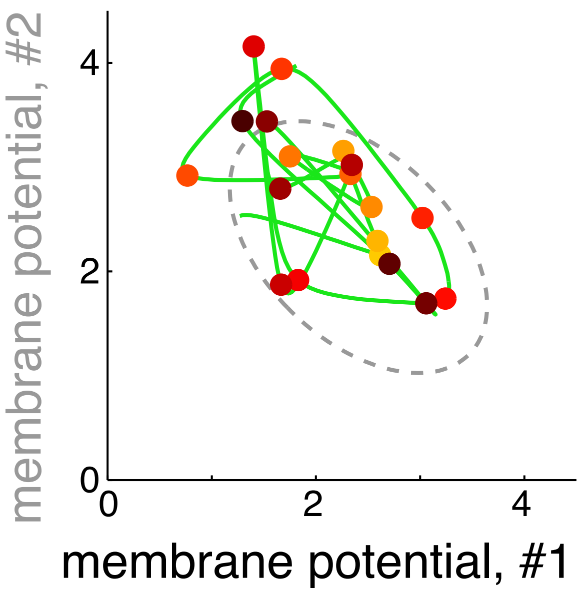
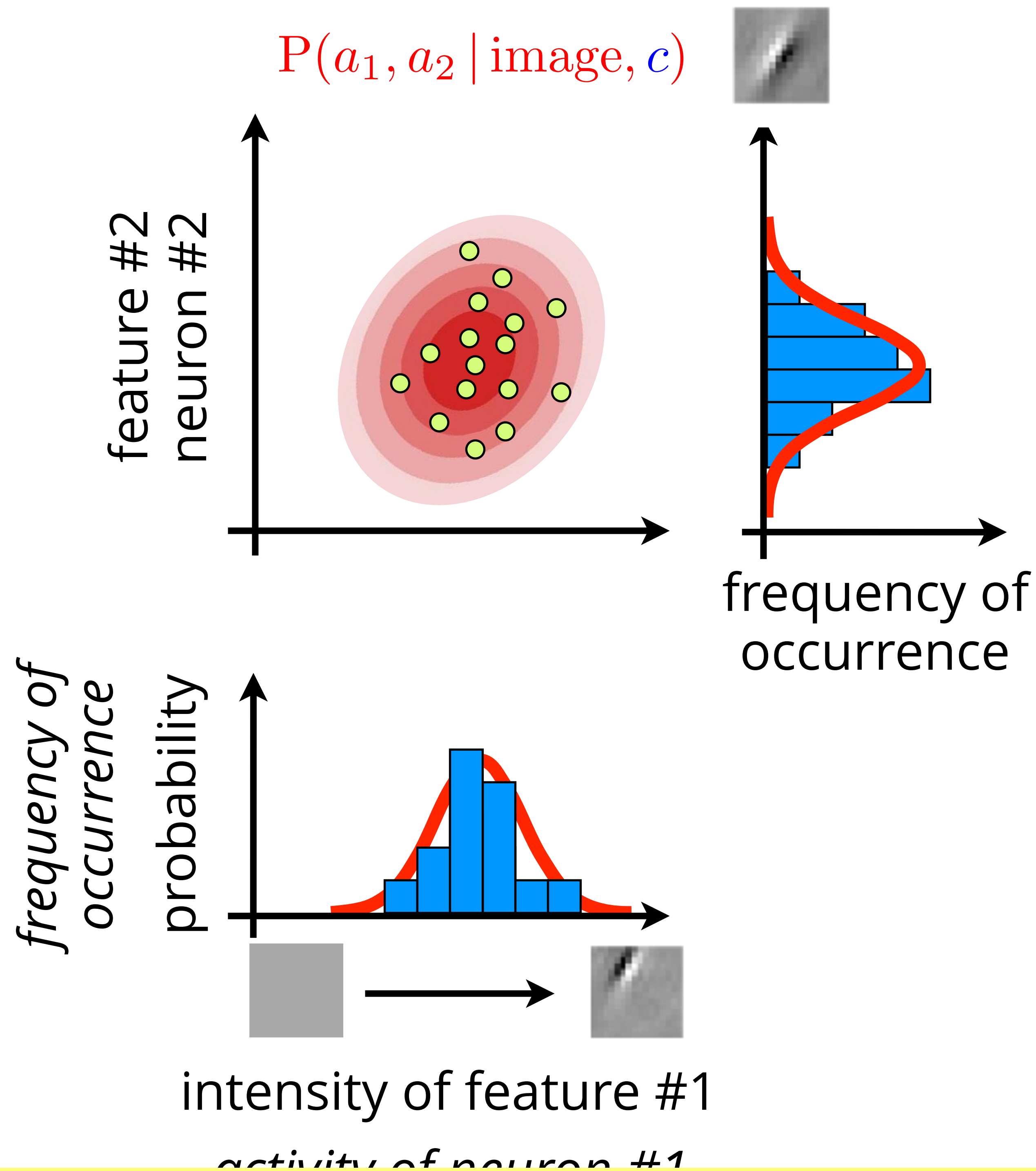




stochastic sampling

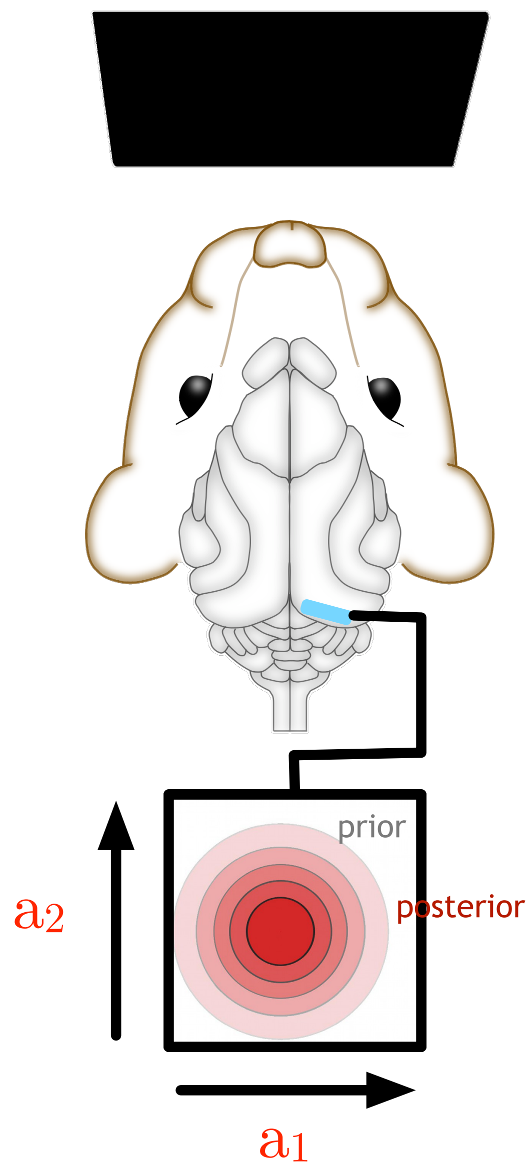




changes in inferences need to be reflected in the response statistics

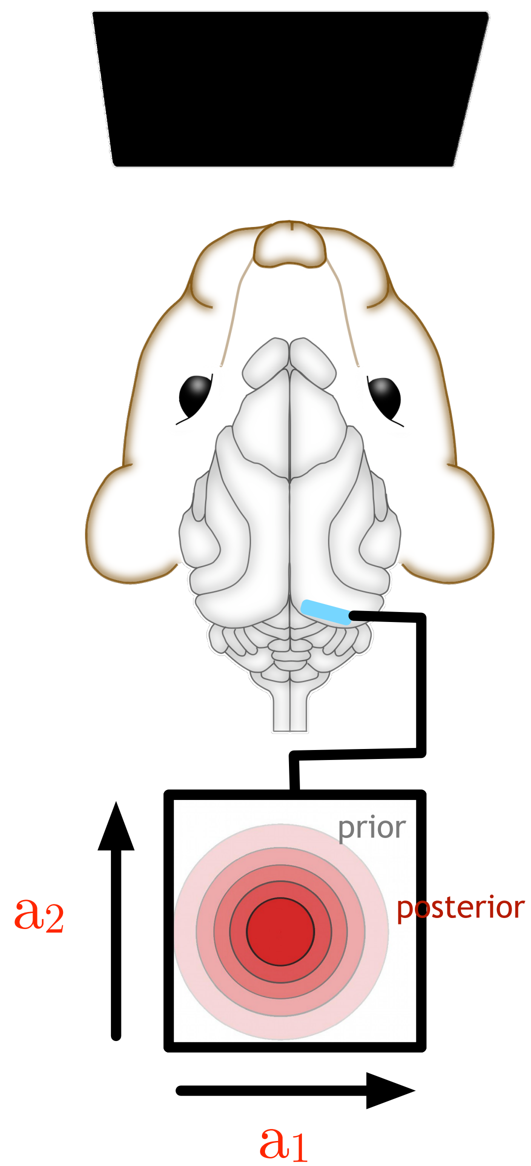
Full response statistics

prior expectations

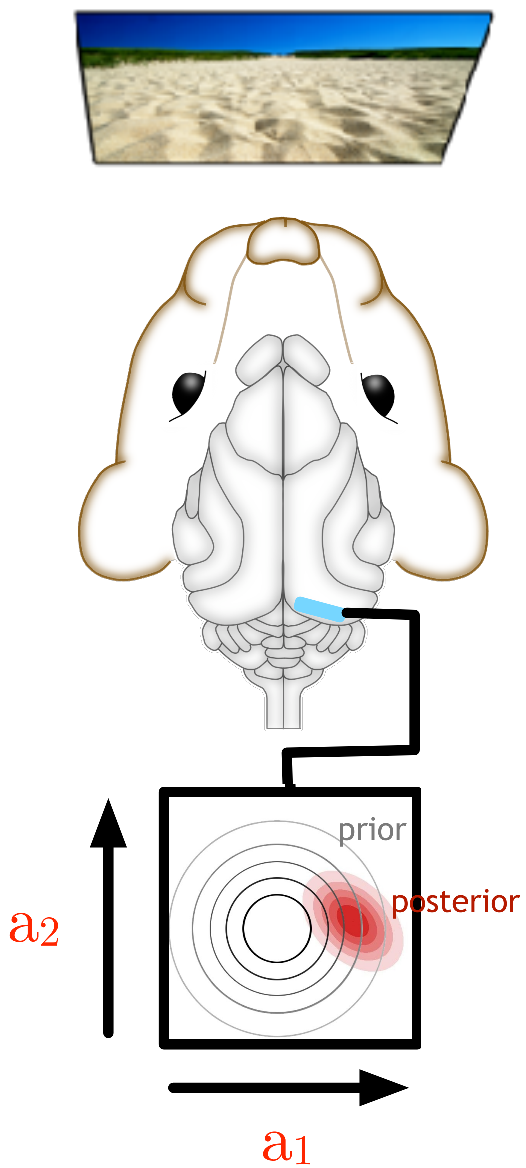


Full response statistics

prior expectations

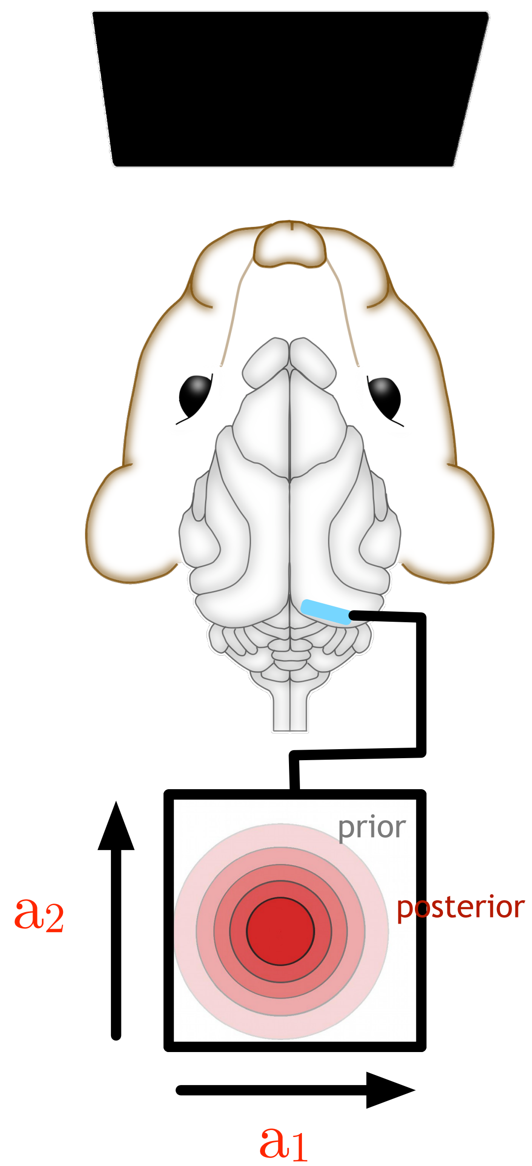


inferences

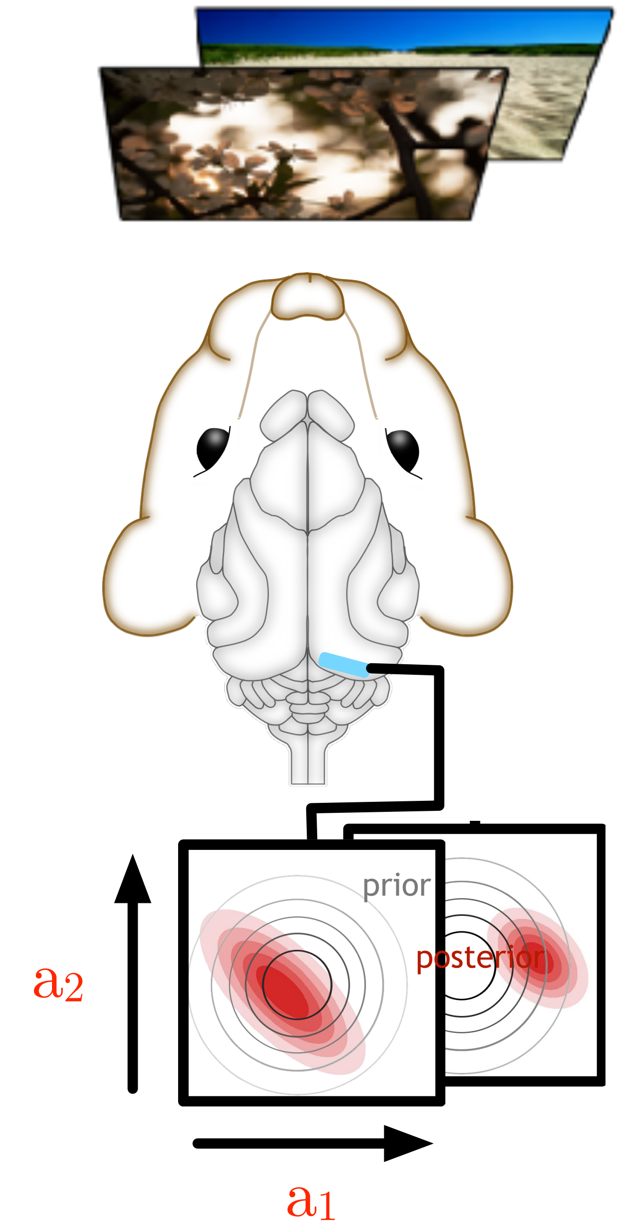


Full response statistics

prior expectations

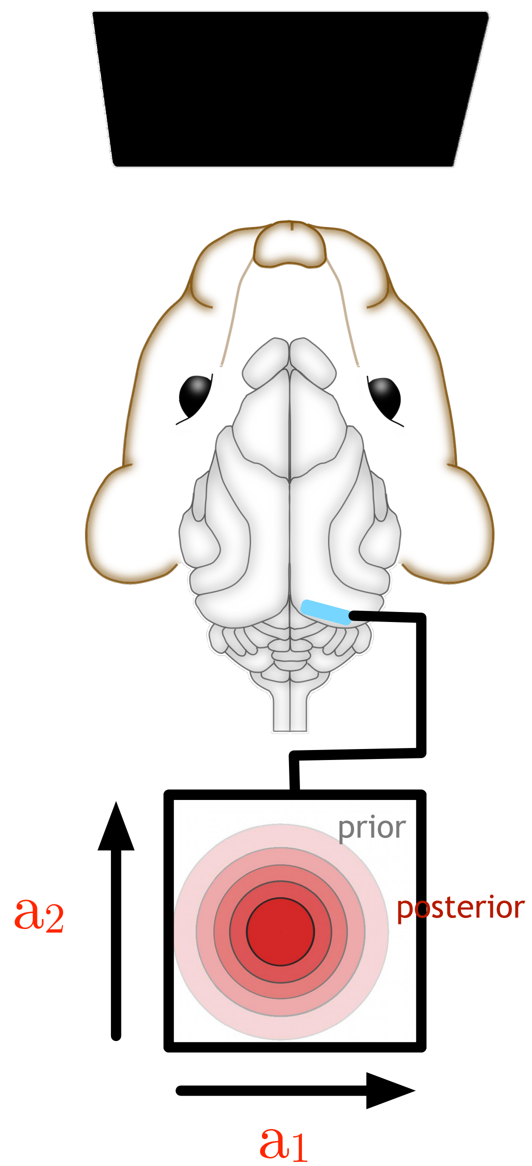


inferences

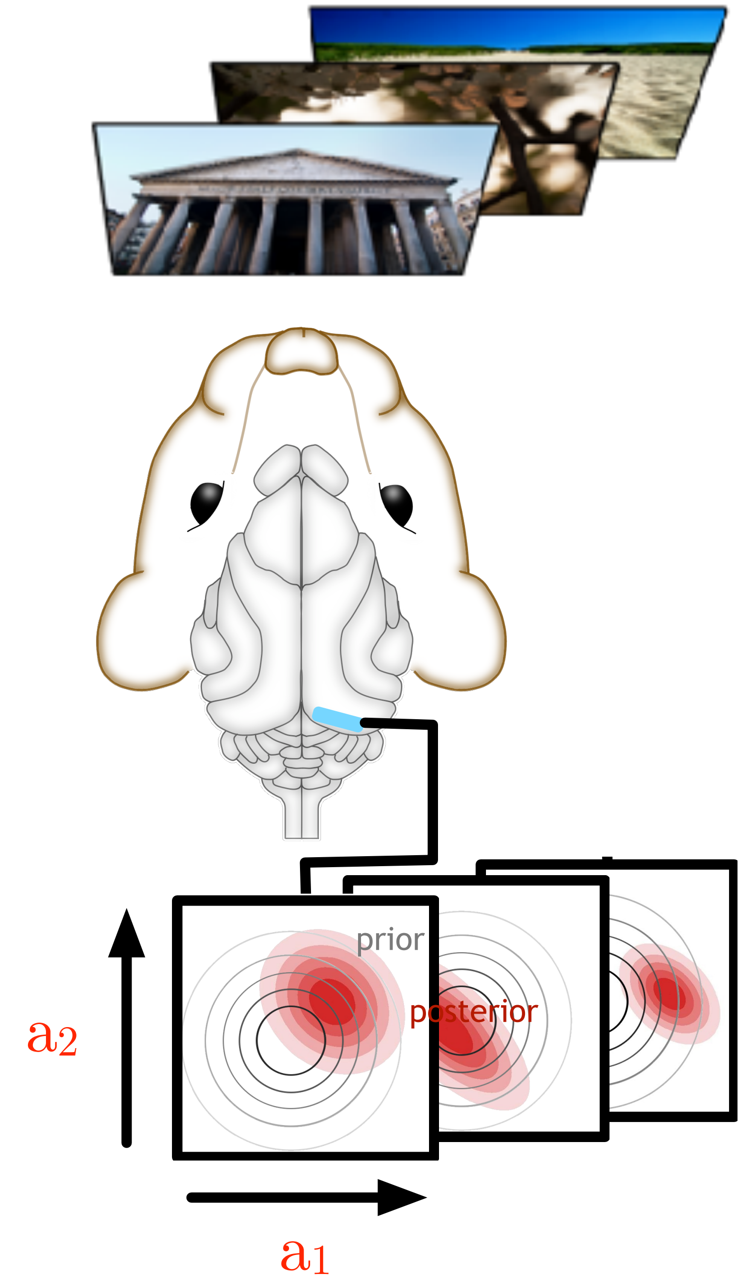


Full response statistics

prior expectations



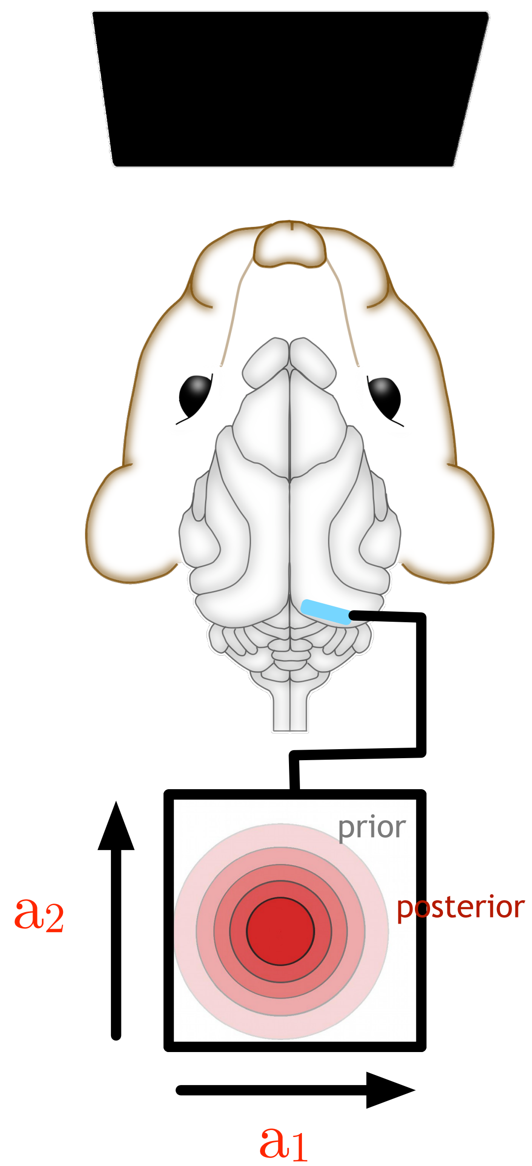
inferences

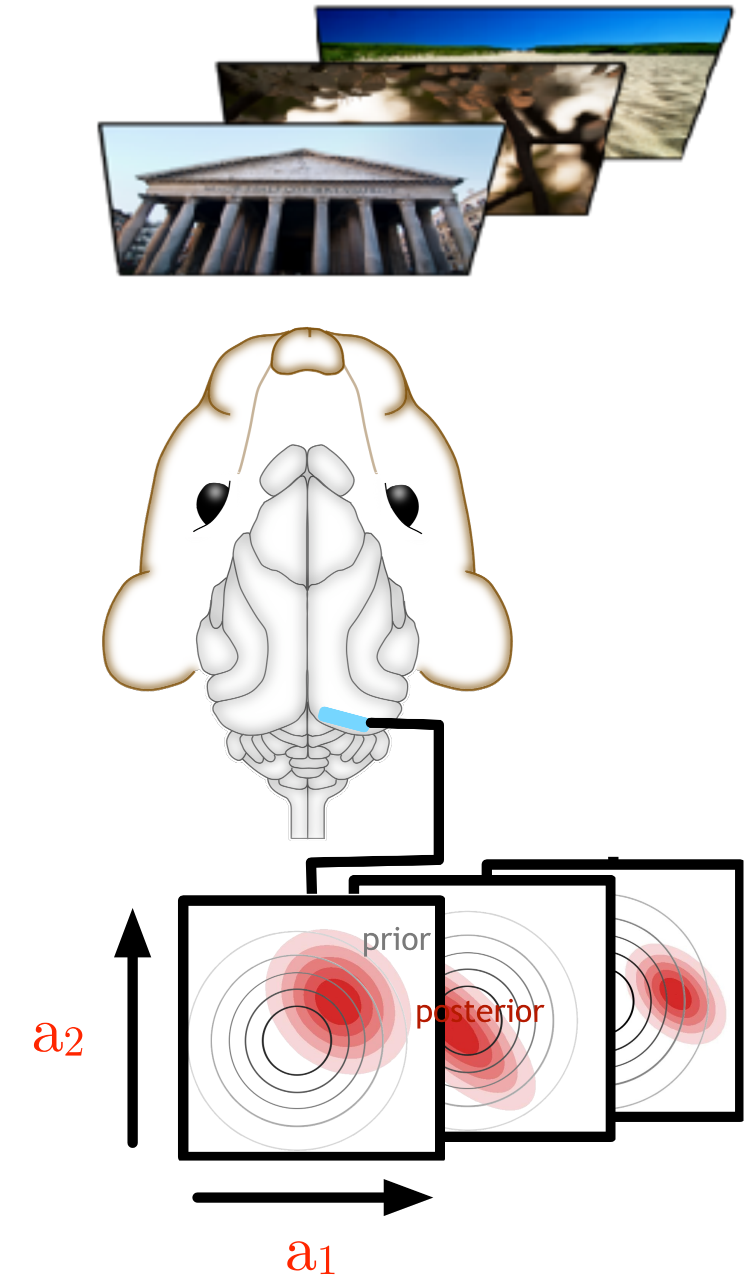


Full response statistics

prior expectations

inferences





spontaneous activity

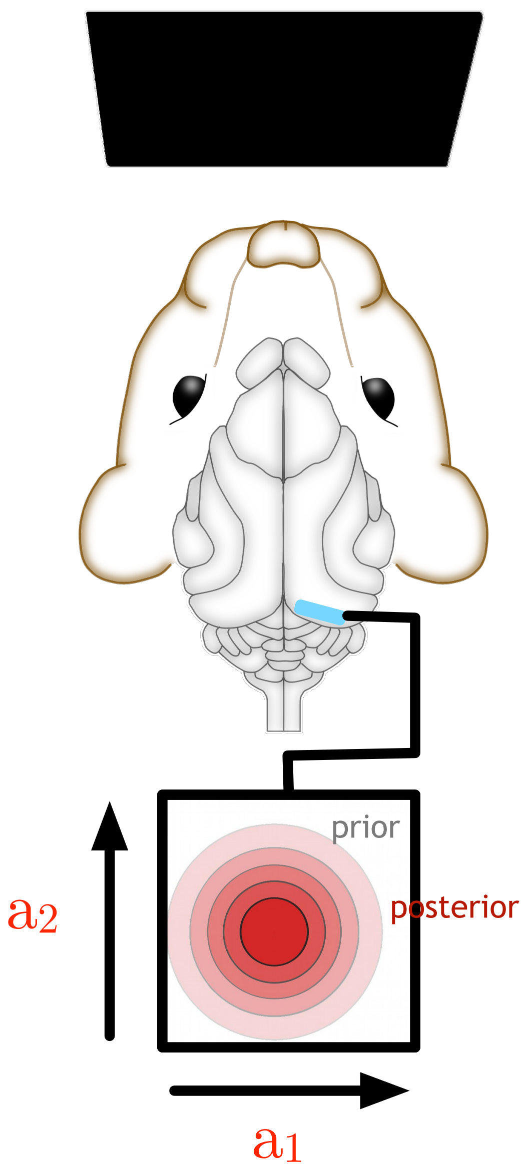
$P(\mathbf{a})$

evoked activity

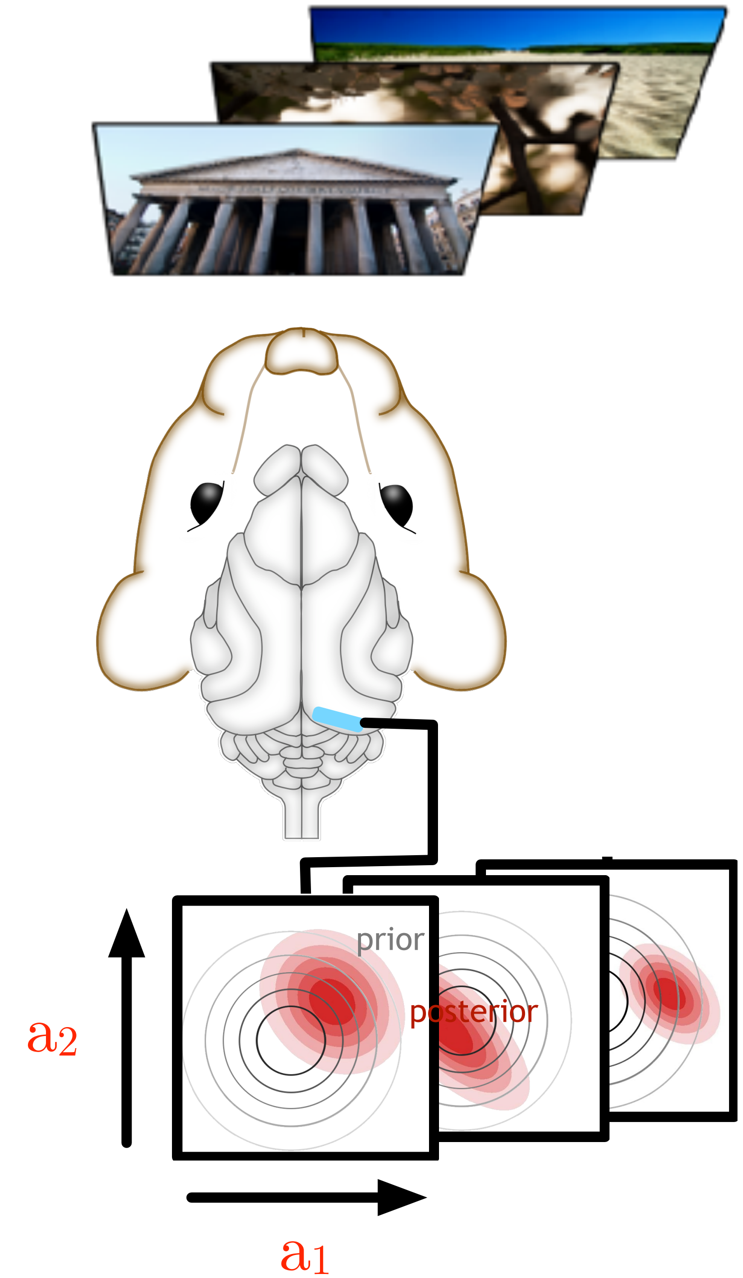
$P(\mathbf{a} \,|\, \mathbf{x})$

Full response statistics

prior expectations



inferences



$$P(\mathbf{a}) = \int dx\, P(\mathbf{a}\,|\,\mathbf{x})\, P(\mathbf{x})$$

spontaneous activity

$P(\mathbf{a})$

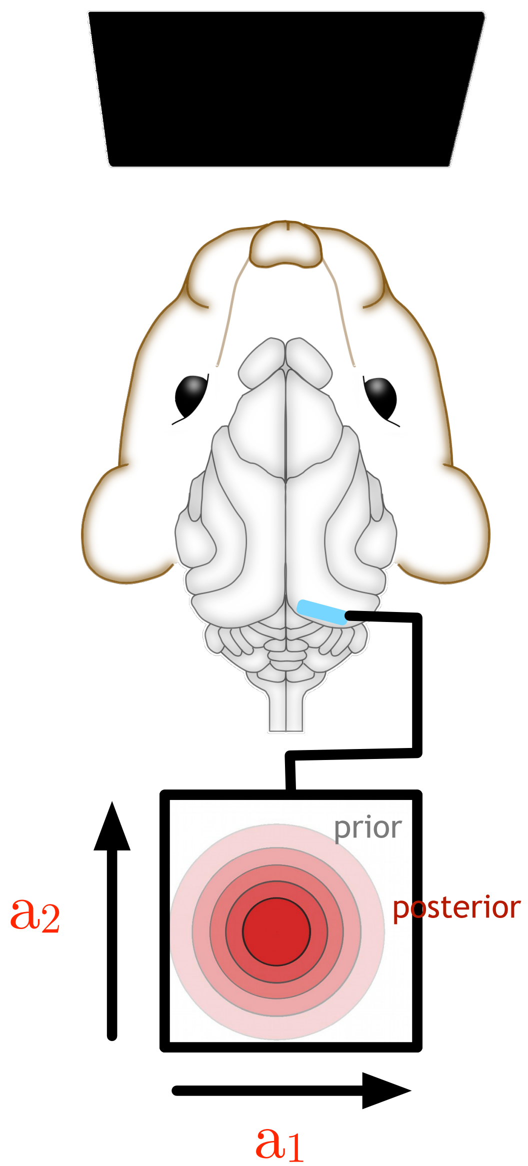
evoked activity

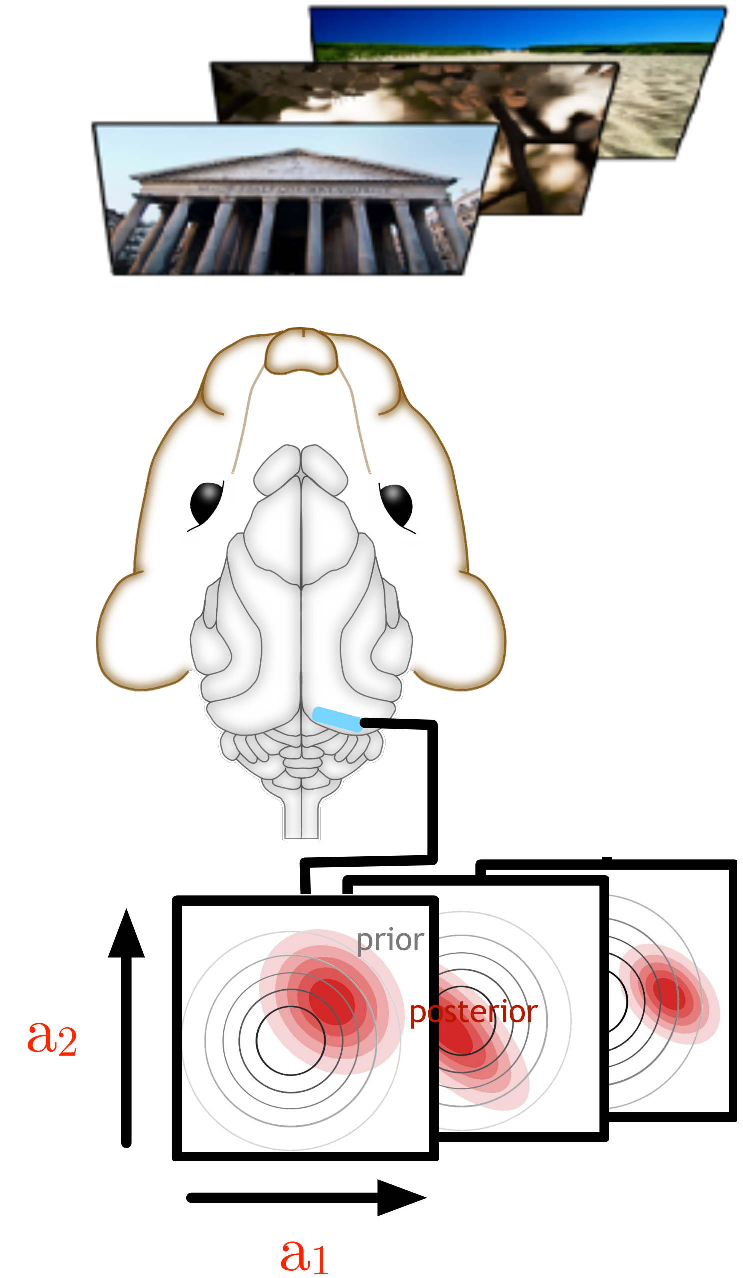
$P(\mathbf{a}\,|\,\mathbf{x})$

Full response statistics

prior expectations

inferences





expectations

$$P(\mathbf{a}) = \int dx\, P(\mathbf{a} \,|\, \mathbf{x})\, P(\mathbf{x})$$

spontaneous activity

$P(\mathbf{a})$

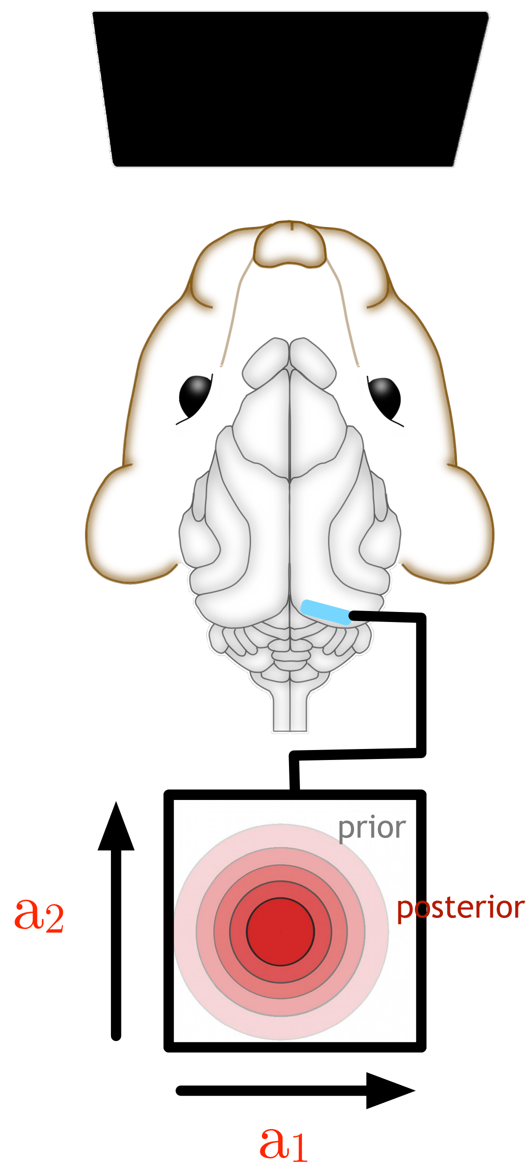
evoked activity

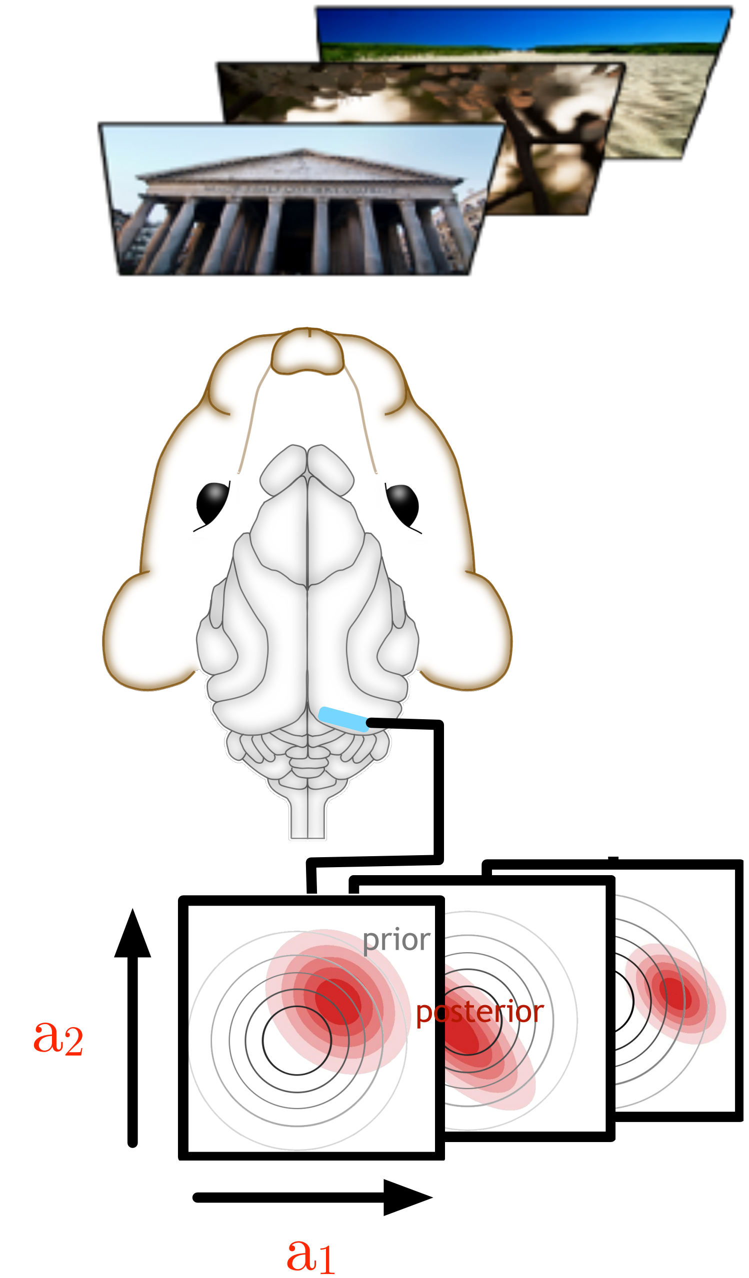
$P(\mathbf{a} \,|\, \mathbf{x})$

Full response statistics

prior expectations

inferences





expectations

inference

$$P(\mathbf{a}) = \int dx\, P(\mathbf{a}\,|\,\mathbf{x})\, P(\mathbf{x})$$

spontaneous activity

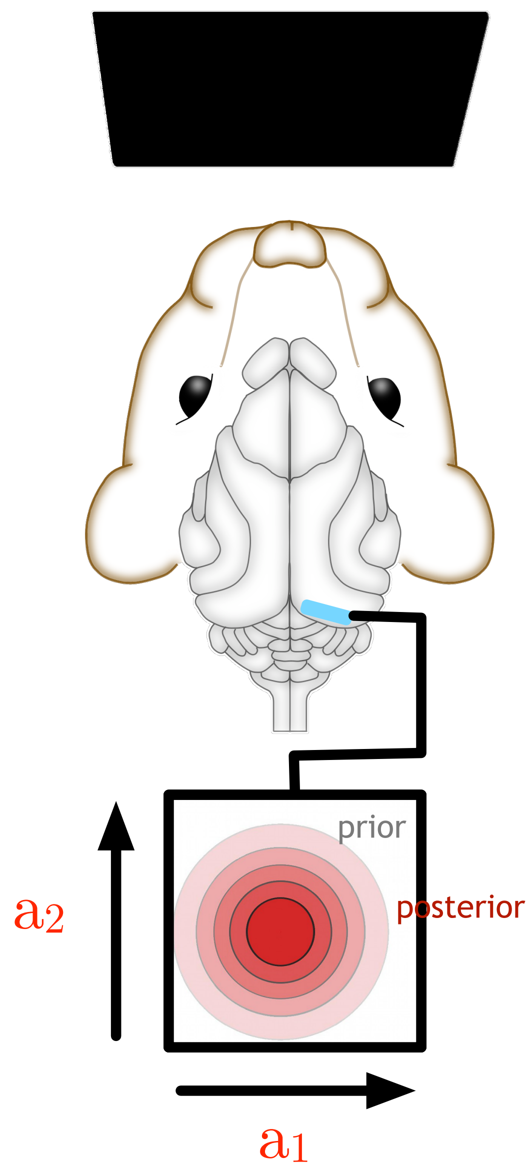
$P(\mathbf{a})$

evoked activity

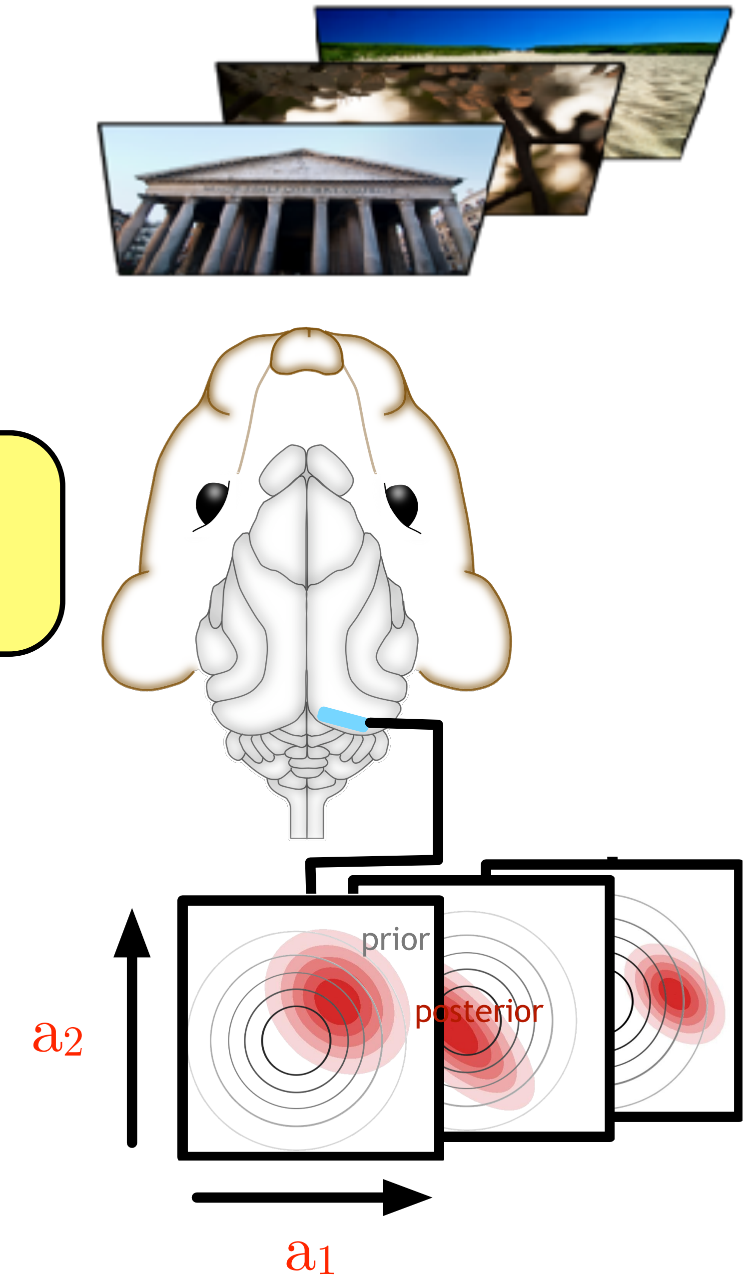
$P(\mathbf{a}\,|\,\mathbf{x})$

Full response statistics

prior expectations



inferences



expectations

inference

stimulus statistics

$$P(\mathbf{a}) = \int dx\, P(\mathbf{a}\,|\,\mathbf{x})\, P(\mathbf{x})$$

spontaneous activity

$P(\mathbf{a})$

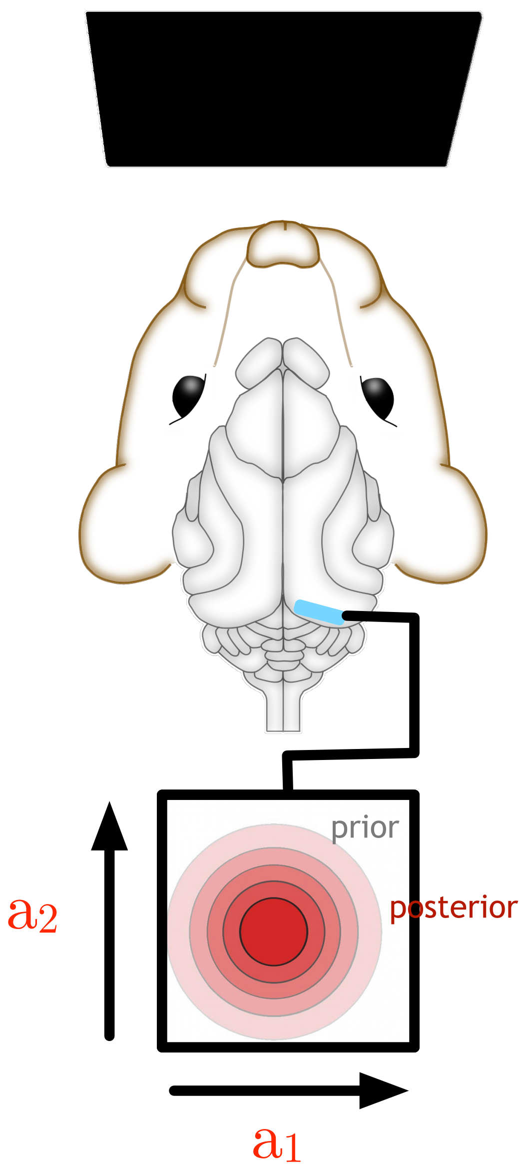
evoked activity

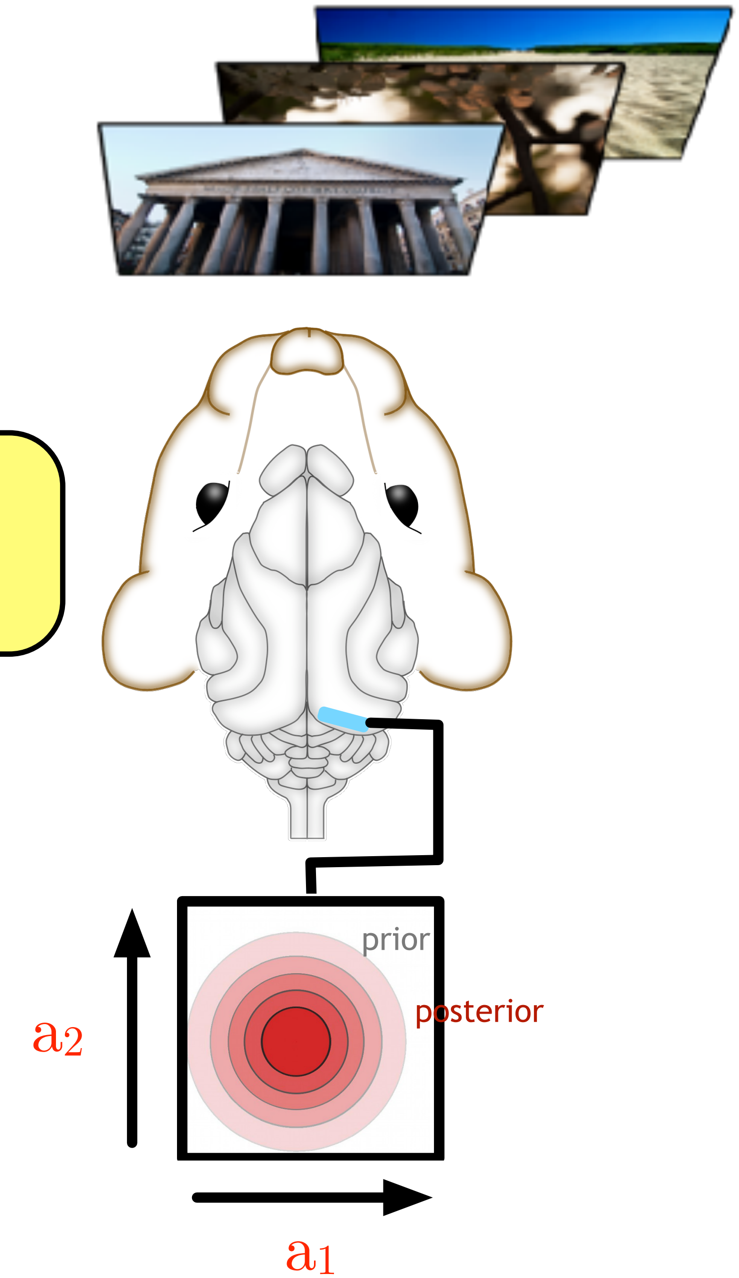
$P(\mathbf{a}\,|\,\mathbf{x})$

Full response statistics

prior expectations

average inferences





expectations — *inference* — stimulus statistics

$$P(\mathbf{a}) = \int dx\, P(\mathbf{a}\,|\,\mathbf{x})\, P(\mathbf{x})$$

spontaneous activity

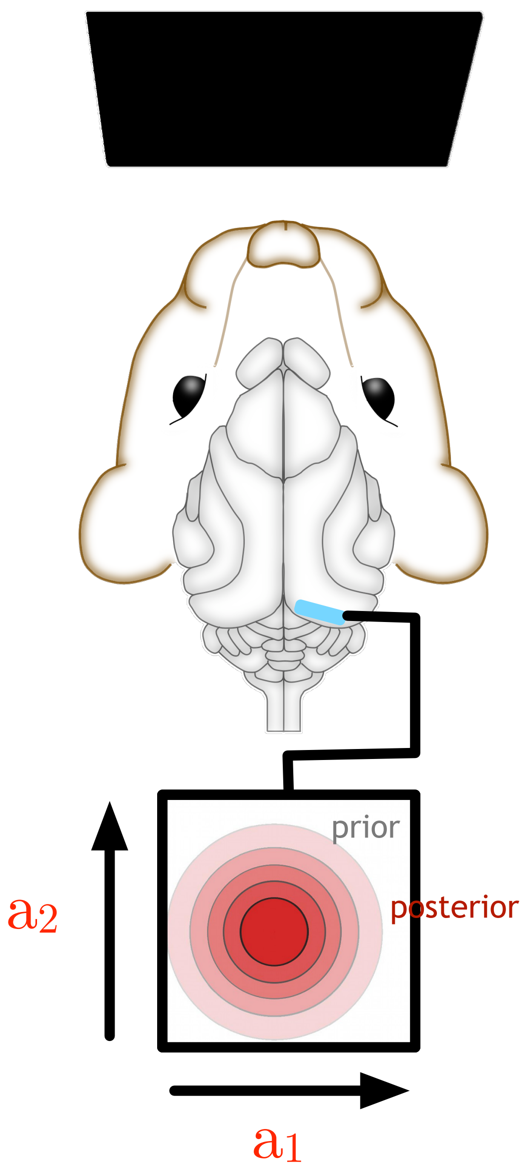
$P(\mathbf{a})$

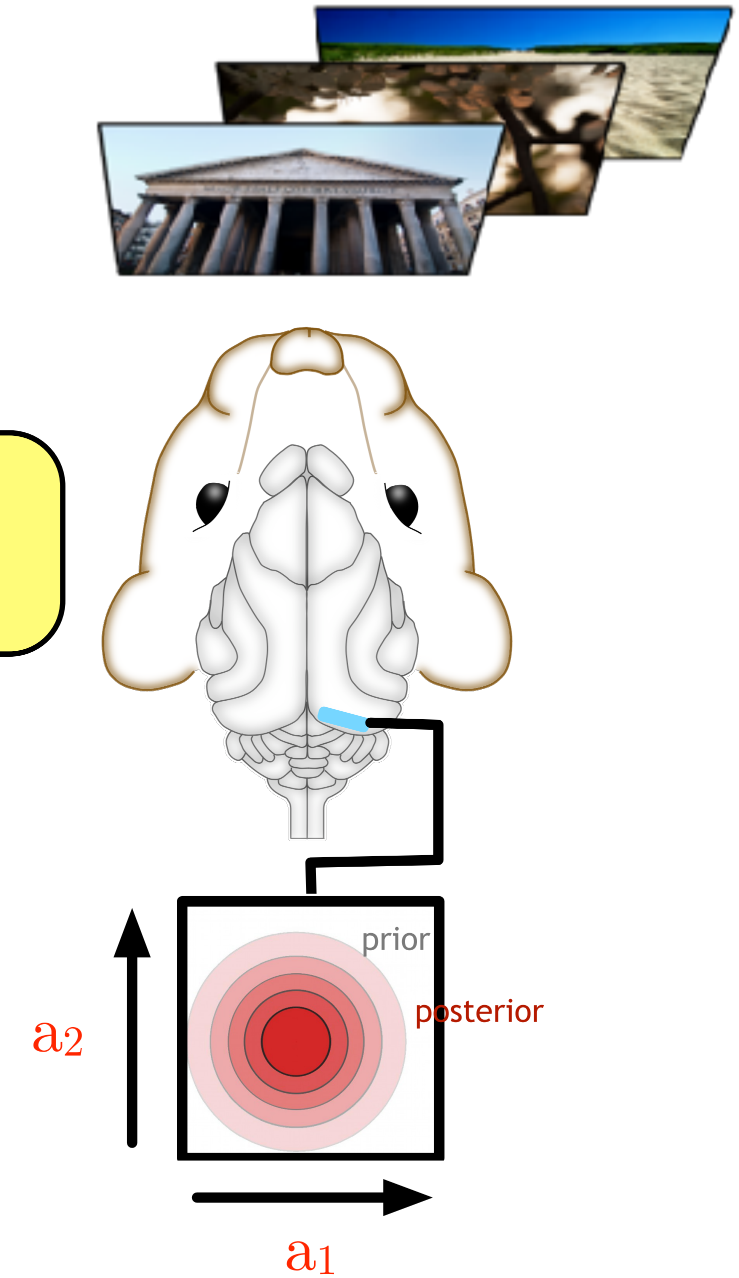
evoked activity

$P(\mathbf{a}\,|\,\mathbf{x})$

Full response statistics

prior expectations | *average inferences*





expectations | *inference* | stimulus statistics

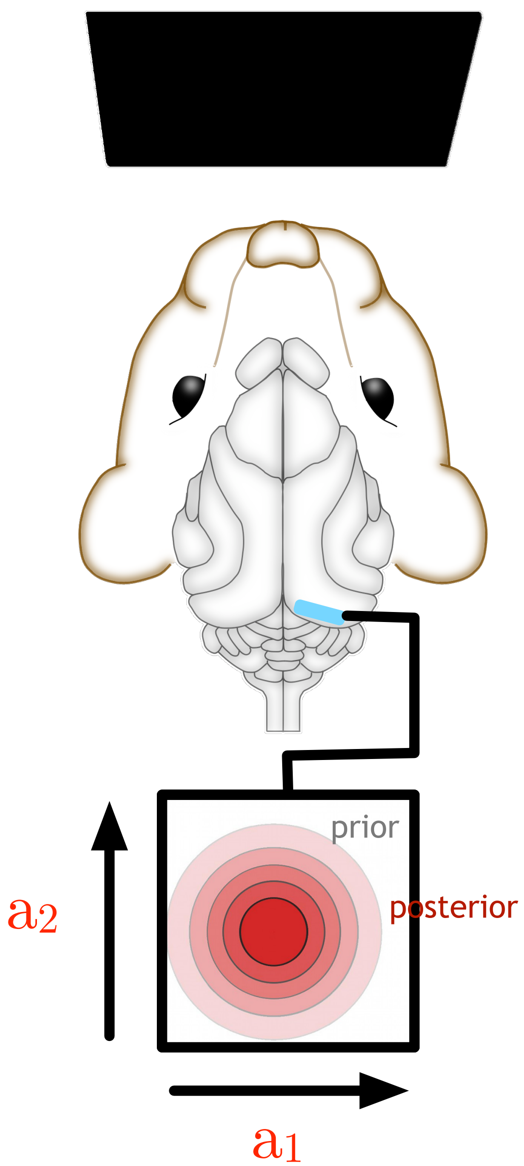
$$P(\mathbf{a}) = \int dx\, P(\mathbf{a}\,|\,\mathbf{x})\, P(\mathbf{x})$$

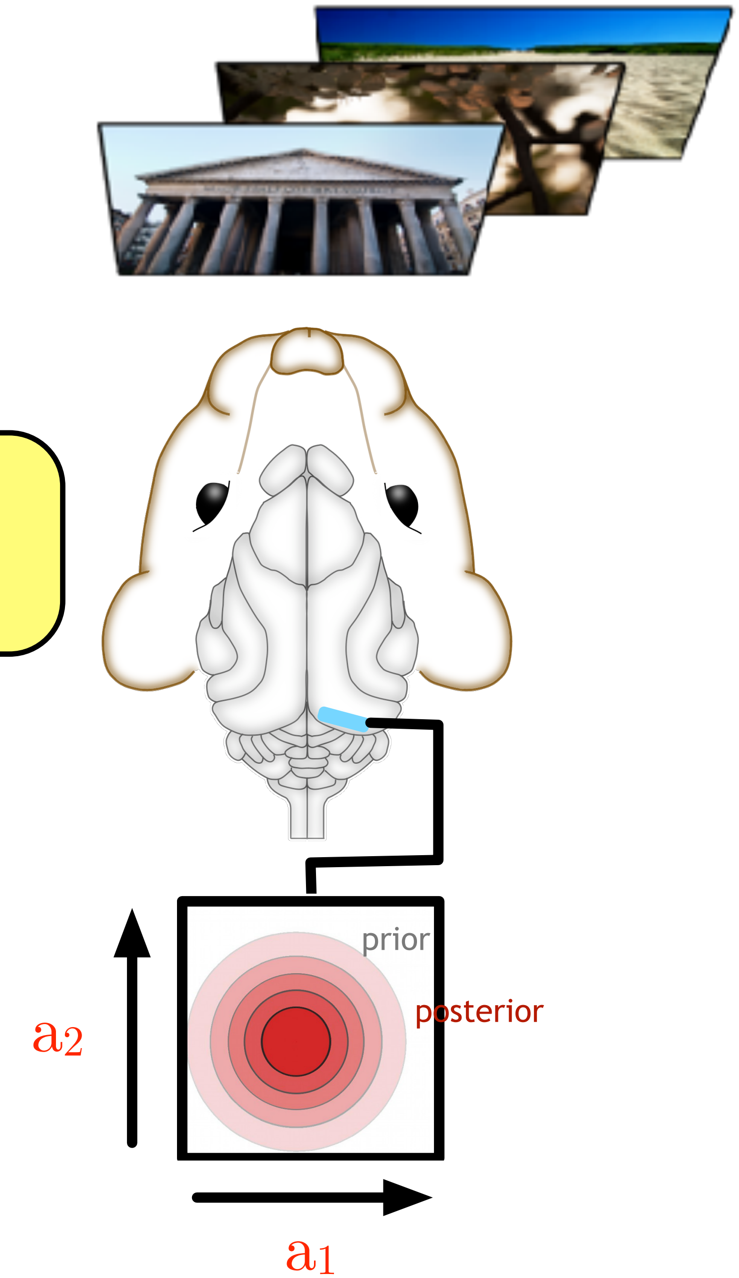
spontaneous activity $\stackrel{?}{=}$ average evoked activity

$P(\mathbf{a})$ | $P(\mathbf{a}\,|\,\mathbf{x})$

Full response statistics

prior expectations — *average inferences*





expectations — *inference* — stimulus statistics

$$P(\mathbf{a}) = \boxed{\int dx\, P(\mathbf{a} \,|\, \mathbf{x})\, P(\mathbf{x})}$$

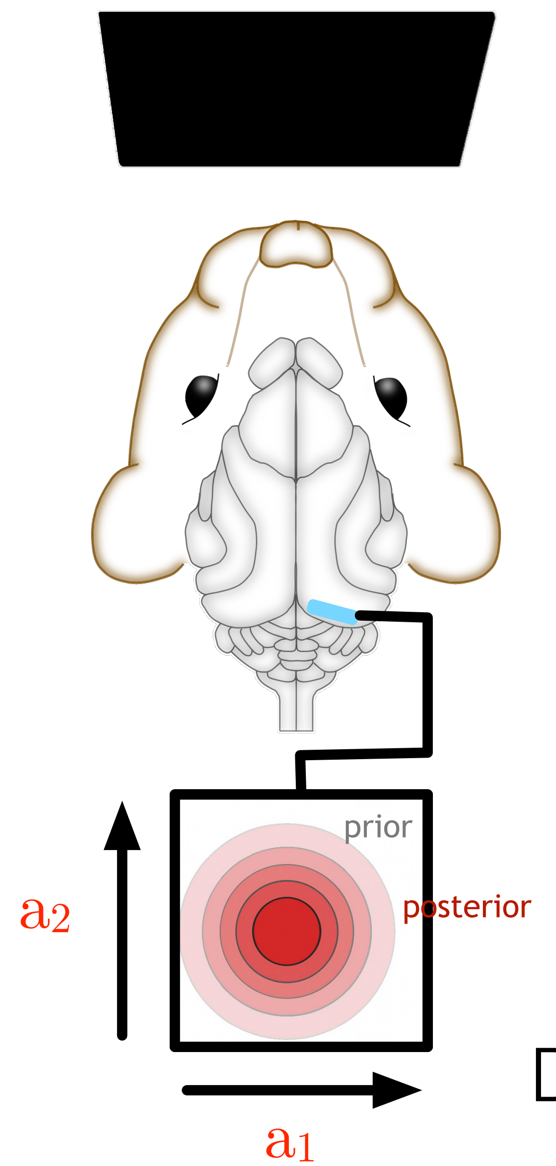
$$\langle P(\mathbf{a} \,|\, \mathbf{x}) \rangle_{P(\mathbf{x})}$$

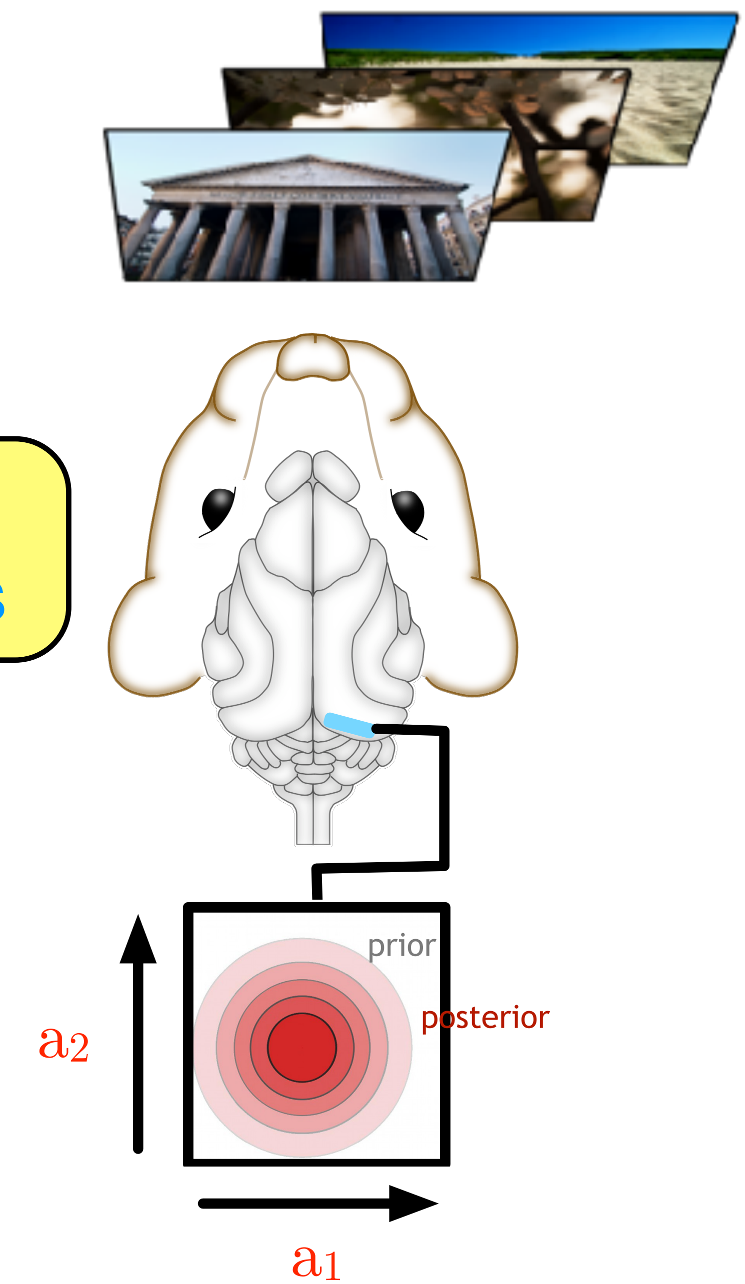
spontaneous activity $\stackrel{?}{=}$ average evoked activity

$P(\mathbf{a})$ — $P(\mathbf{a} \,|\, \mathbf{x})$

Full response statistics

prior expectations *average inferences*





expectations *inference* stimulus statistics

$$P(\mathbf{a}) = \int dx\, P(\mathbf{a}\,|\,\mathbf{x})\, P(\mathbf{x})$$

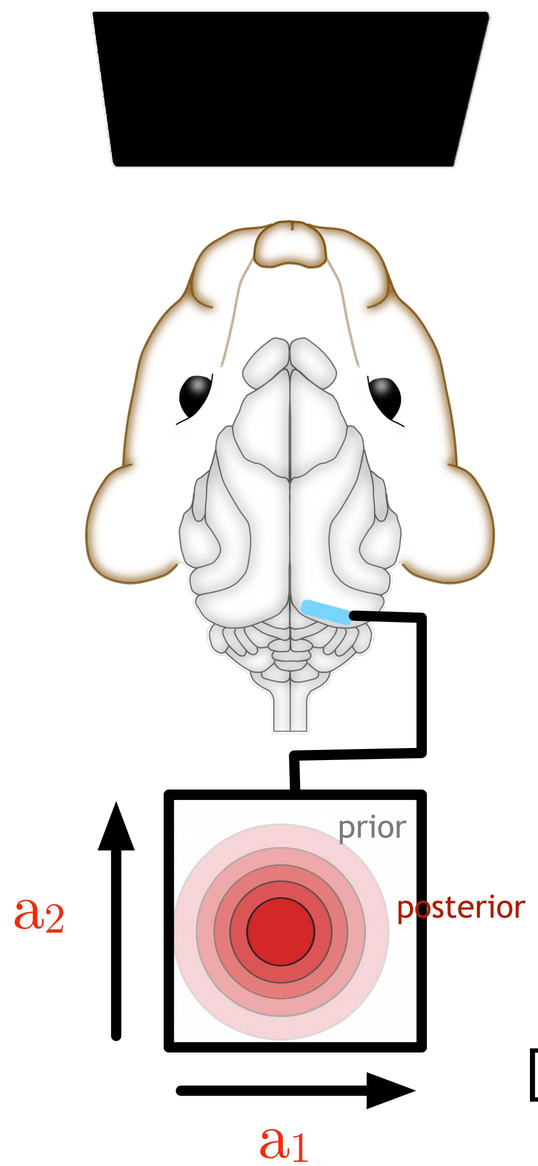
Dissimilarity: $\mathrm{KL}[P(\mathbf{a})\;||\langle P(\mathbf{a}\,|\,\mathbf{x})\rangle_{P(\mathbf{x})}]$

spontaneous activity $\overset{?}{=}$ average evoked activity

$P(\mathbf{a})$ $P(\mathbf{a}\,|\,\mathbf{x})$

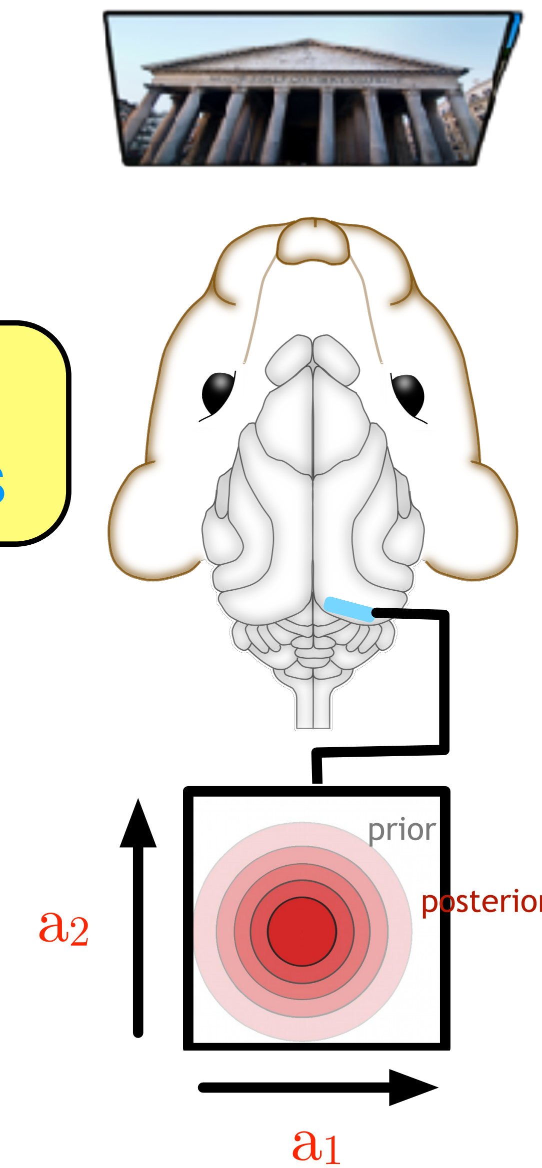
Full response statistics

prior expectations *average inferences*



expectations *inference* stimulus statistics

$$P(\mathbf{a}) = \int dx\, P(\mathbf{a}\,|\,\mathbf{x})\, P(\mathbf{x})$$



Dissimilarity: $\mathrm{KL}[P(\mathbf{a})\ ||\langle P(\mathbf{a}\,|\,\mathbf{x})\rangle_{P(\mathbf{x})}]$

spontaneous activity $\stackrel{?}{=}$ average evoked activity

$P(\mathbf{a})$ $P(\mathbf{a}\,|\,\mathbf{x})$

Full response statistics

$$P(\mathbf{a}) = \int dx\, P(\mathbf{a}\,|\,\mathbf{x})\, P(\mathbf{x})$$

spontaneous activity $\stackrel{?}{=}$ average evoked activity

$P(\mathbf{a})$ $P(\mathbf{a}\,|\,\mathbf{x})$

Full response statistics

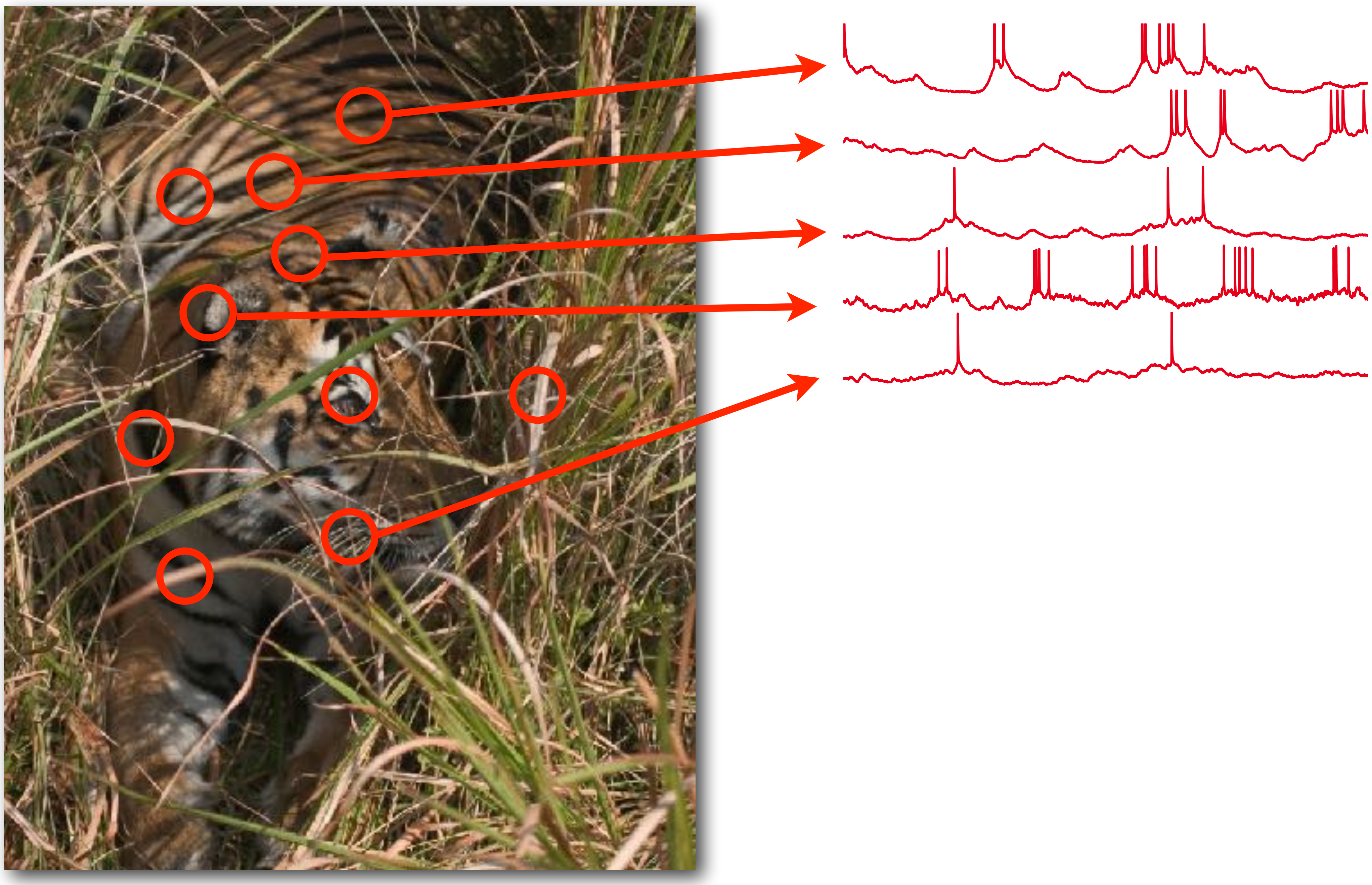
- ★ the model has been adapted to the appropriate model of the world
- ★ the stimulus statistics tested is appropriate

$$P(\mathbf{a}) = \int dx \, P(\mathbf{a} \,|\, \mathbf{x}) \, P(\mathbf{x})$$

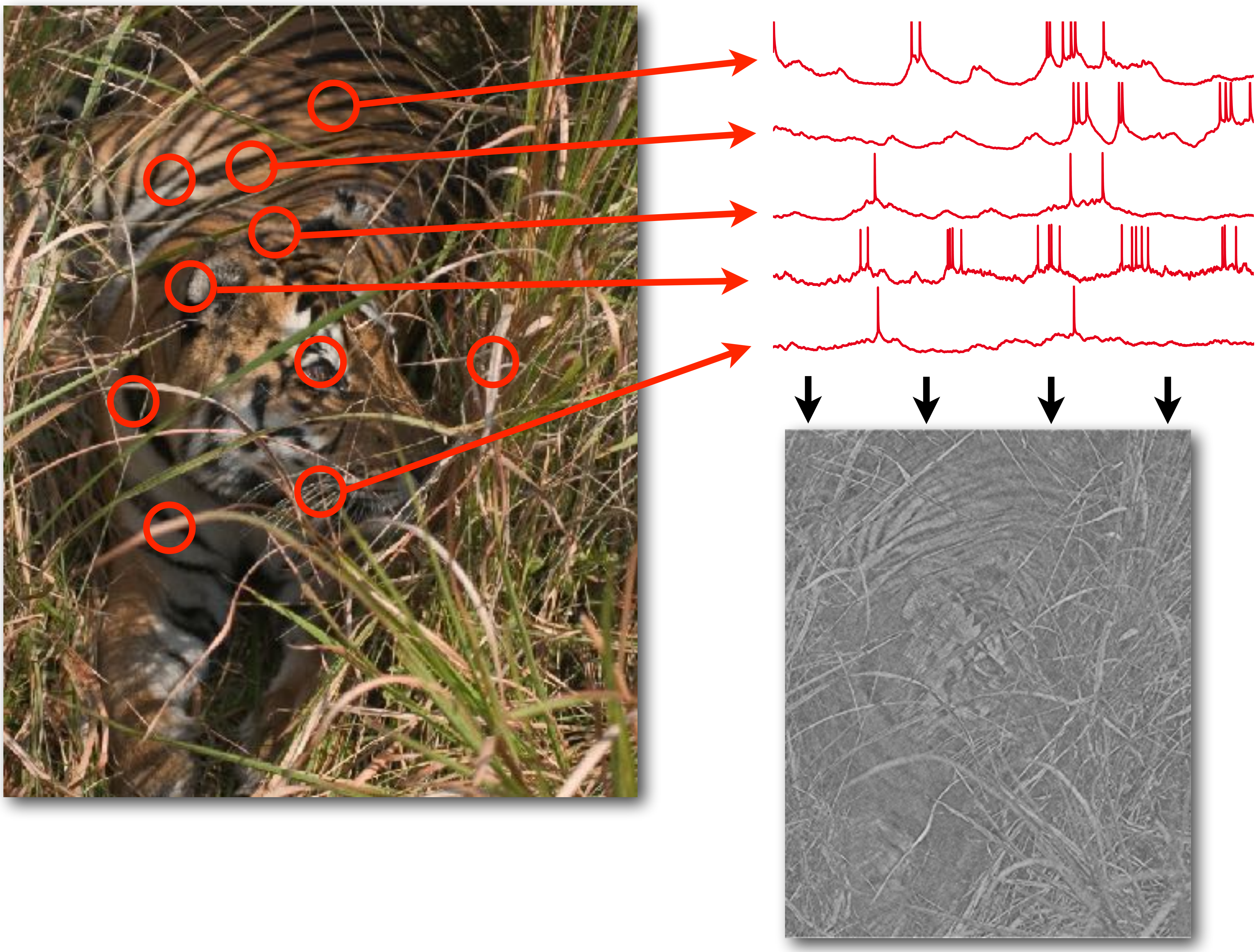
spontaneous activity $\stackrel{?}{=}$ average evoked activity

$P(\mathbf{a})$ $P(\mathbf{a} \,|\, \mathbf{x})$

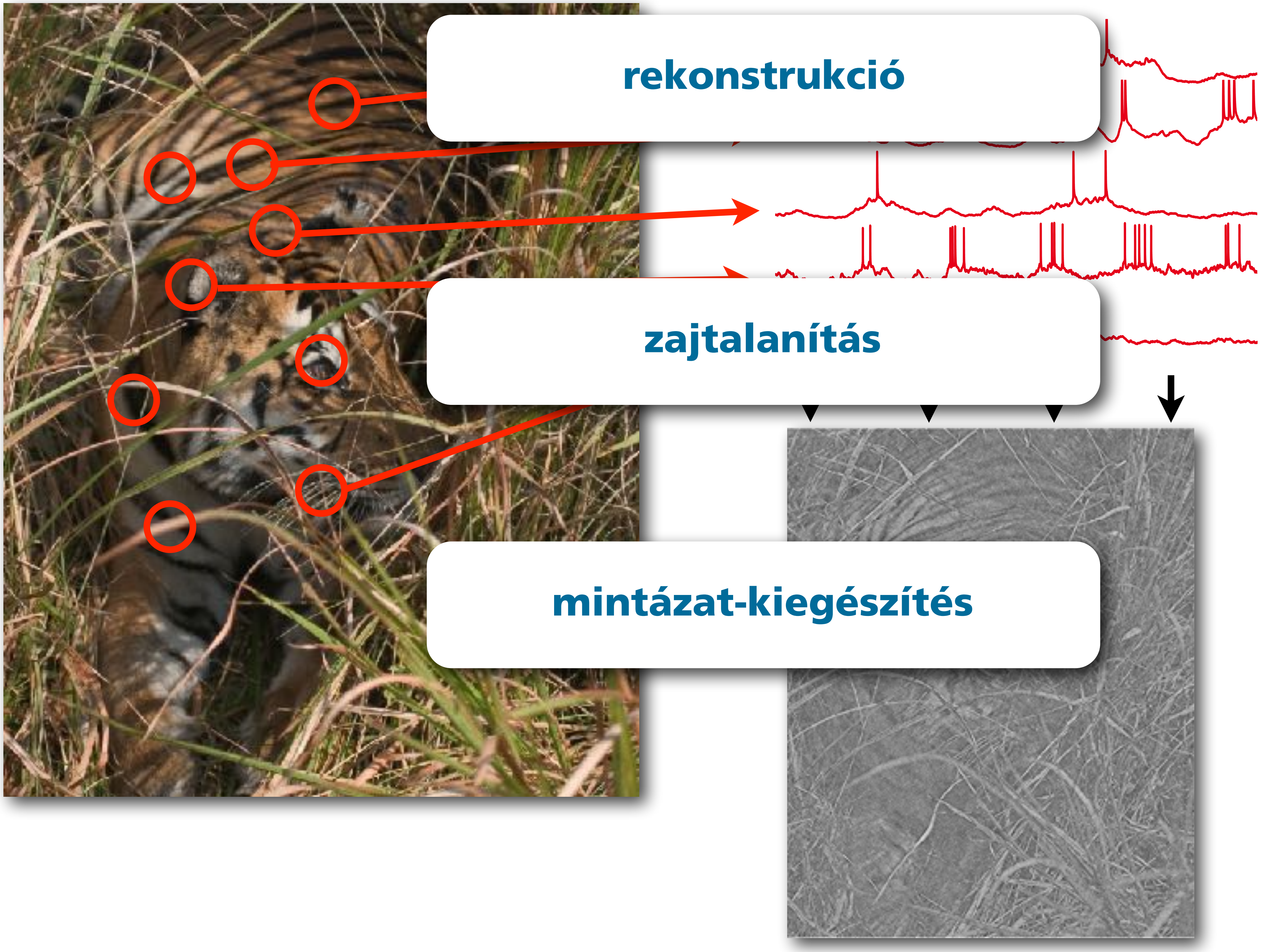
Idegsejtek és információelmélet

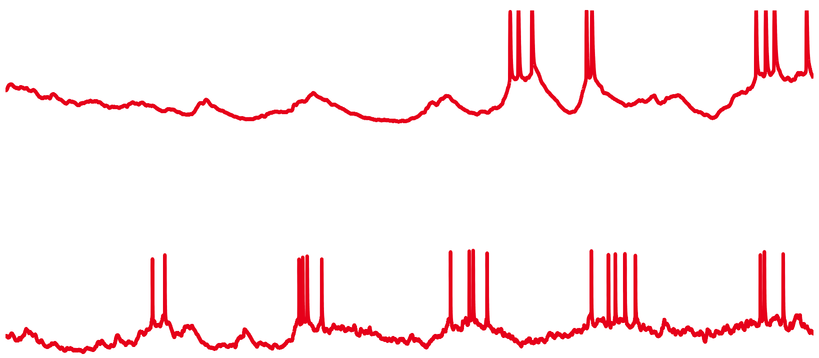


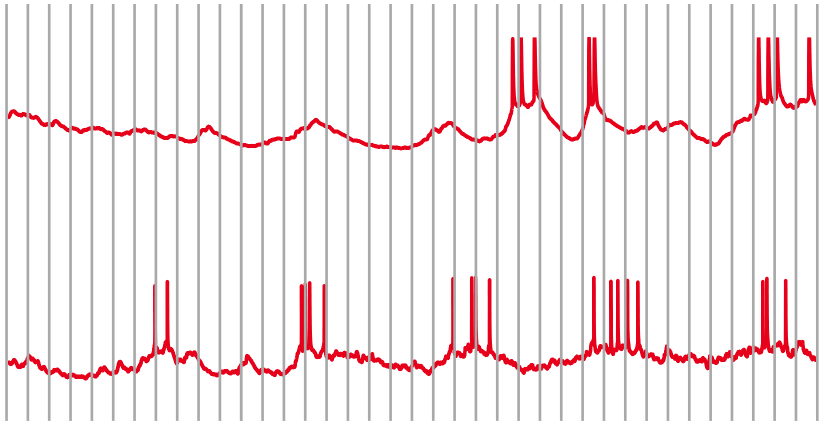
Idegsejtek és információelmélet

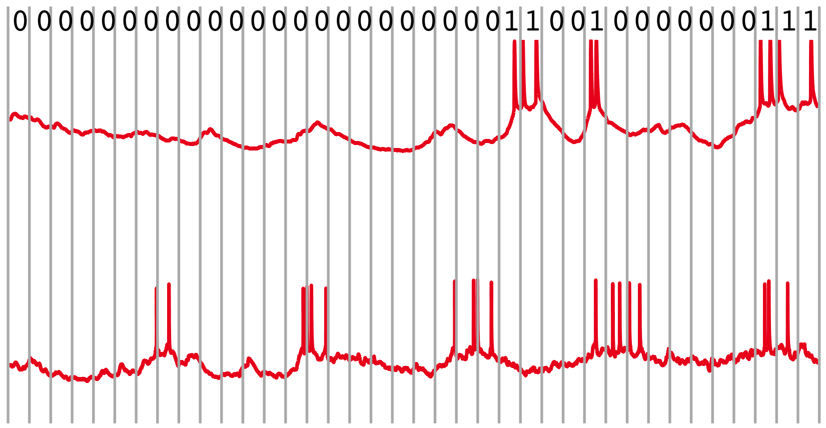


Idegsejtek és információelmélet





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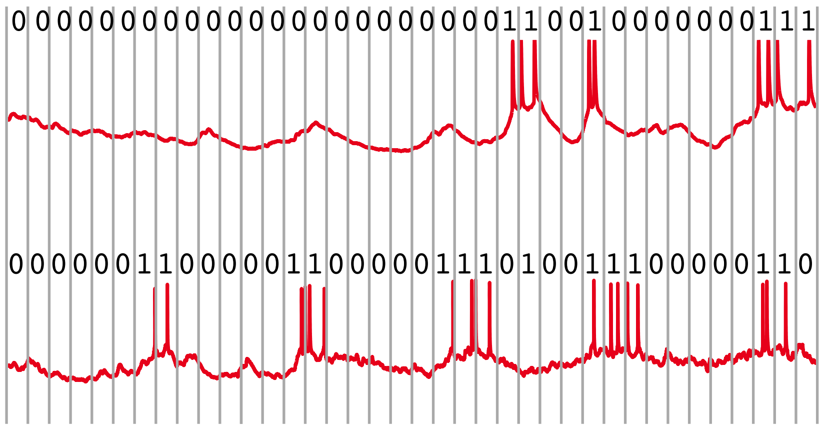
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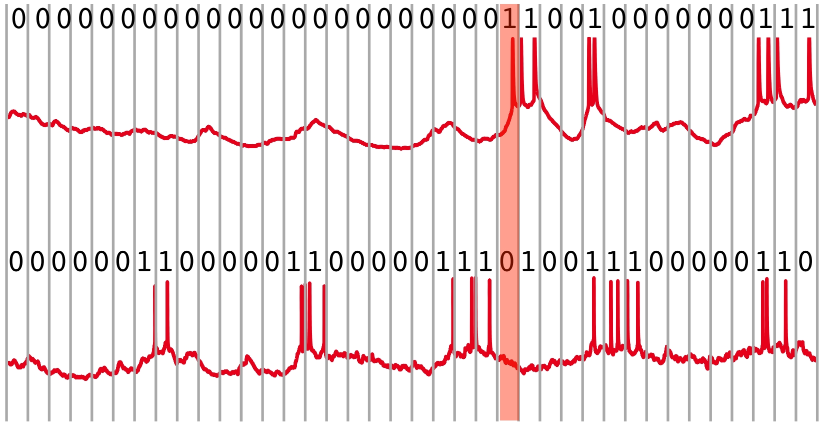
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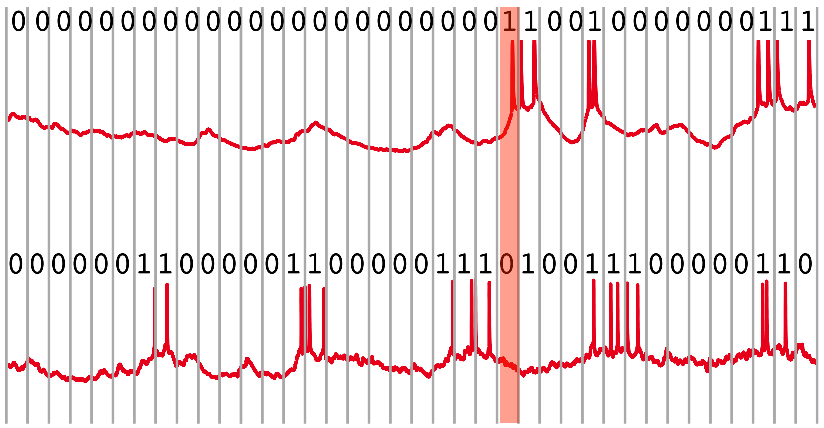

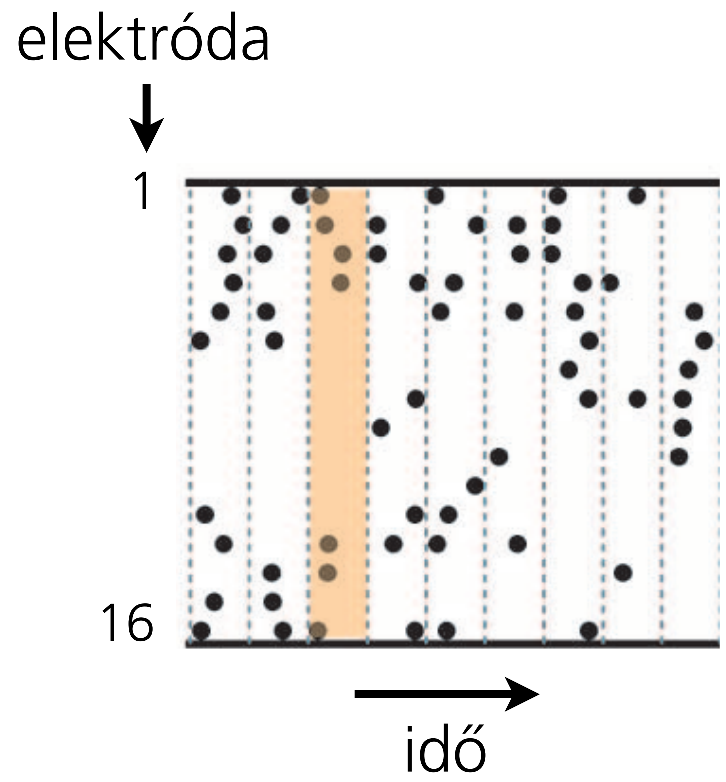
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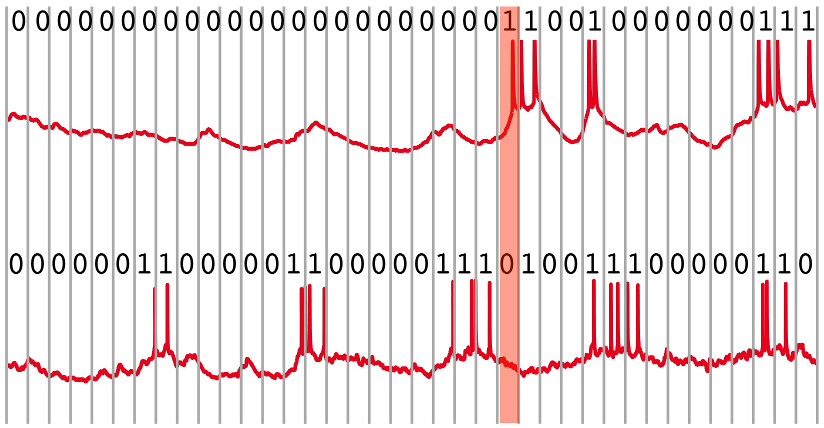
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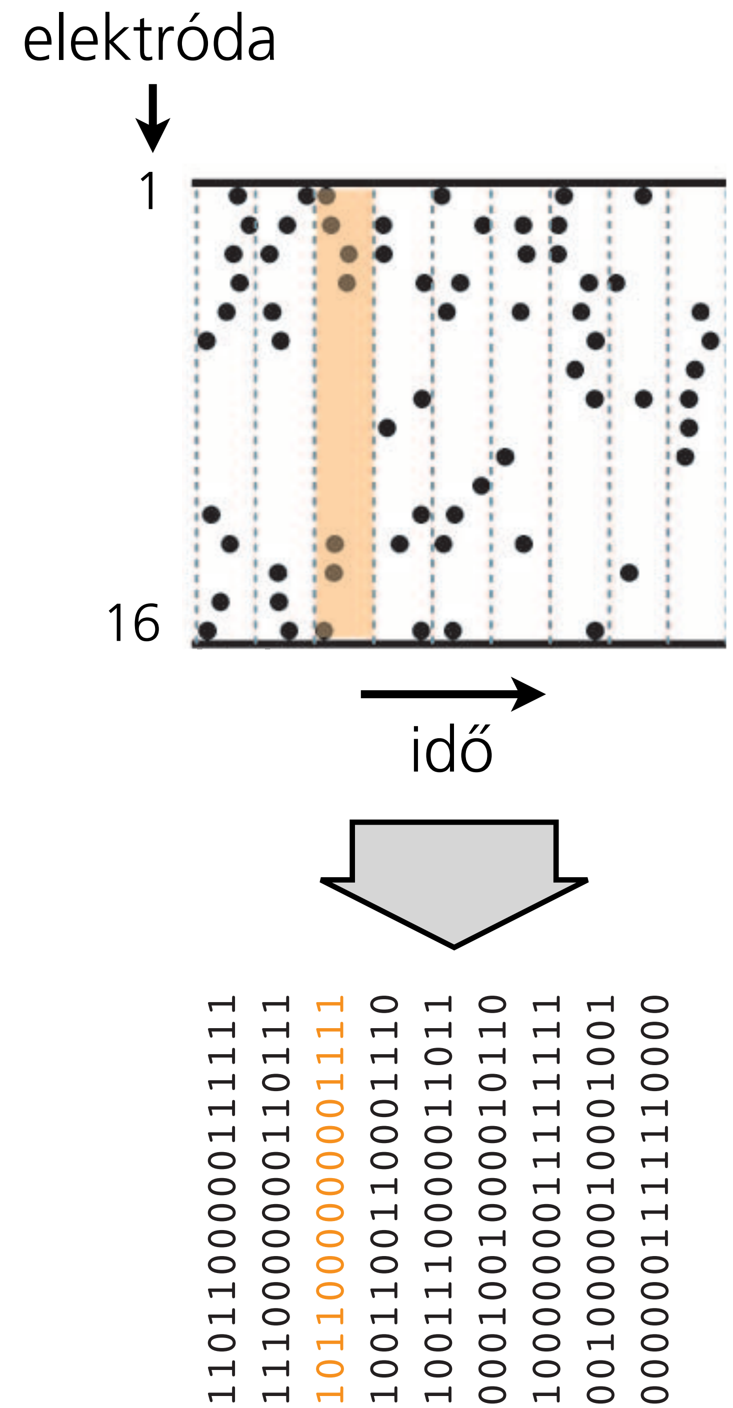
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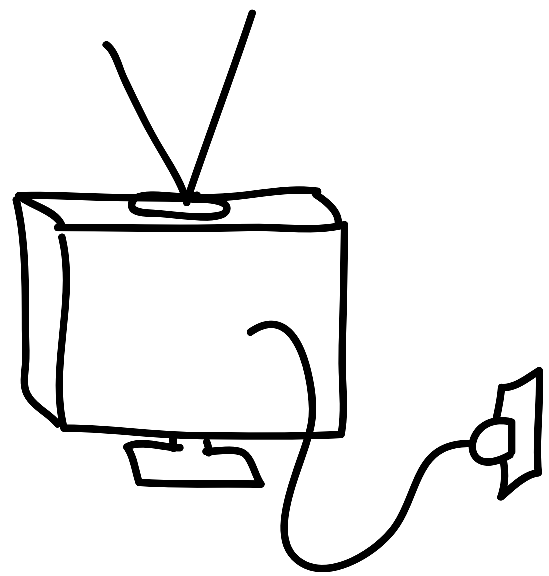

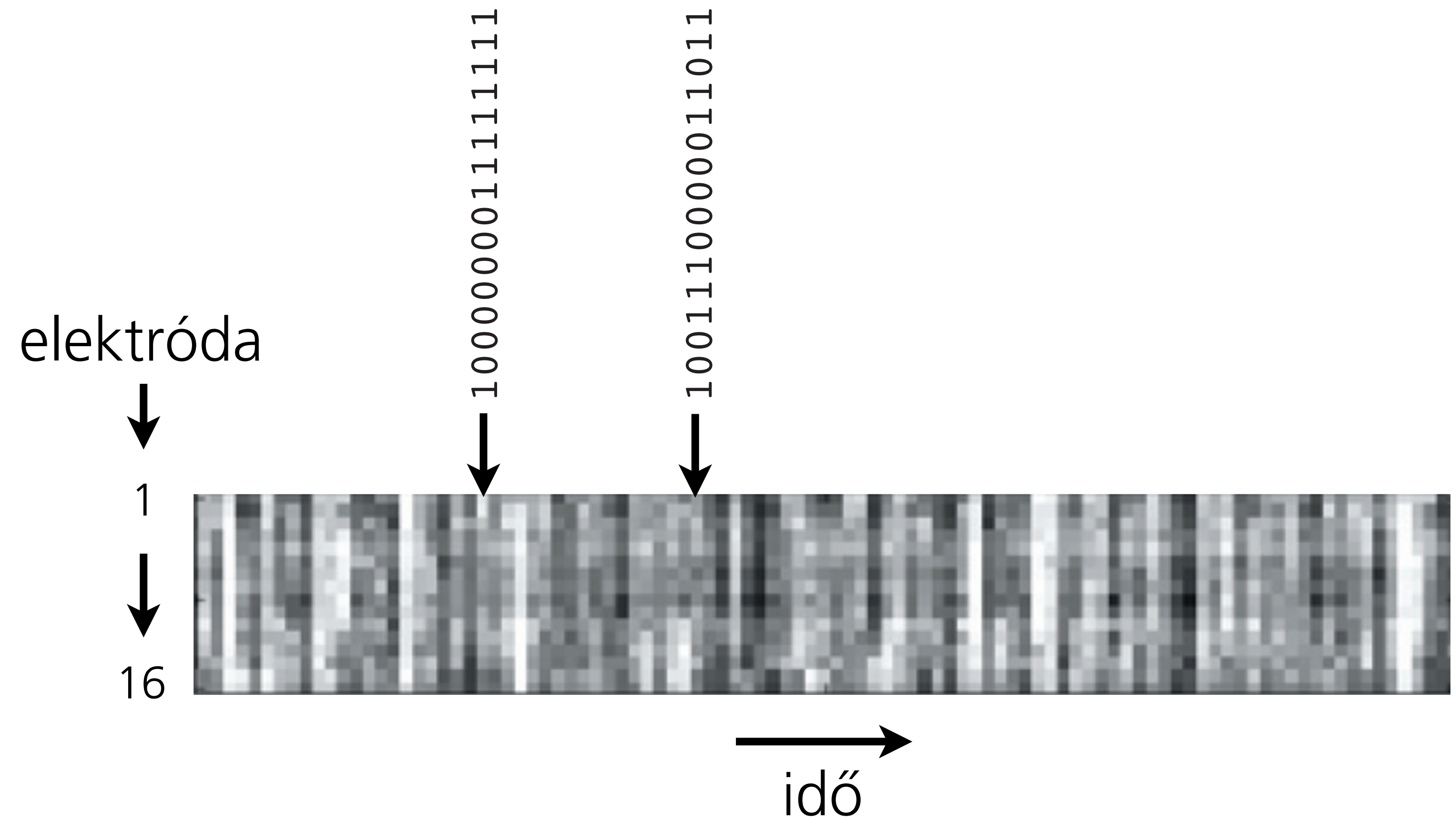
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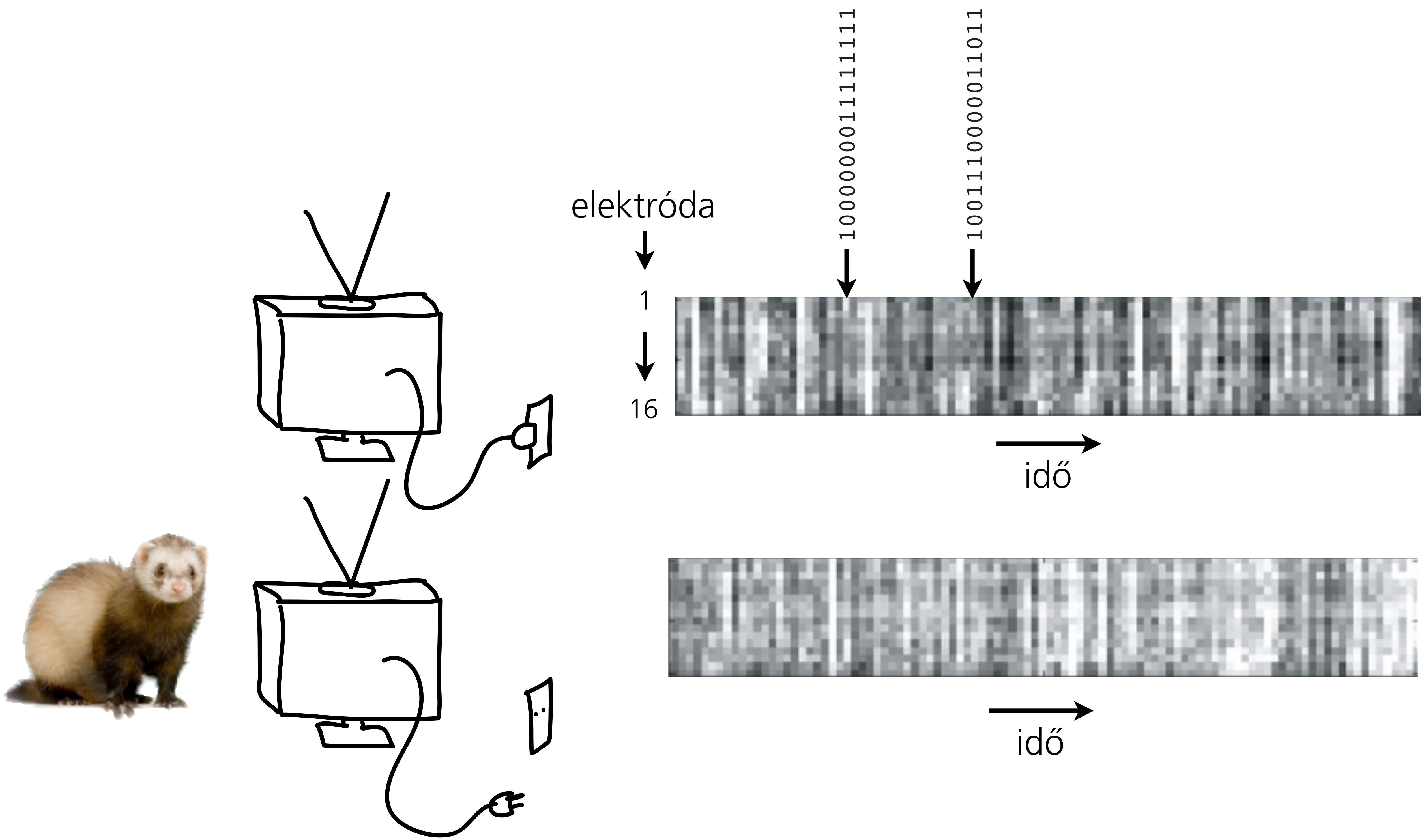
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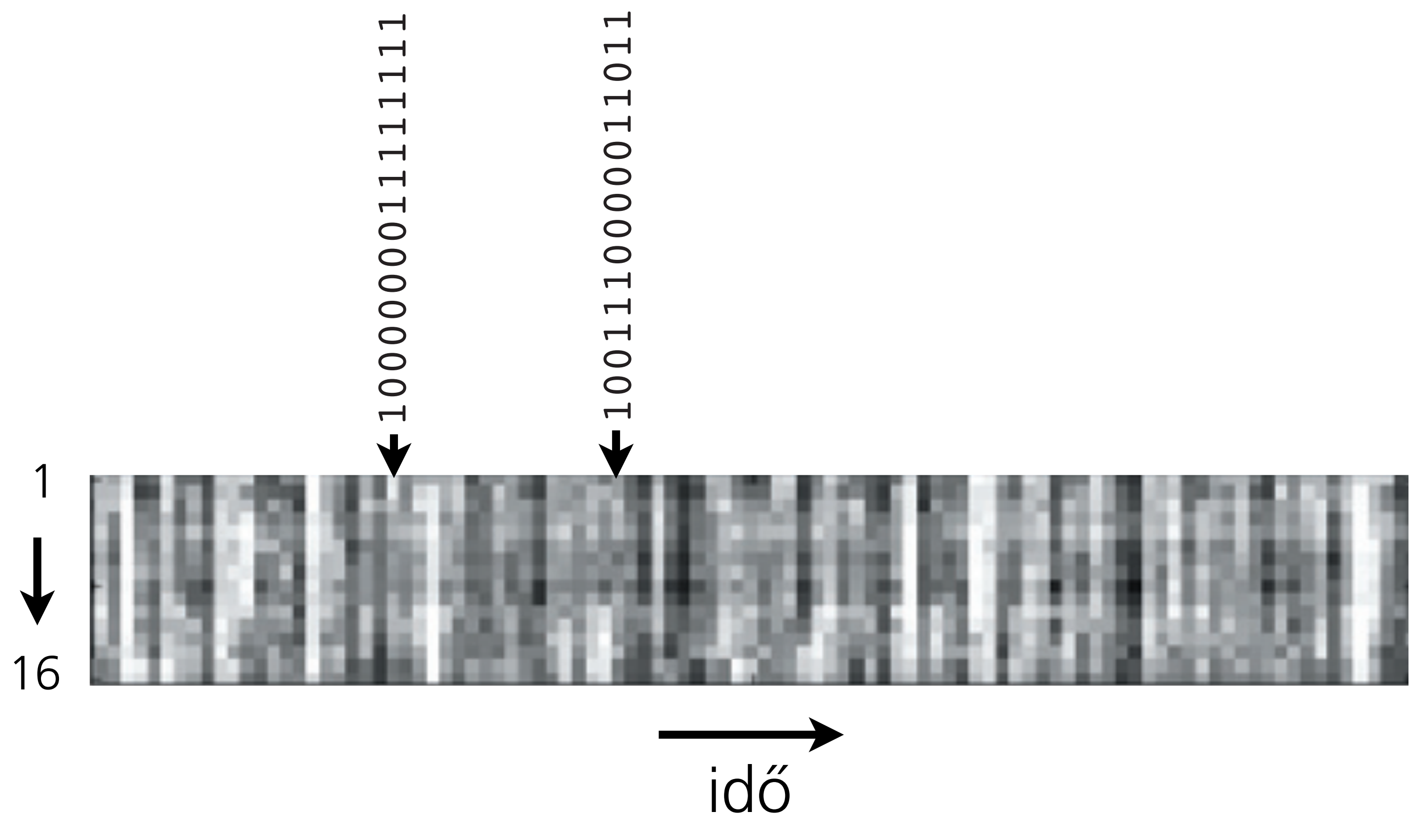
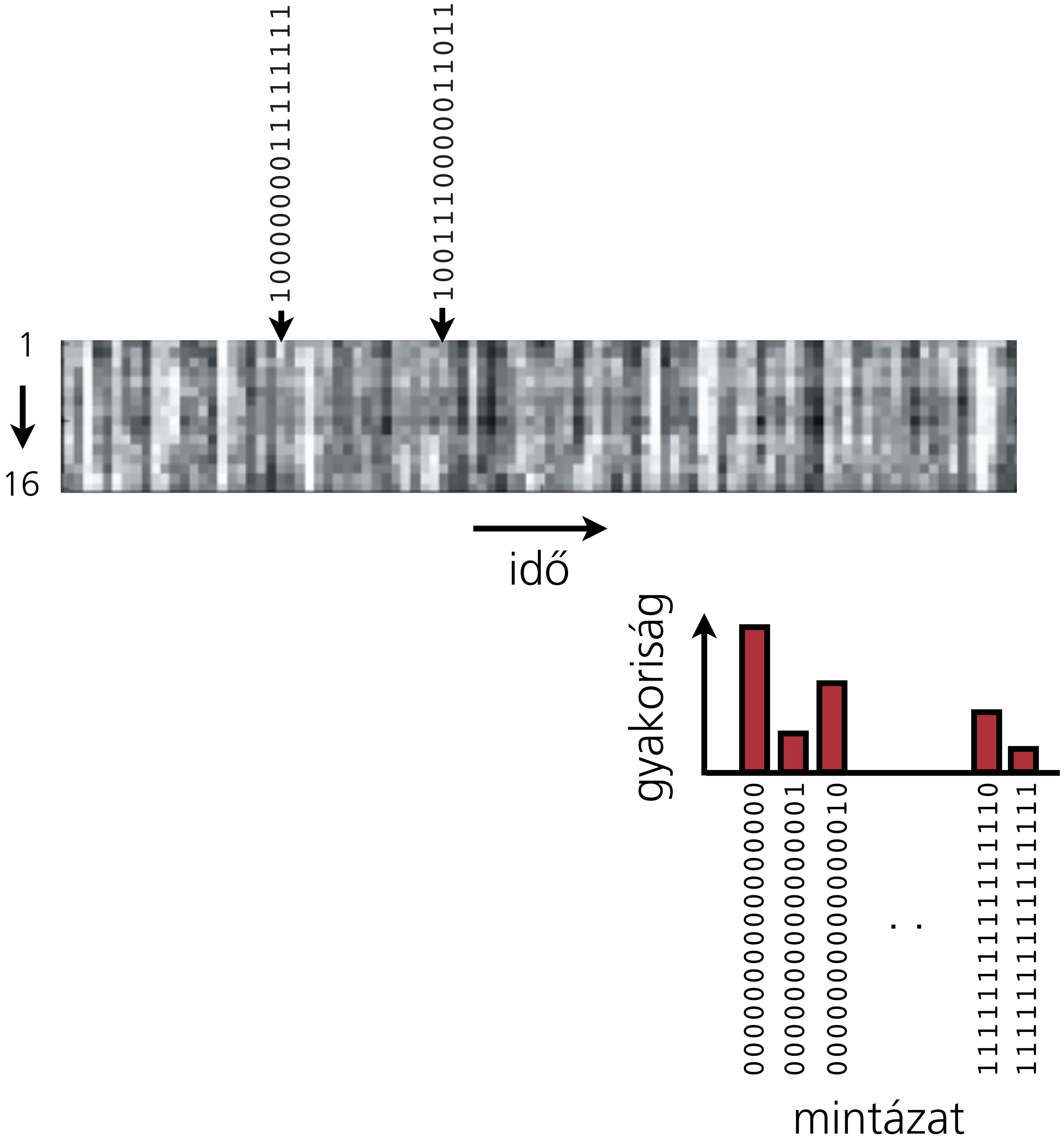
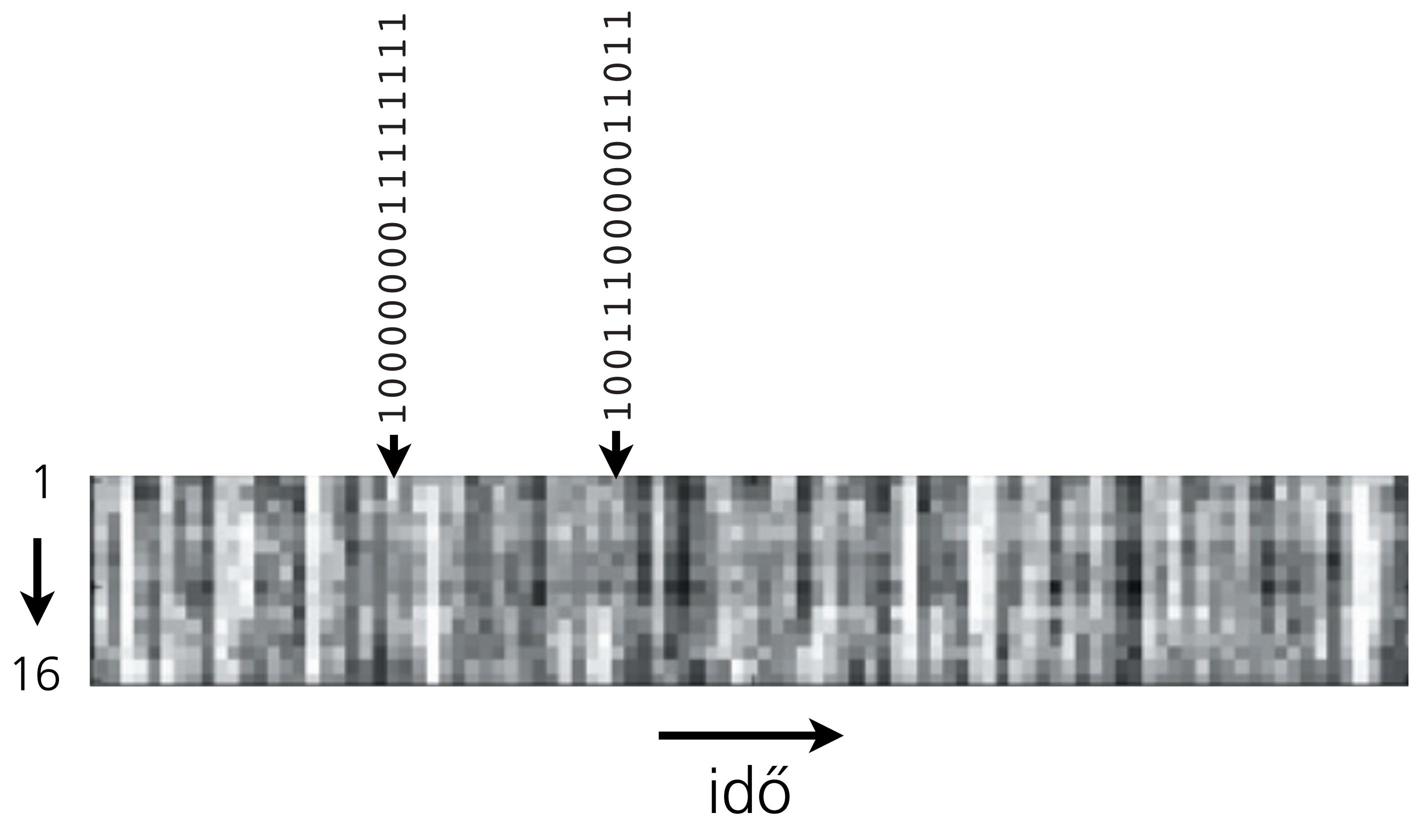
Hatékonyság?

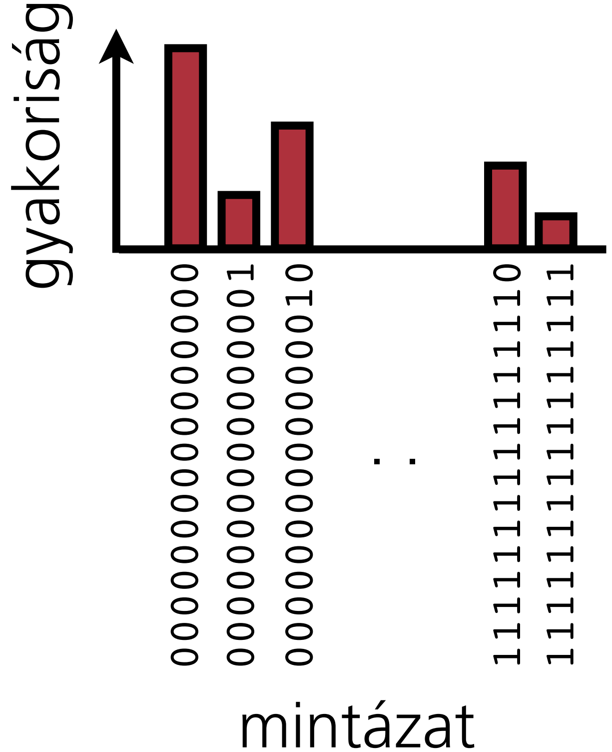


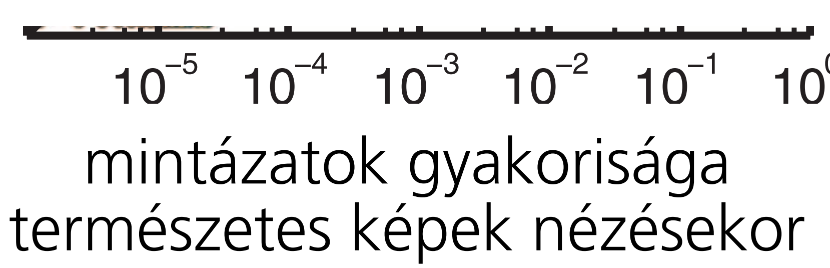
Hatékonyság?



Hatékonyság?







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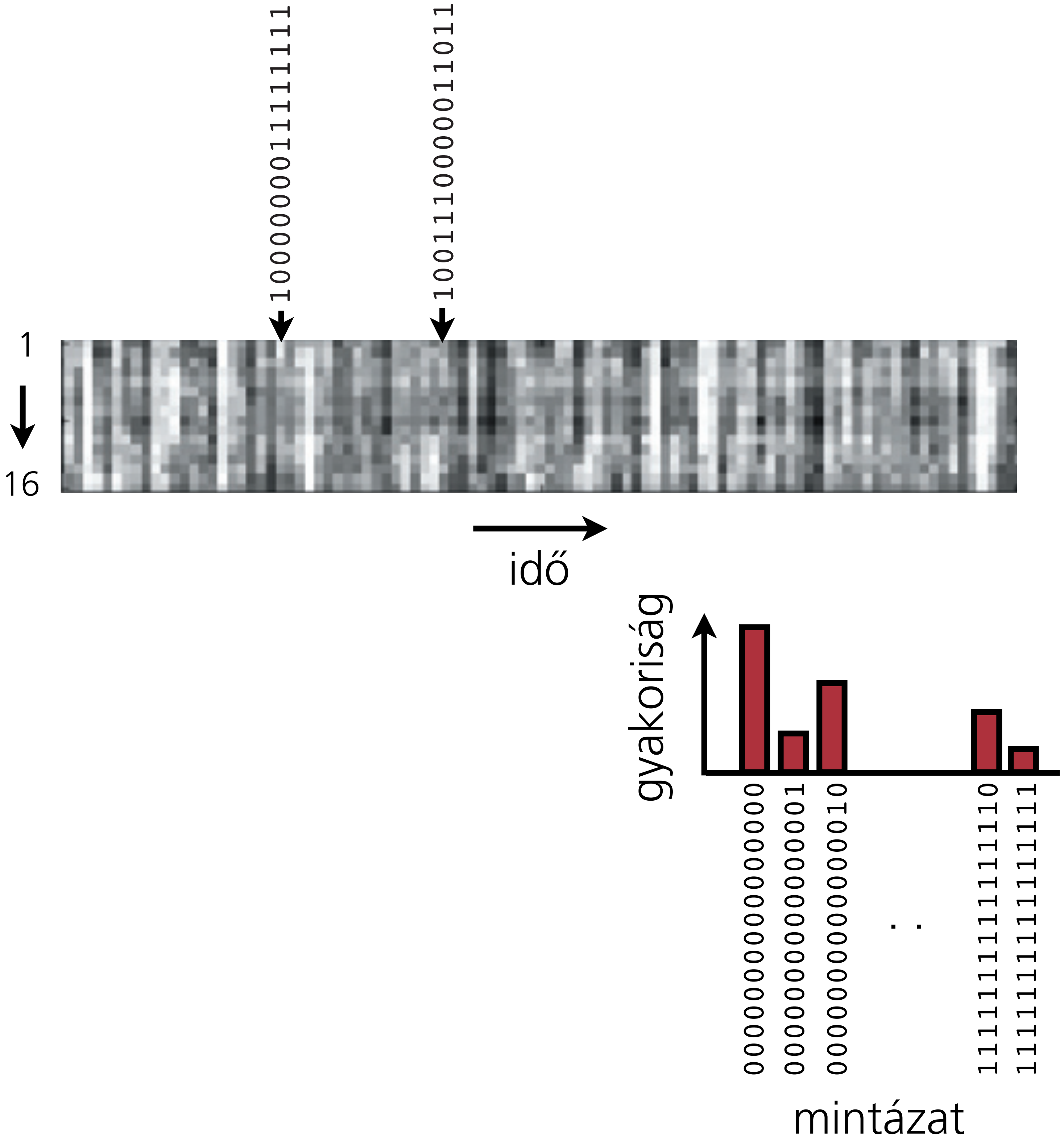
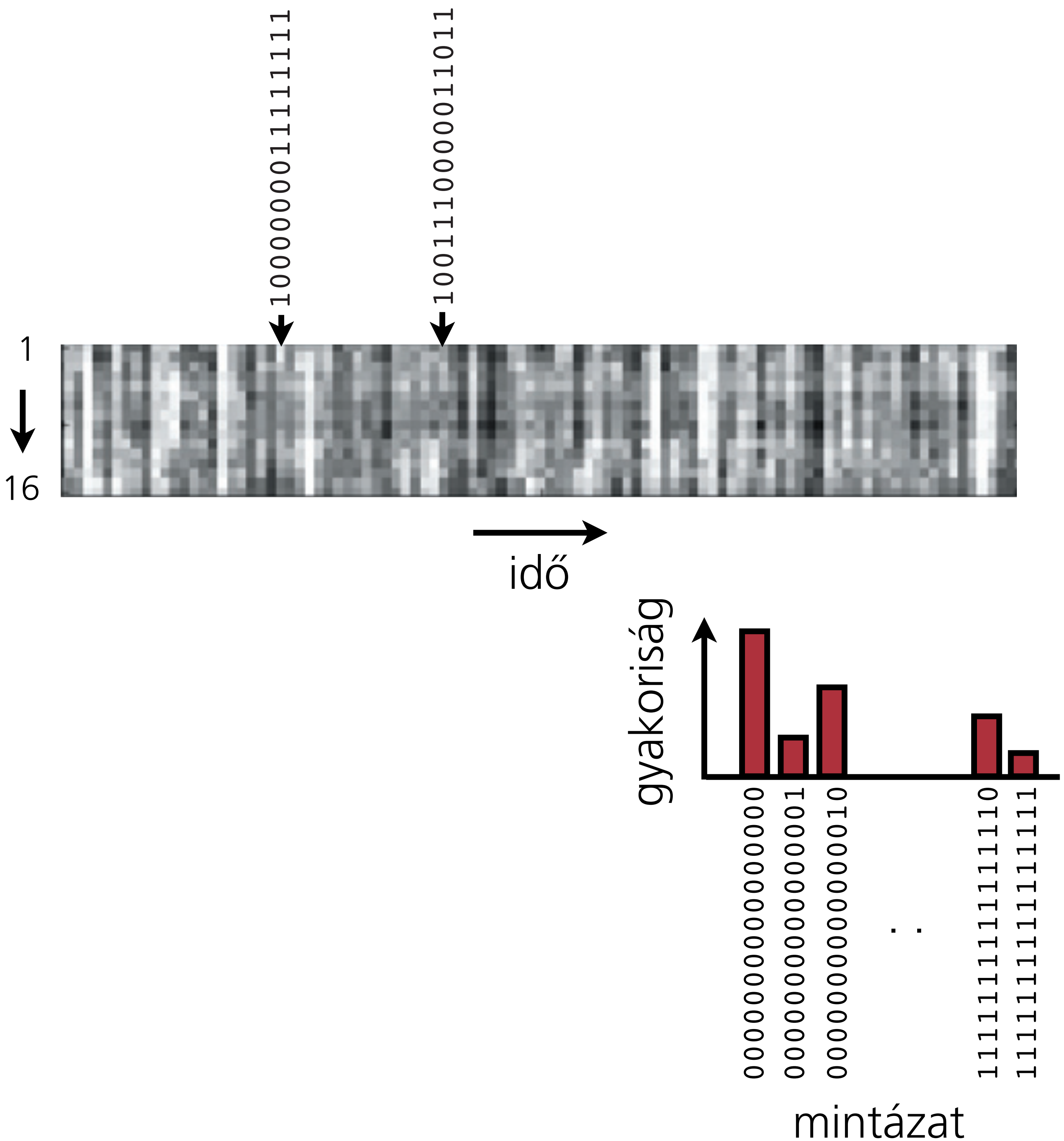
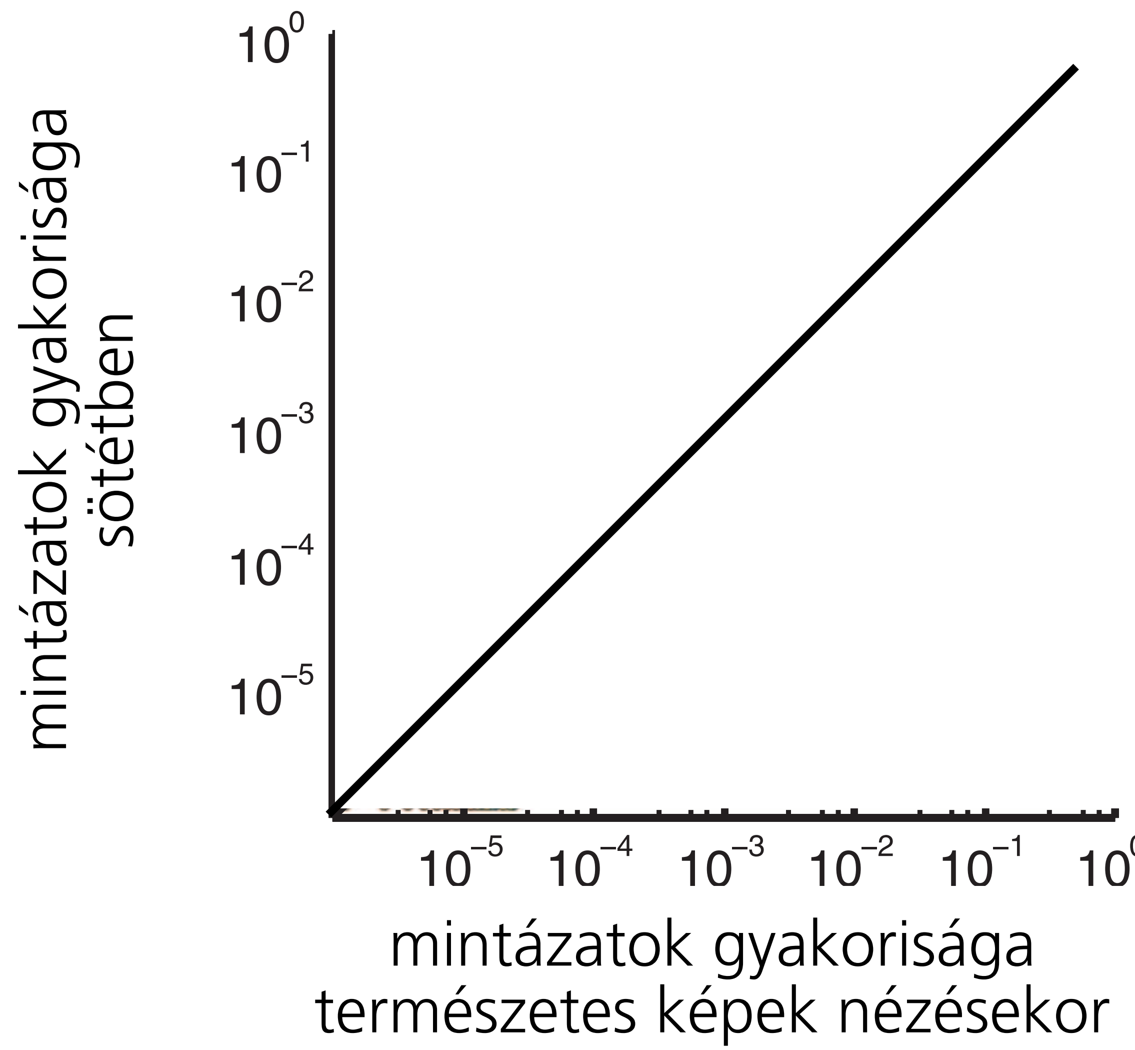
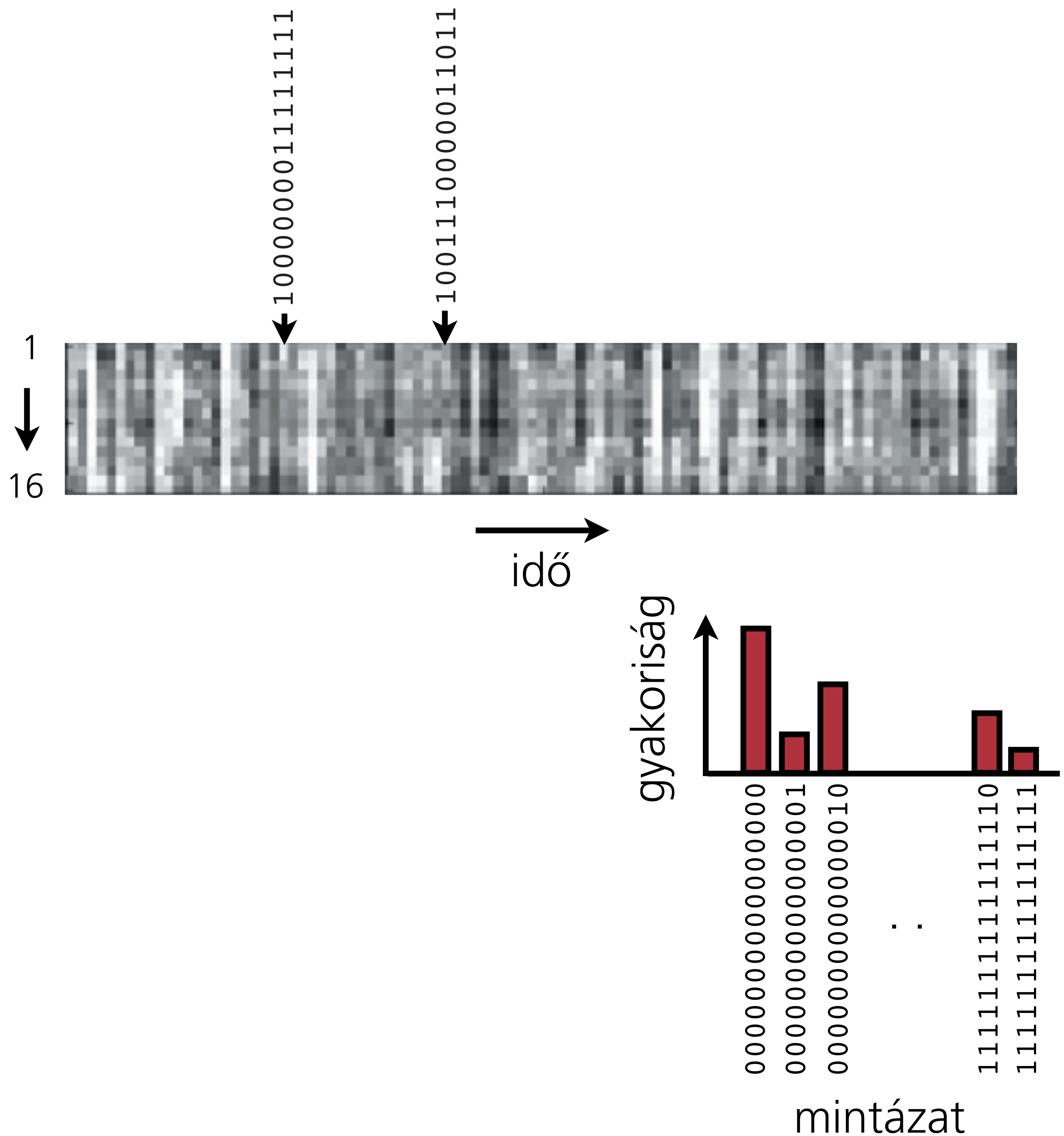


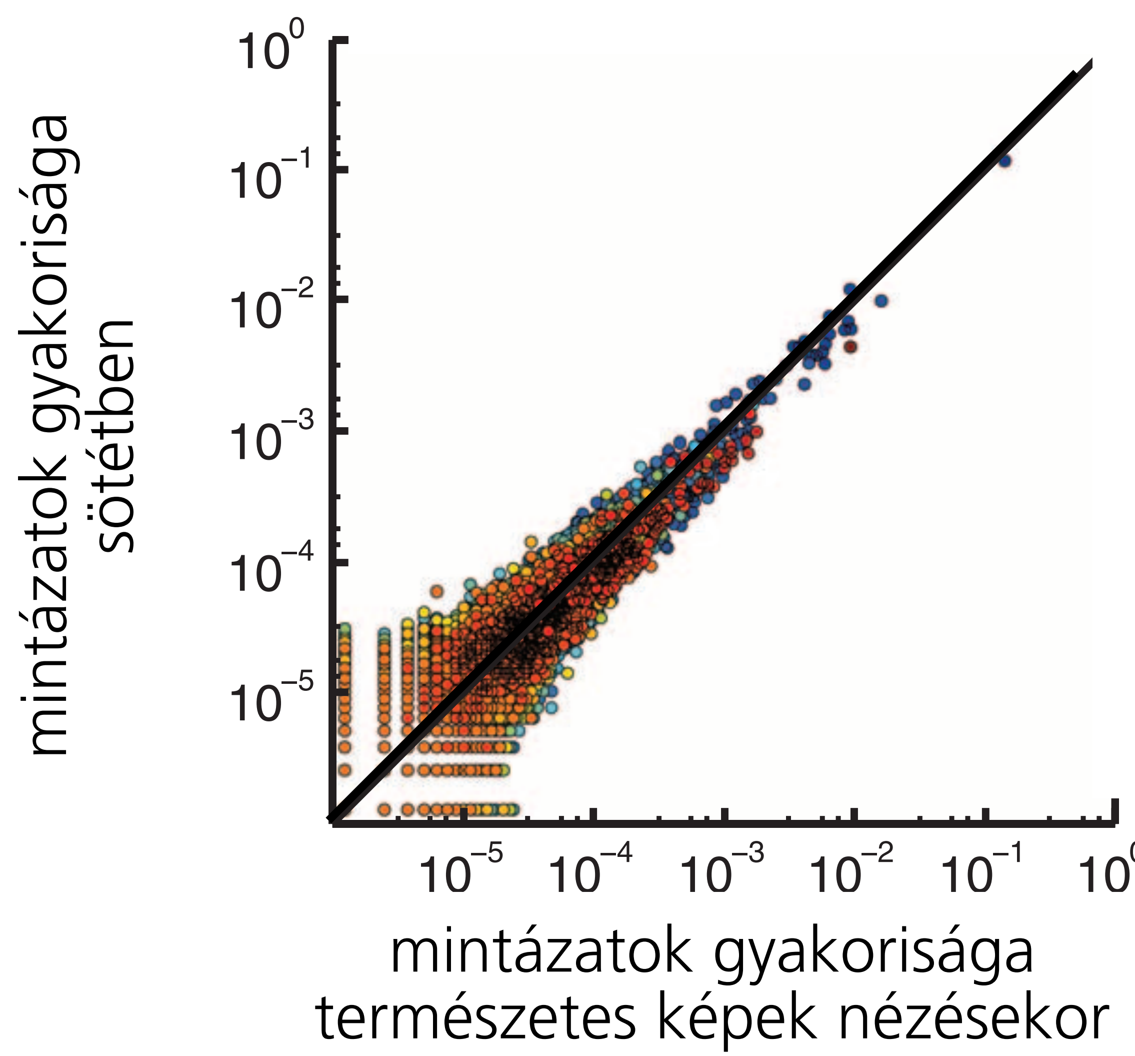
Hatékonyság?



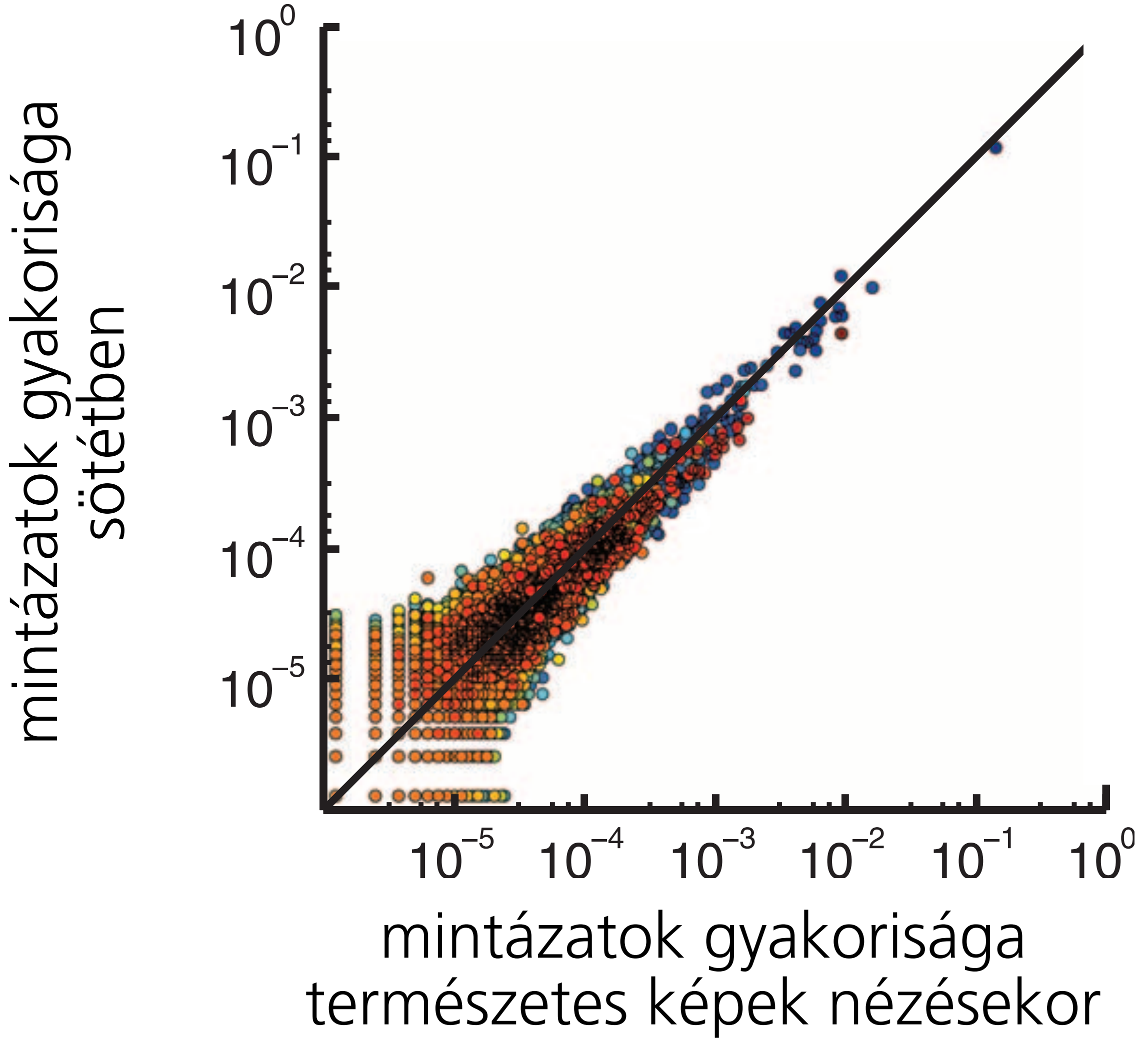


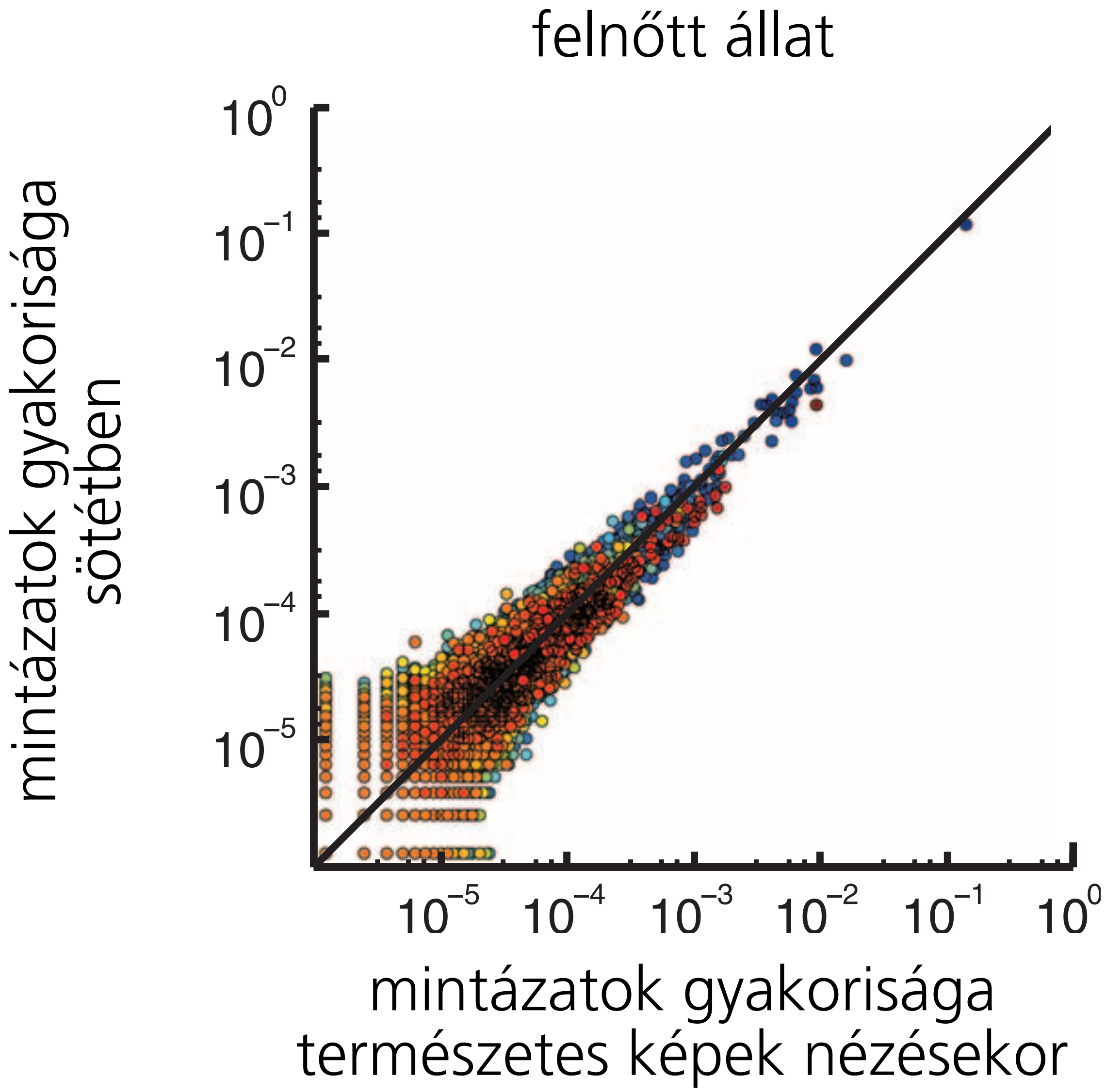
Hatékonyság?



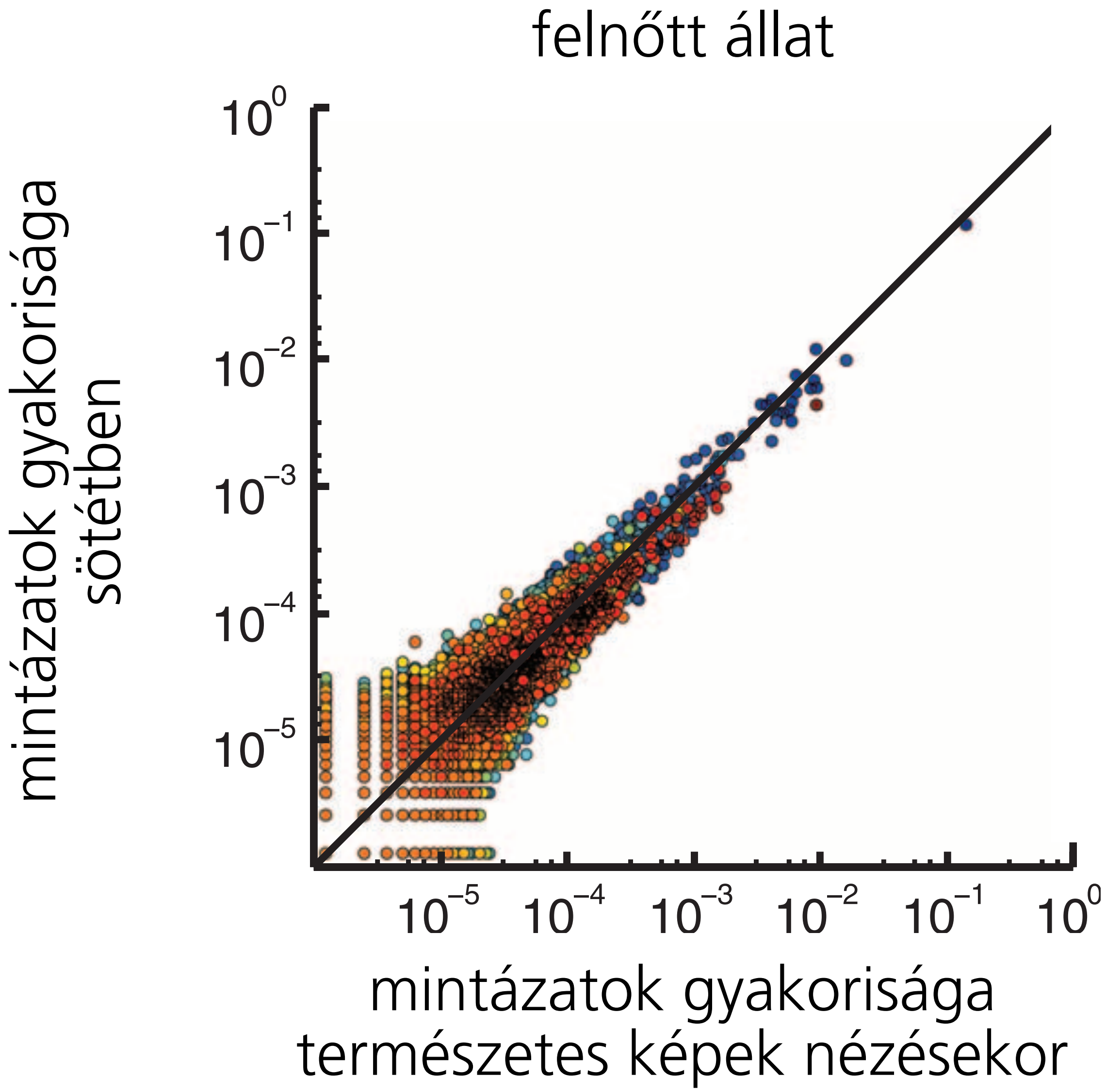


- Ha
- az idegrendszer ismeri a világ szerkezetét,
- akkor
- az elvárásai nem különböznek attól,
- amit általában érzékel

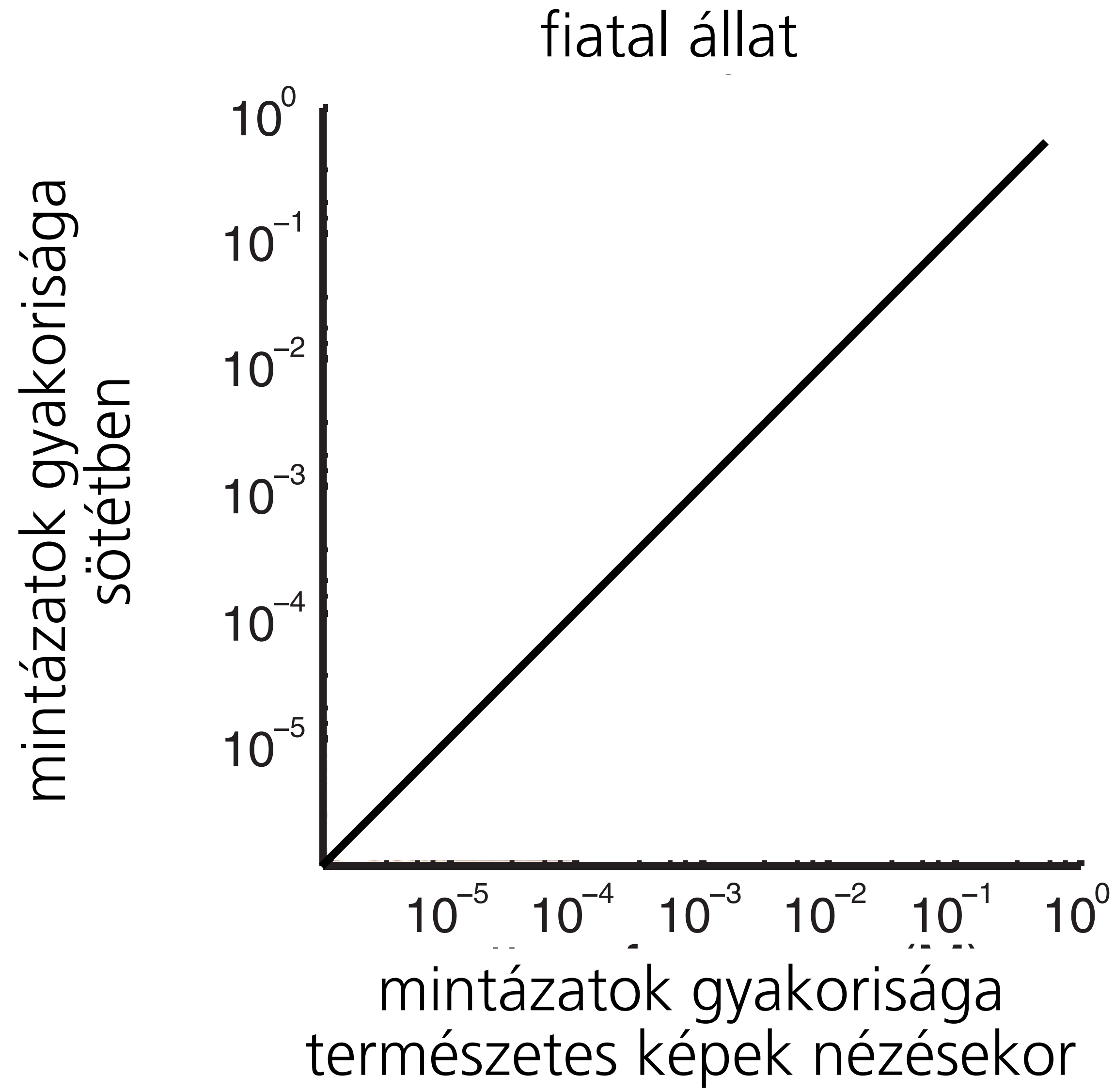

mintázatok gyakorisága
sötétben
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0
mintázatok gyakorisága
természetes képek nézésekor



felnőtt állat
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
mintázatok gyakorisága sötétben
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0}
mintázatok gyakorisága természetes képek nézésekor



felnőtt állat
mintázatok gyakorisága sötétben
mintázatok gyakorisága természetes képek nézésekor
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}



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mintázatok gyakorisága természetes képek nézésekor
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10^{-2}
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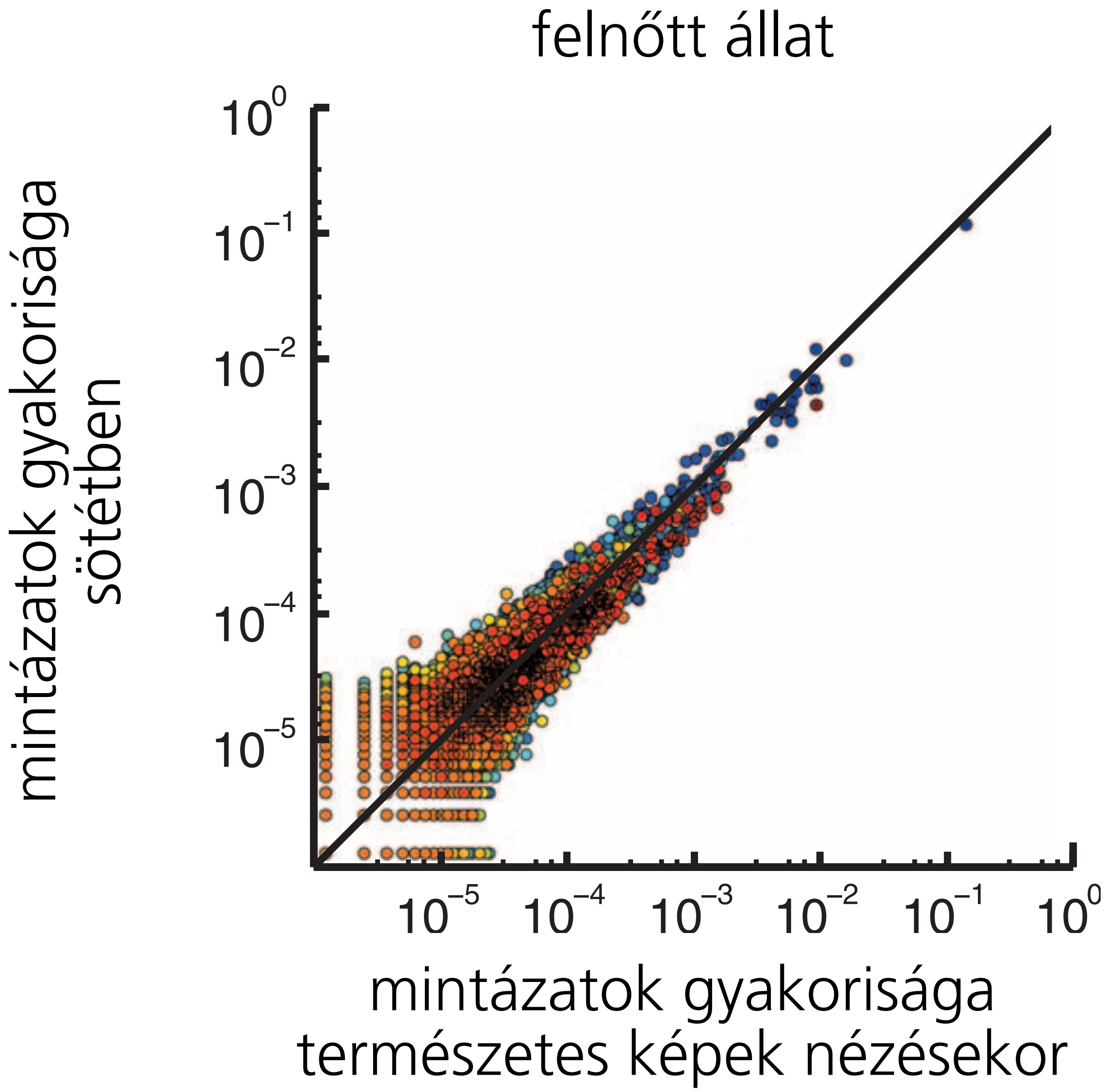
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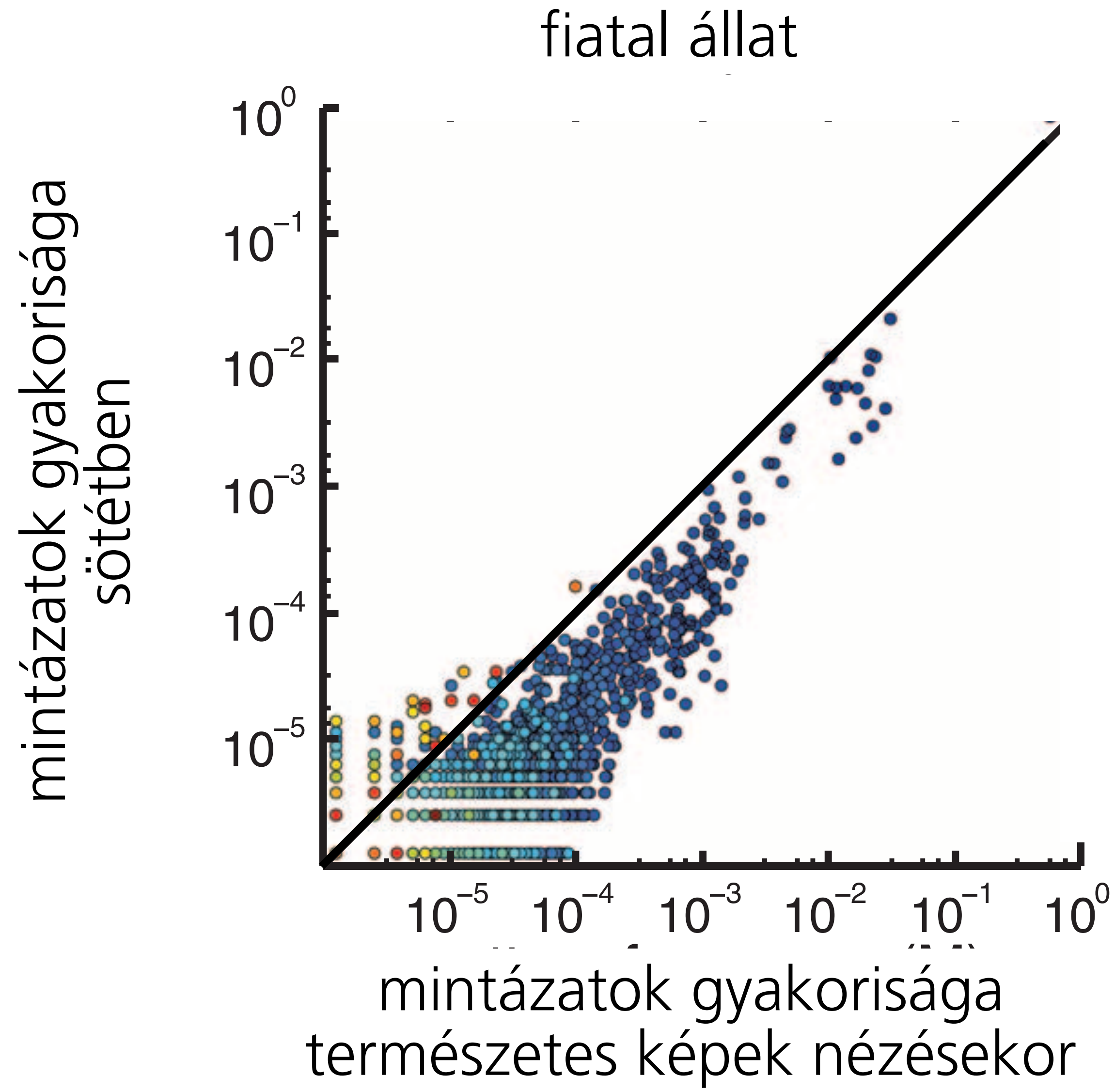
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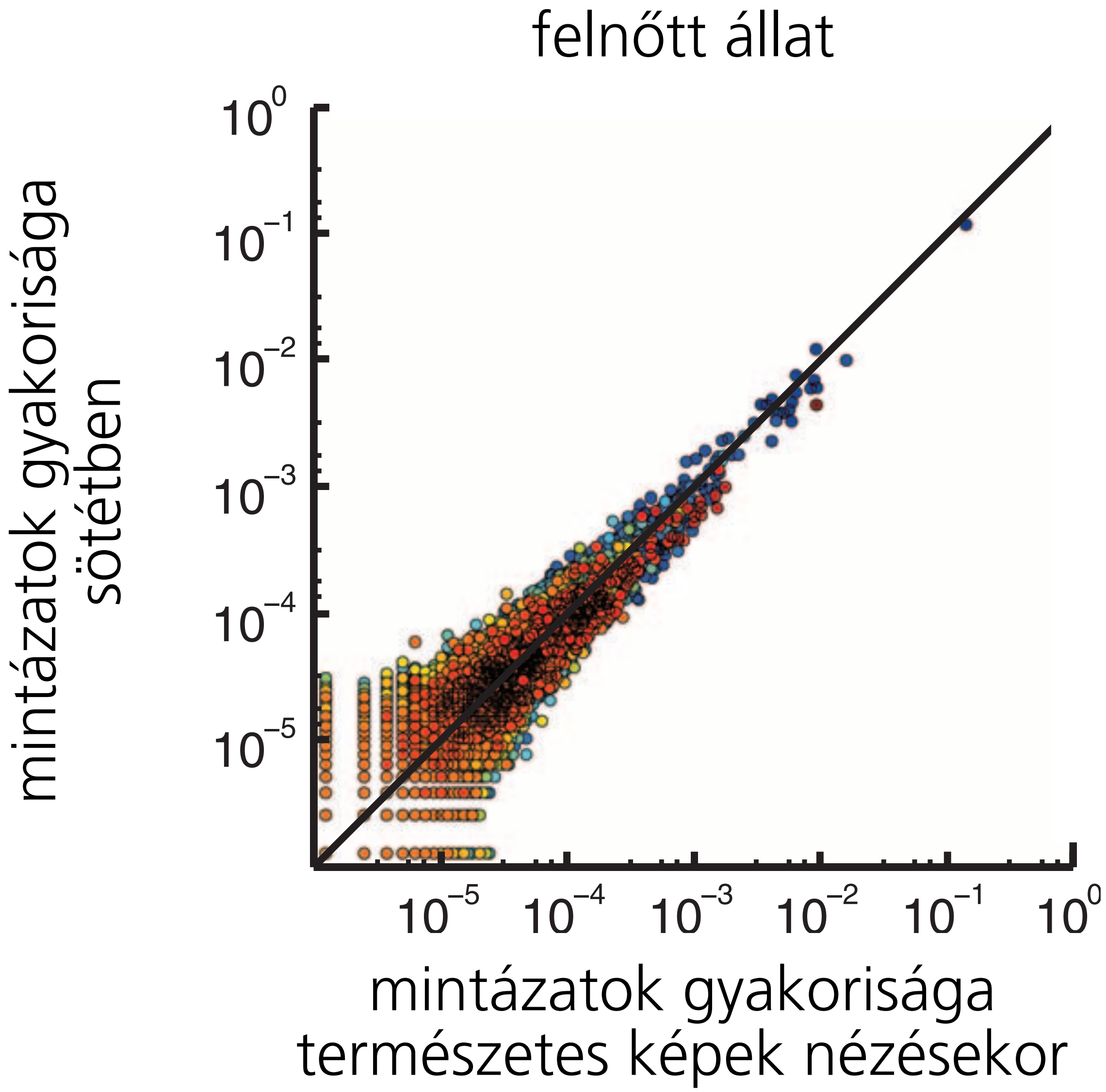
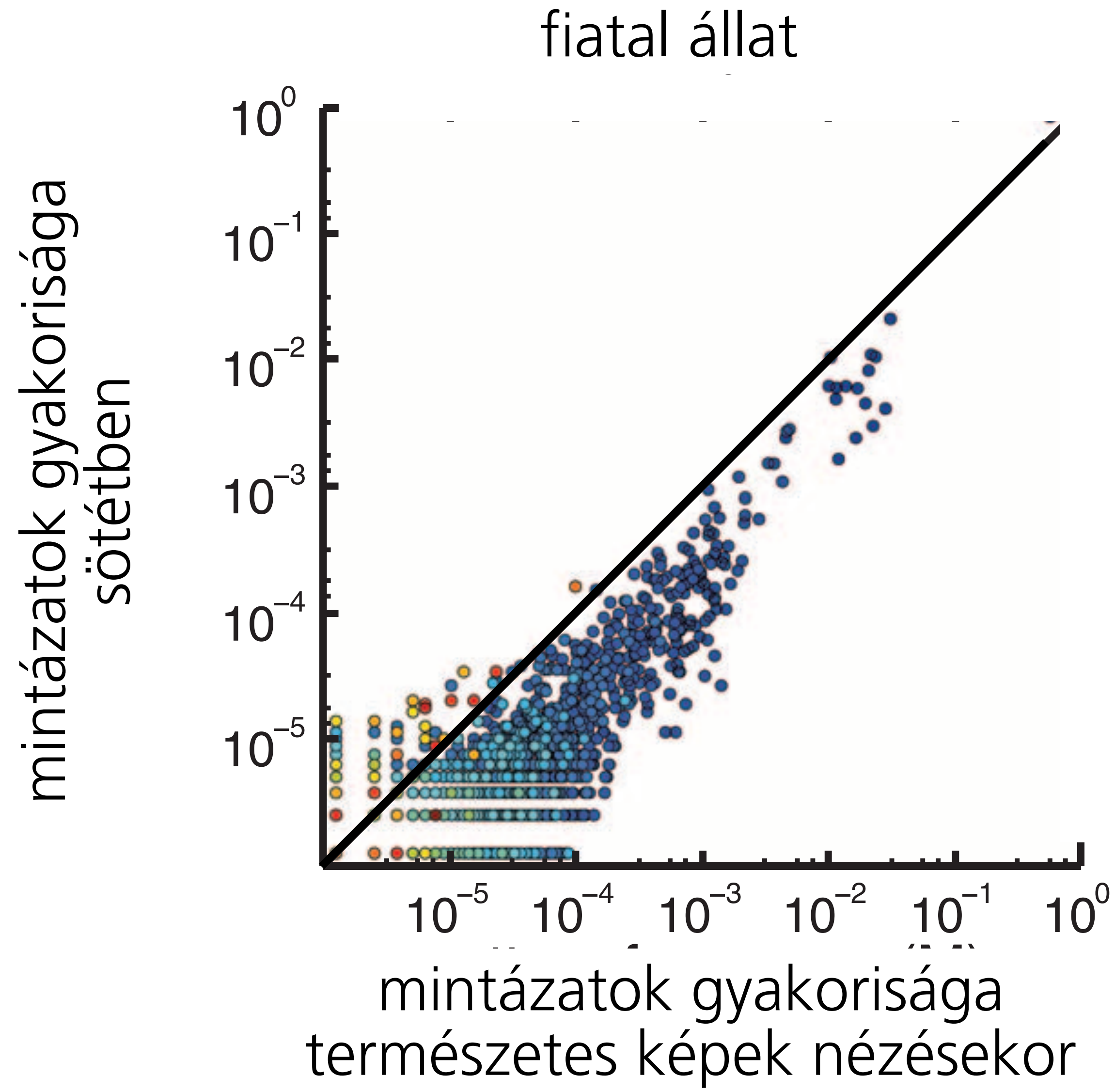
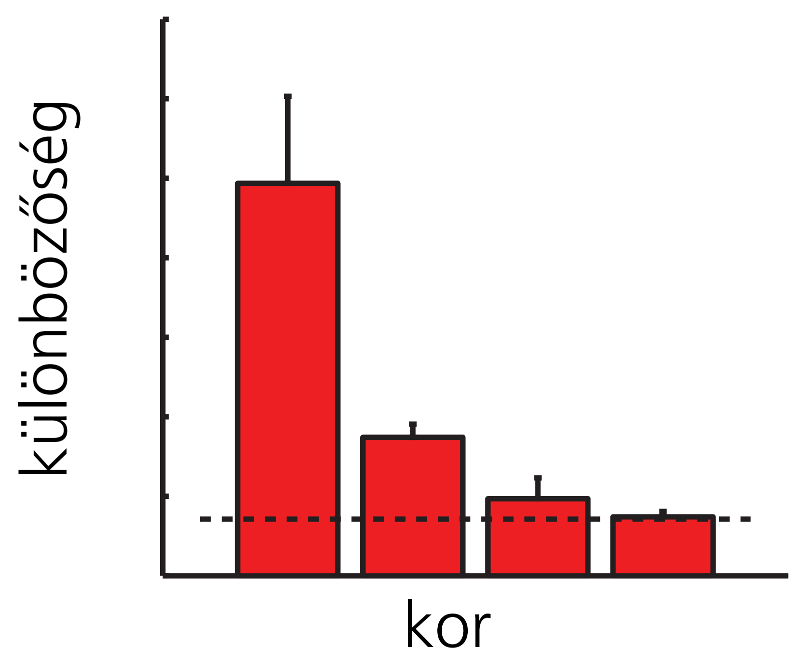
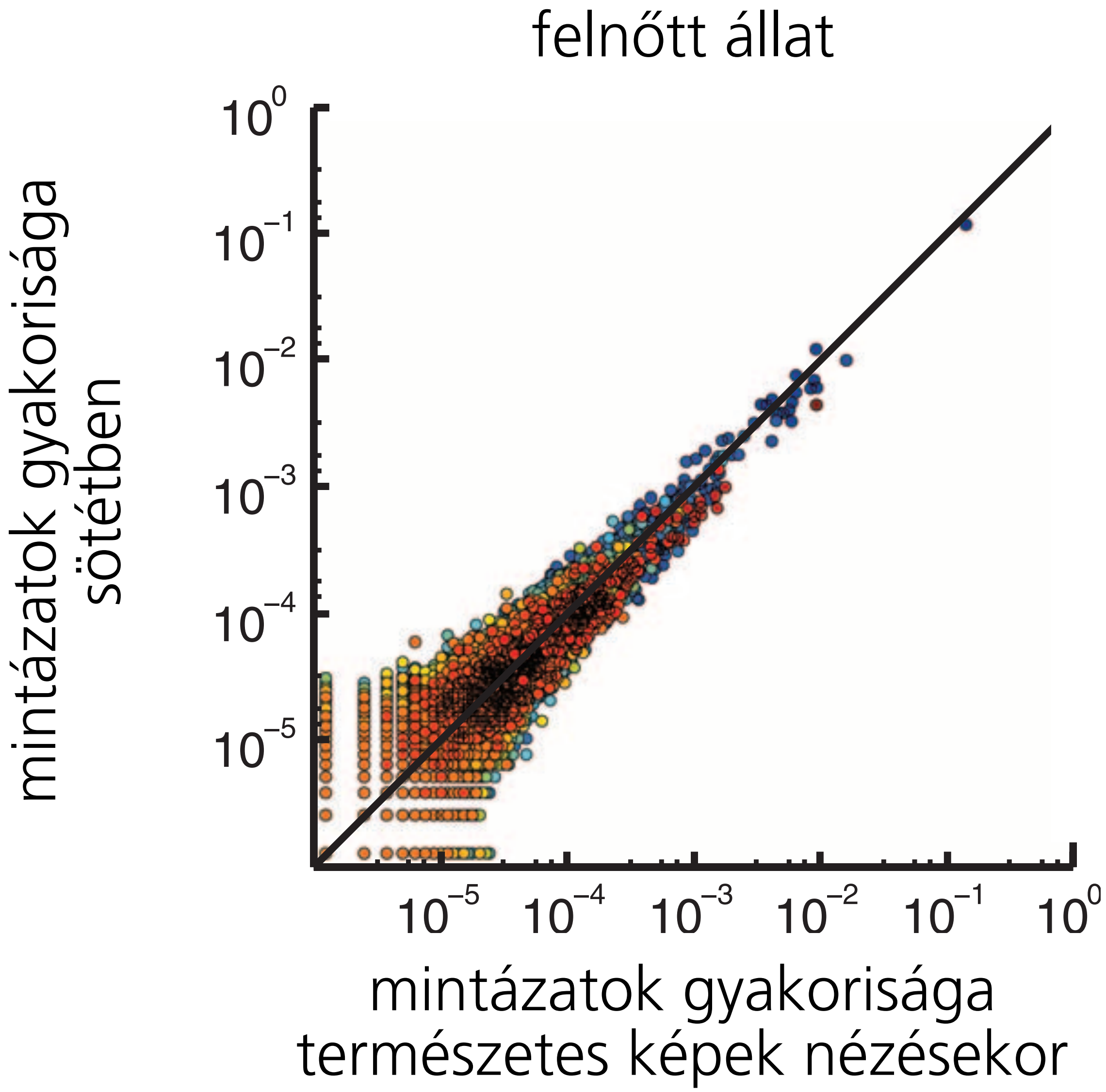
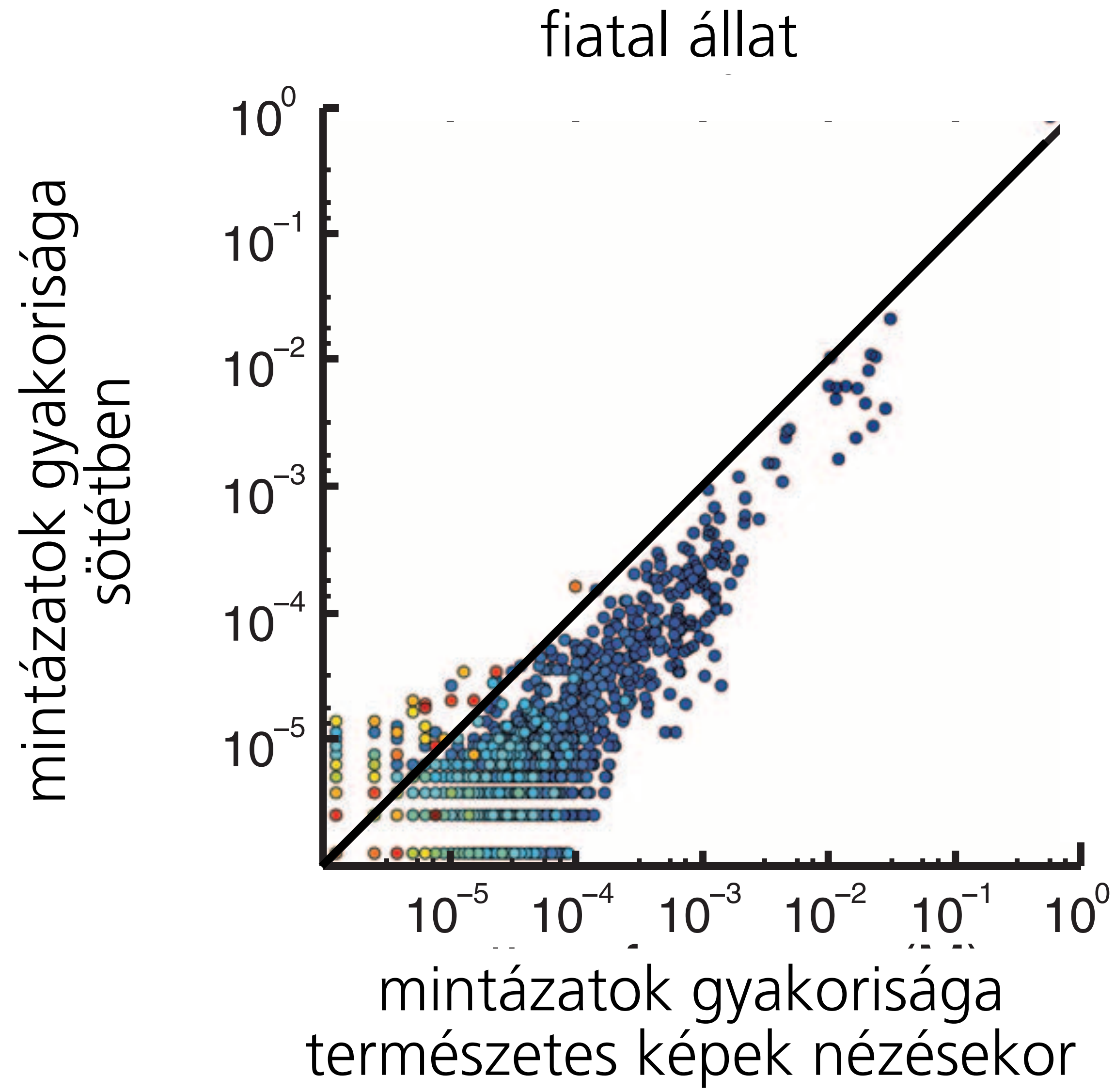


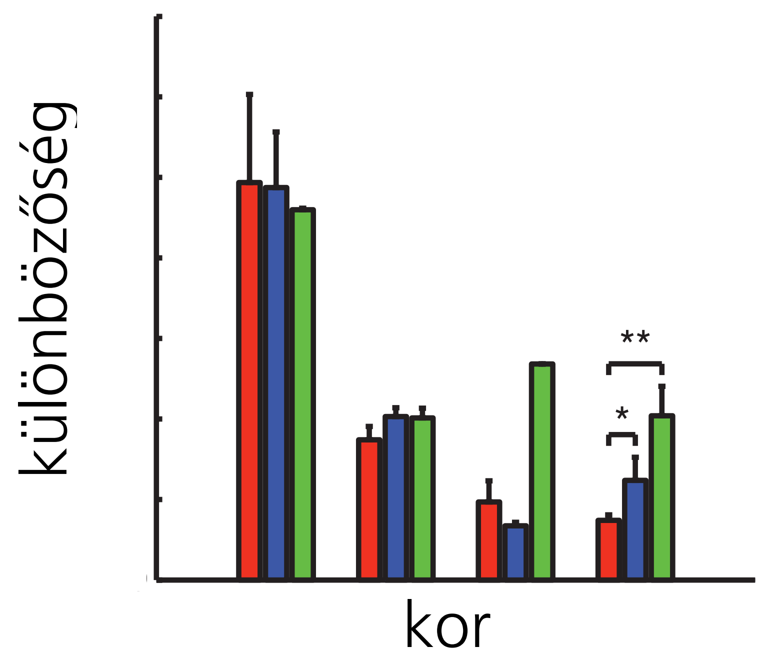




természetes képek
zaj
periodikusan sávozott képek







természetes képek
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periodikusan sávozott képek