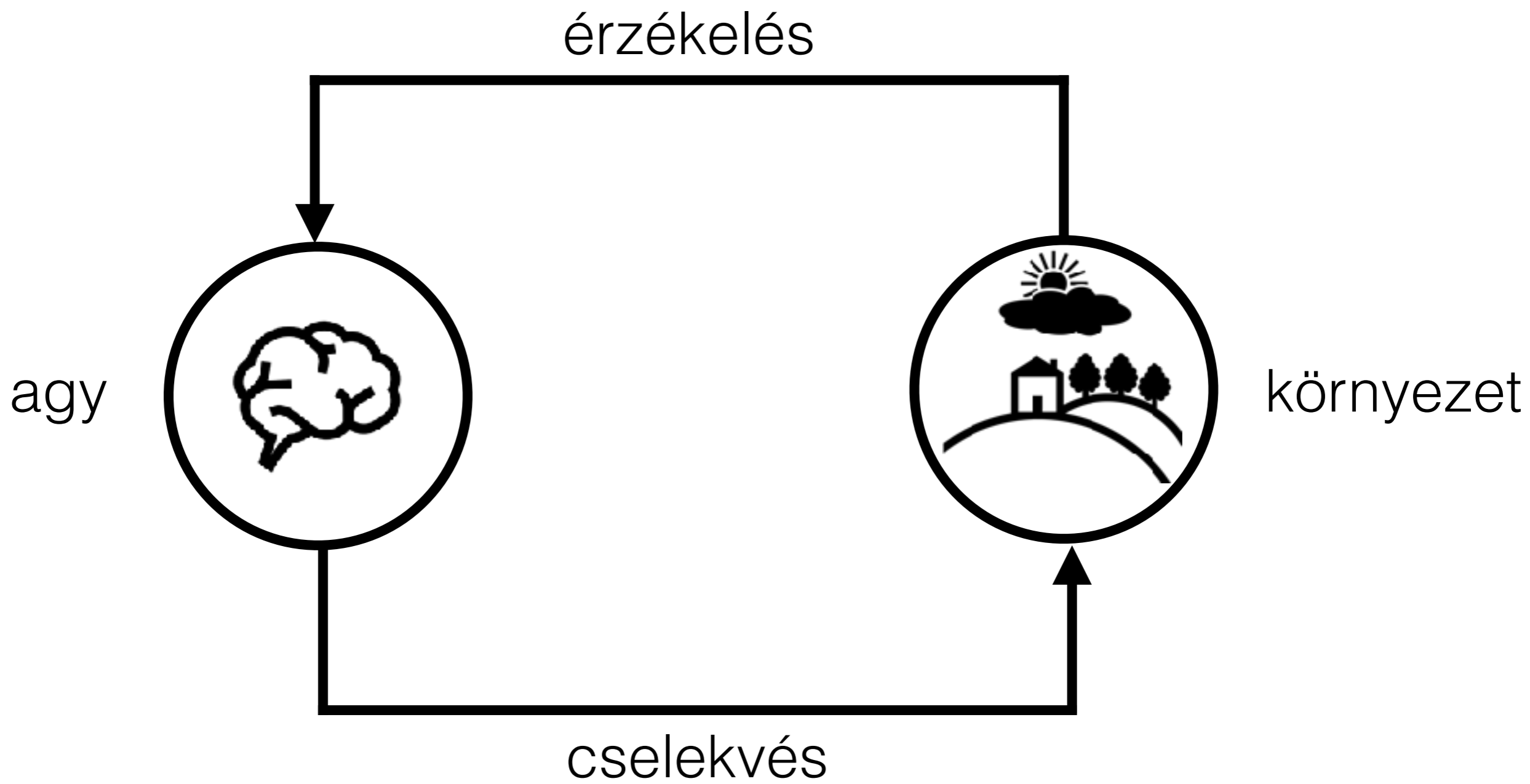


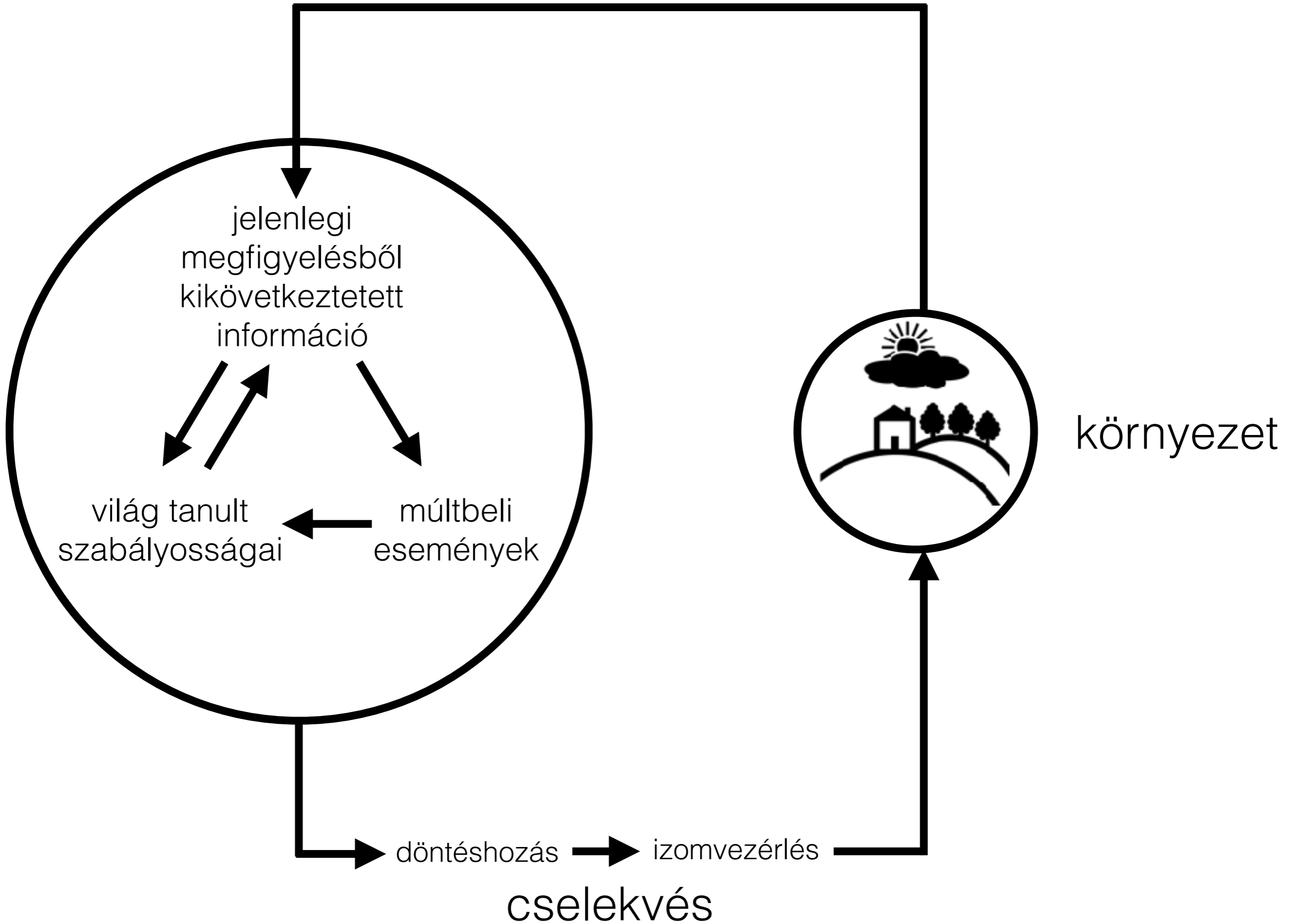
Statisztikus tanulás az idegrendszerben

ORBÁN GERGŐ

golab.wigner.mta.hu



érzékelés



környezet

cselekvés

érzékelés

**percepció
(észlelés)**

jelenlegi
megfigyelésből
kikövetkeztetett
információ

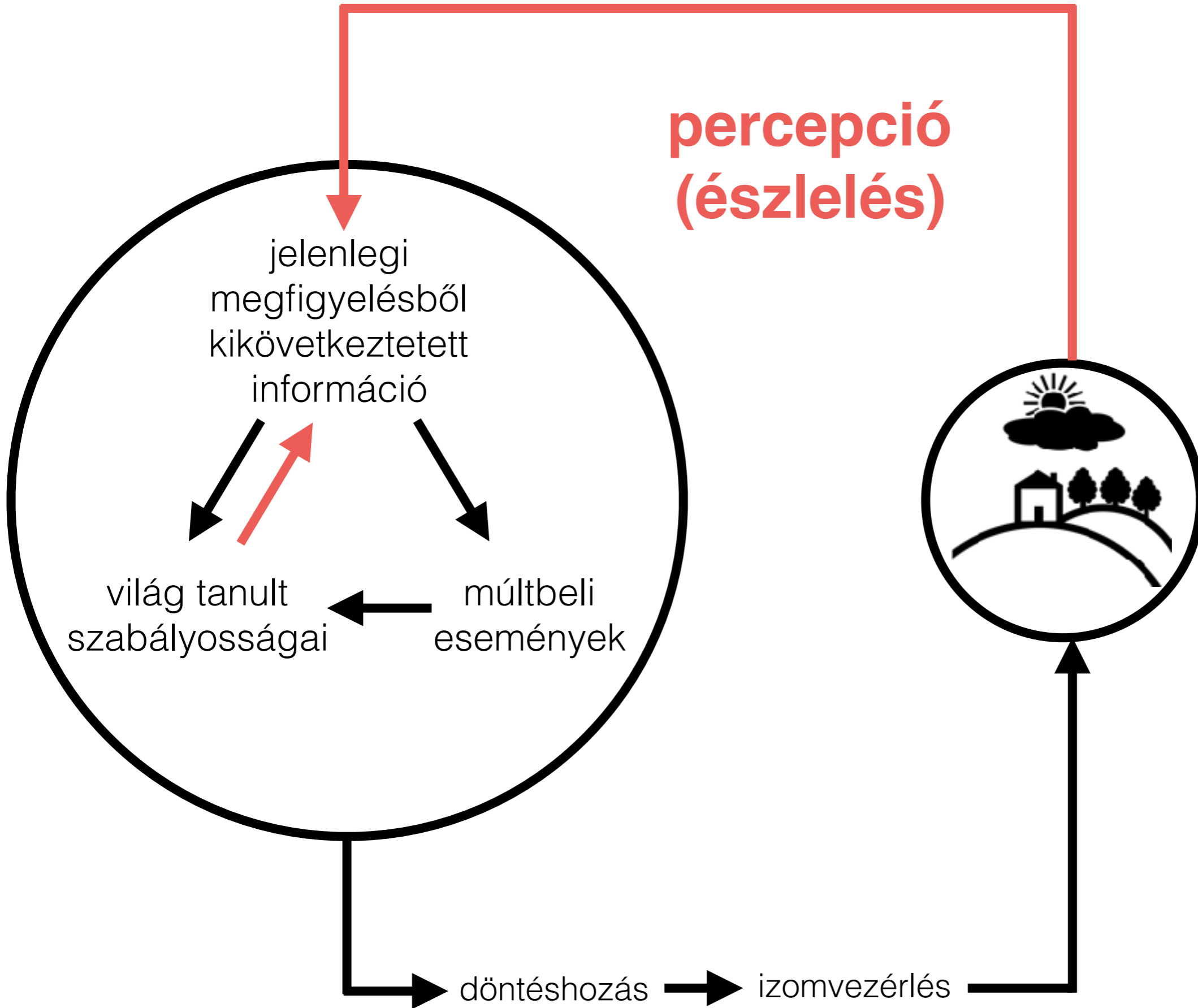
világ tanult
szabályosságai

múltbeli
események

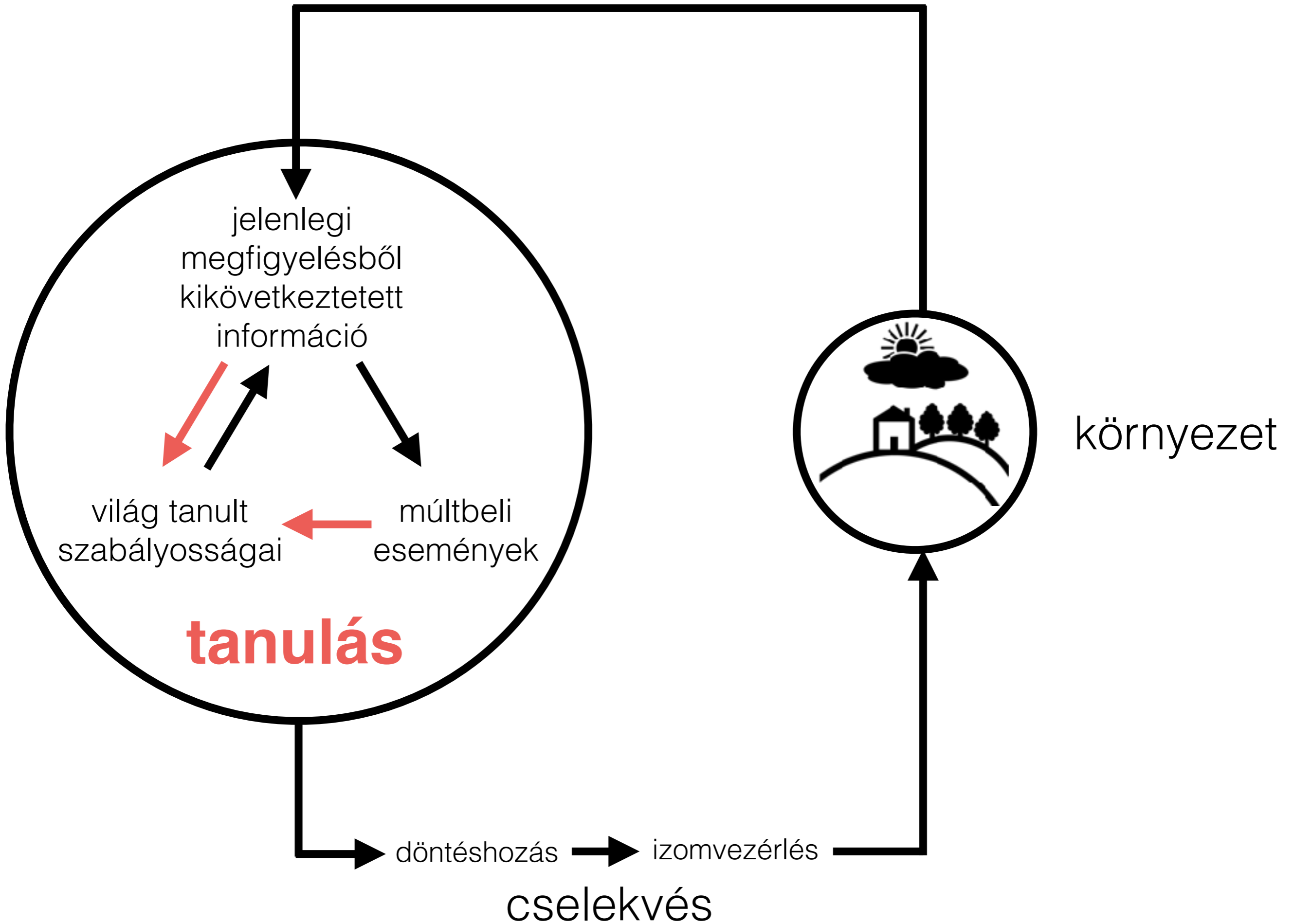
környezet

döntéshozás → izomvezérlés

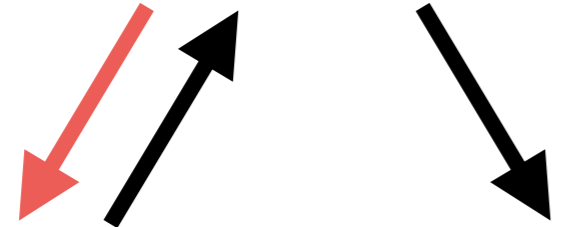
cselekvés



érzékelés



jelenlegi
megfigyelésből
kikövetkeztetett
információ



világ tanult
szabályosságai

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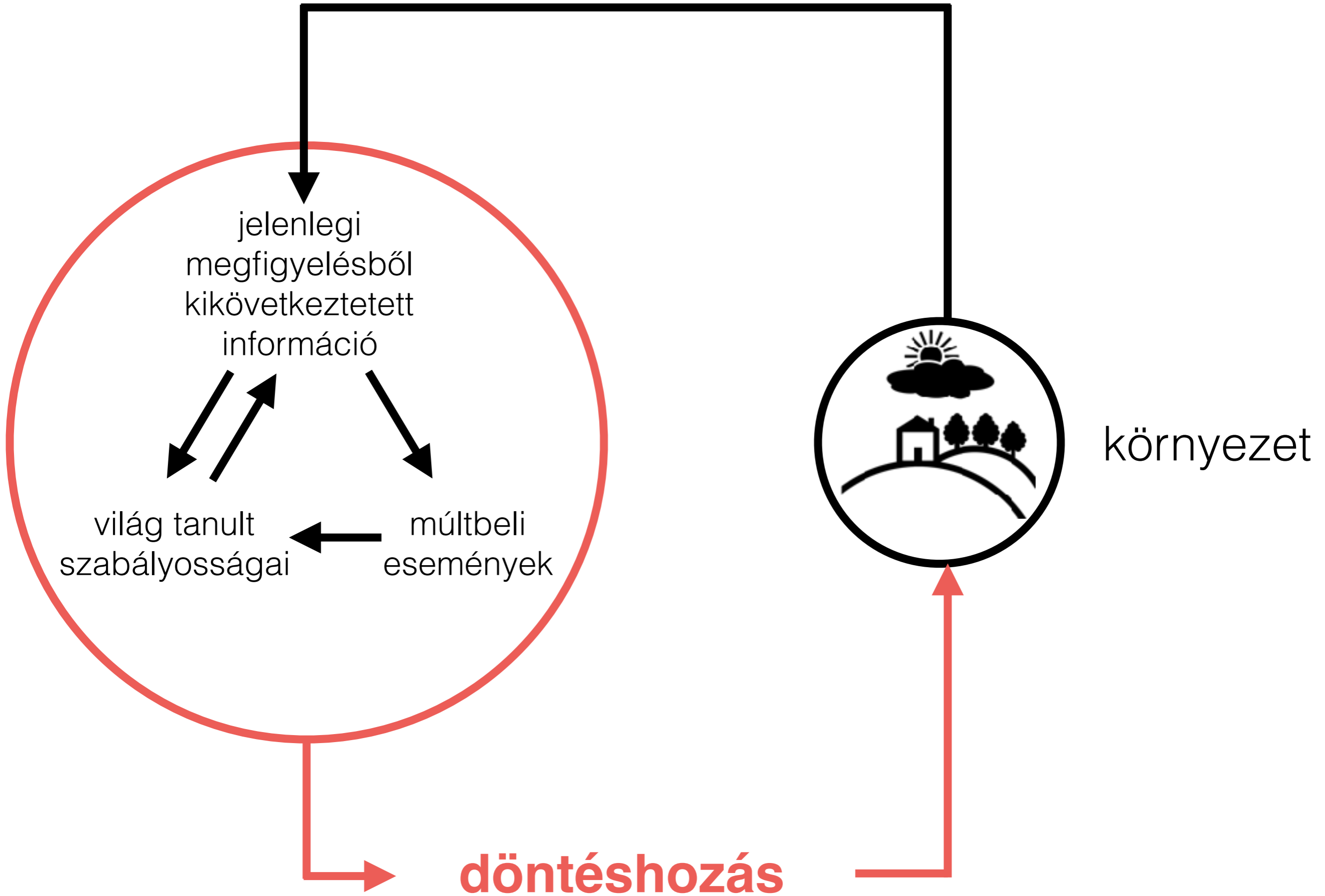
világ tanult
szabályosságai

múltbeli
események



környezet

döntéshozás



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kognitív

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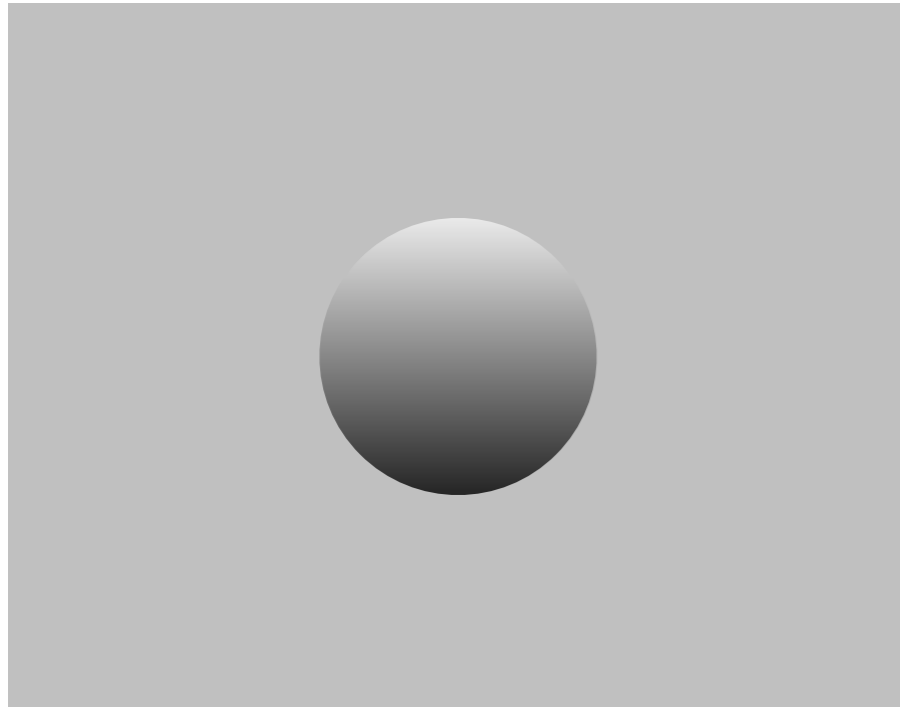
neurális

Structure learning

Vision II

Decision making and reinforcement learning

RECAP: role of priors



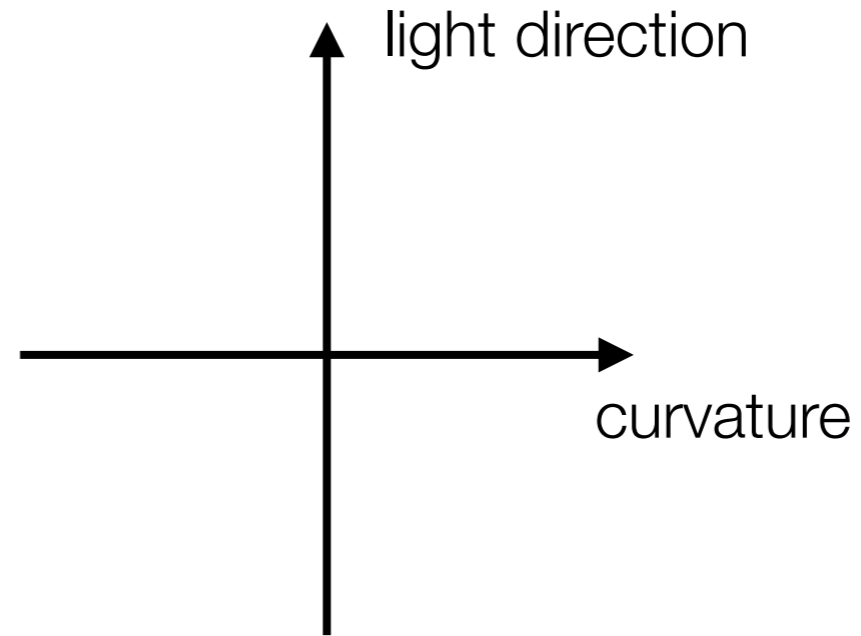
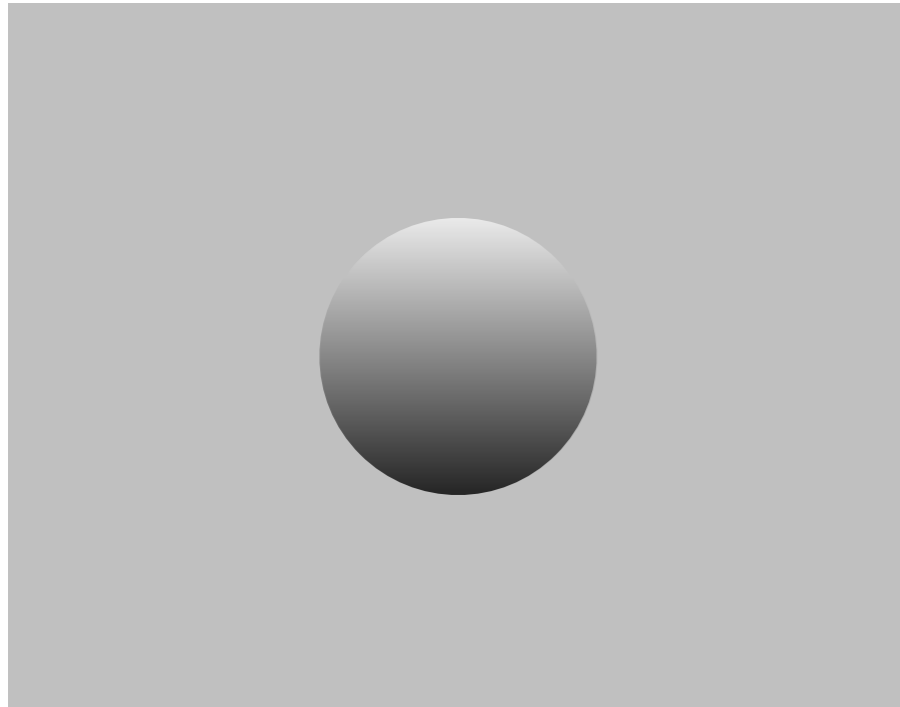
$$P(\text{feature} \mid \text{stimulus}) \propto P(\text{stimulus} \mid \text{feature}) \times P(\text{feature})$$

posterior: inference

likelihood: evidence

prior : expectations

RECAP: role of priors



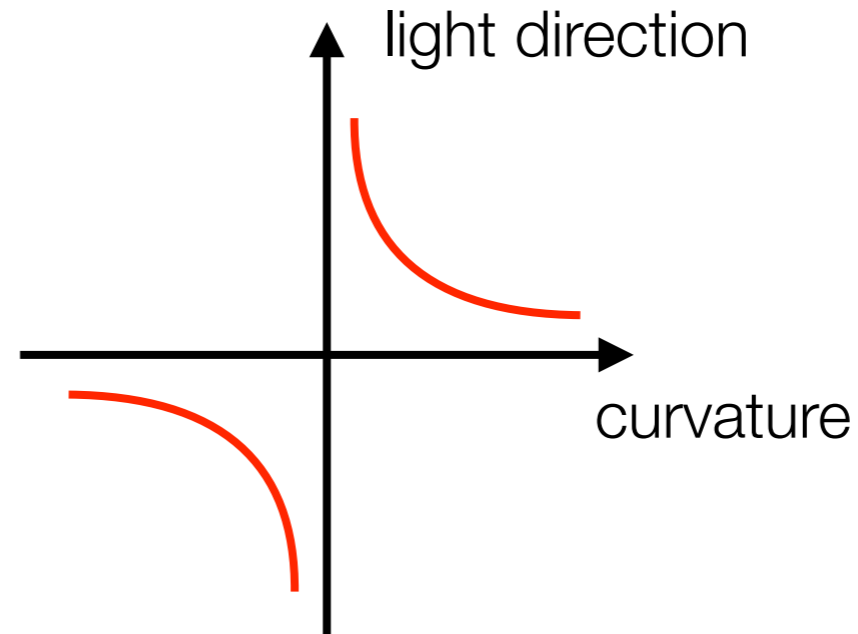
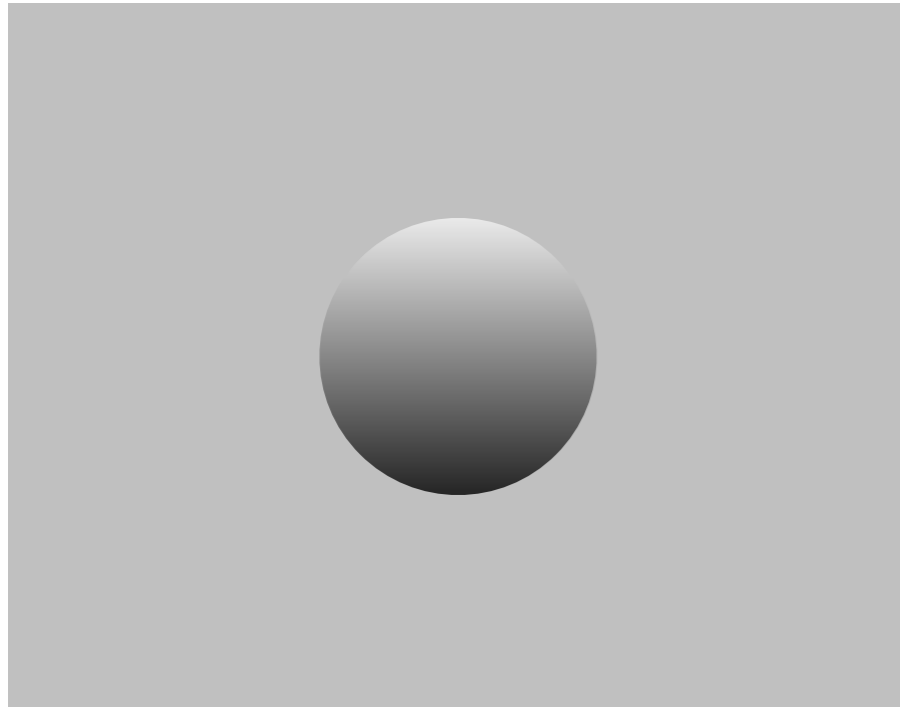
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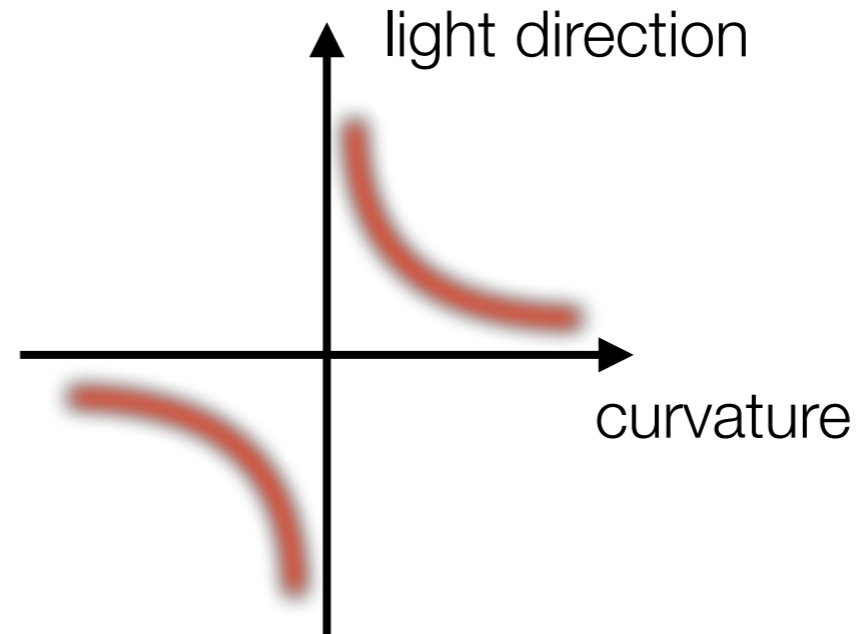
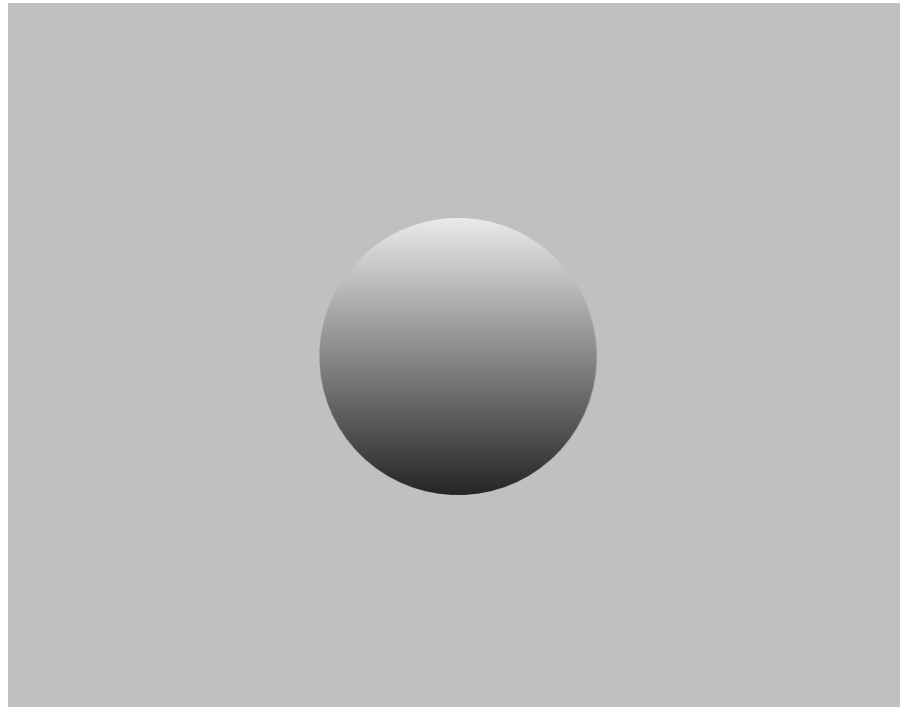
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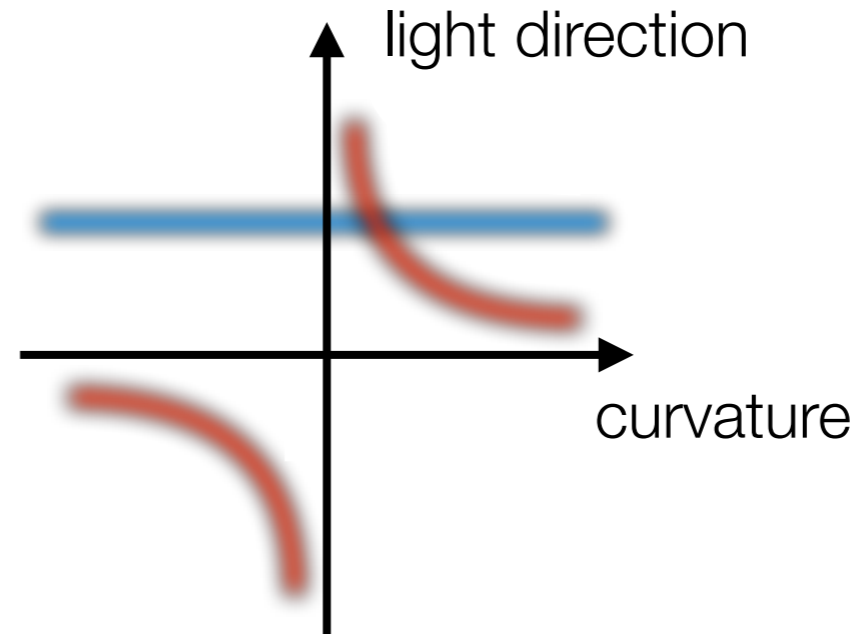
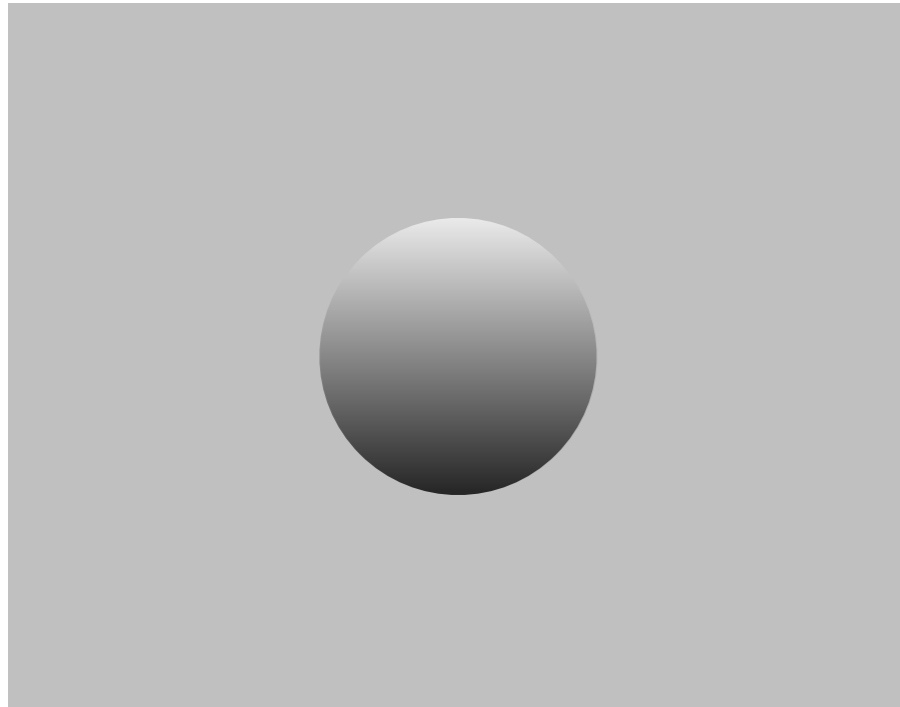
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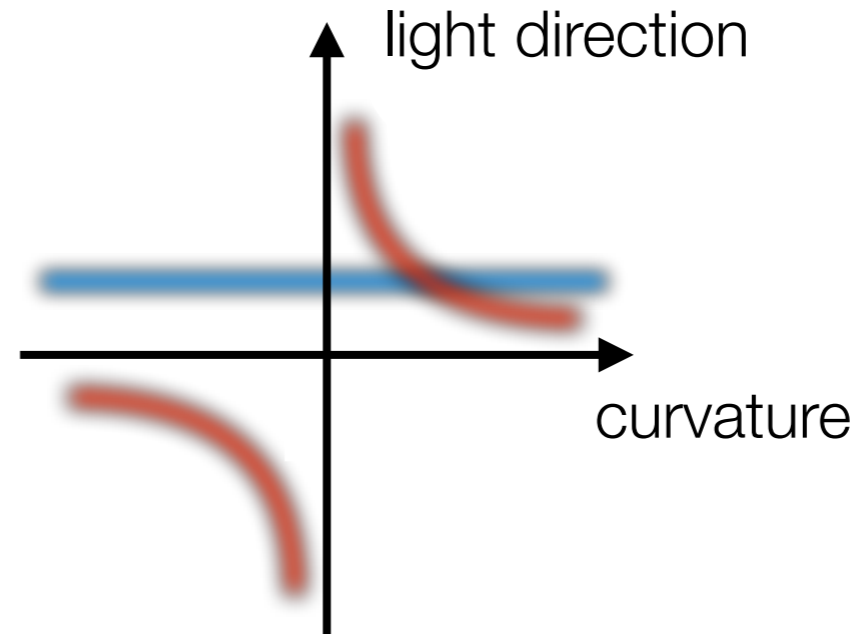
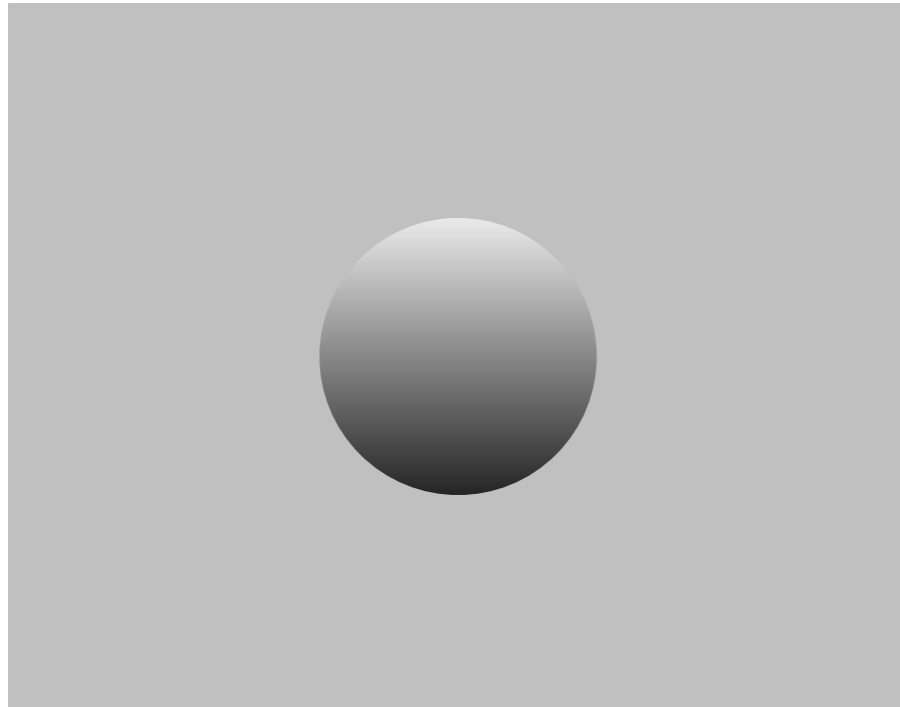
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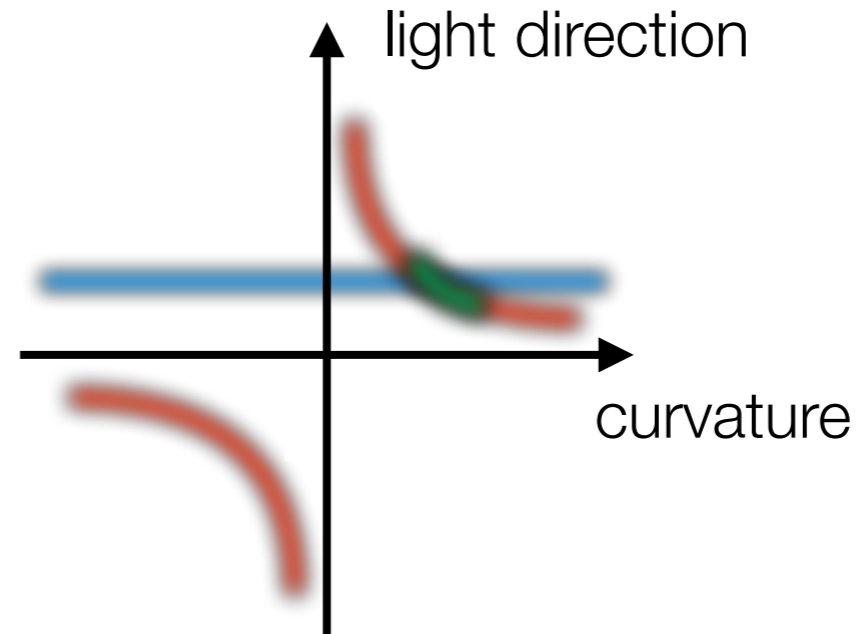
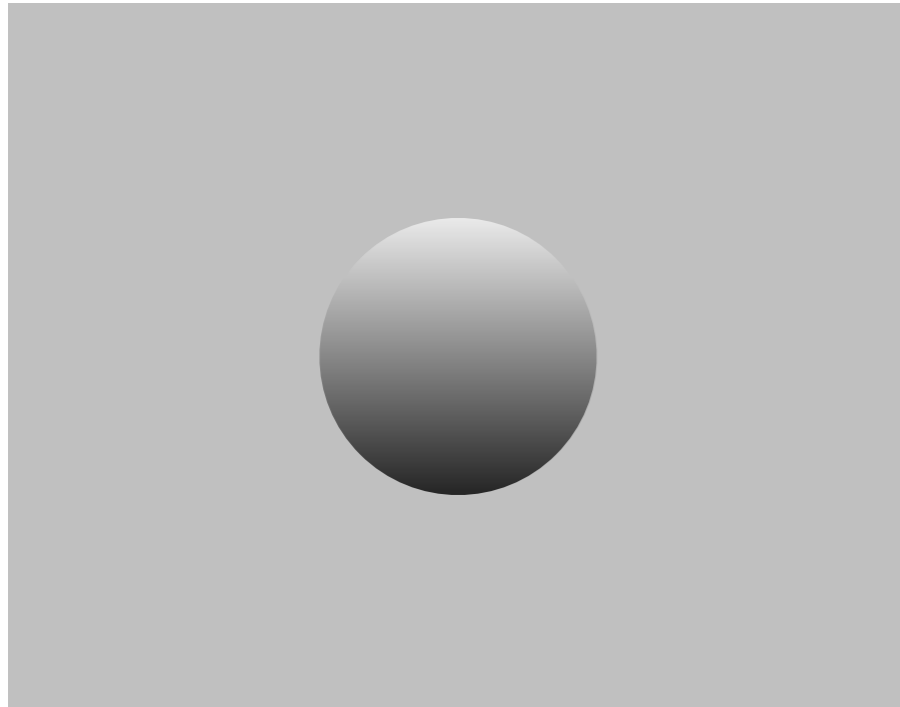
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Bayes inferencia

Miért érdekes a poszterior eloszlás?

Bayes inferencia

Miért érdekes a poszterior eloszlás?

stimulus

perception

action

Bayes inferencia

Miért érdekes a poszterior eloszlás?



Bayes inferencia

Miért érdekes a poszterior eloszlás?

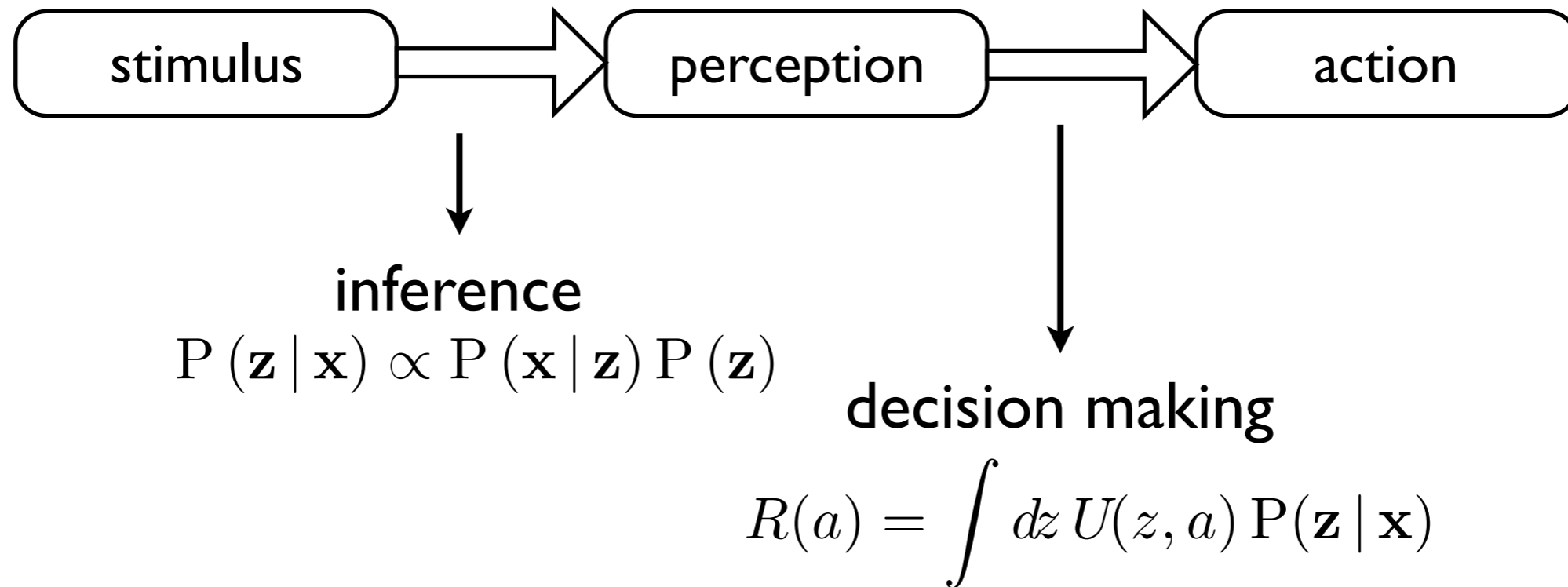


inference

$$P(\mathbf{z} | \mathbf{x}) \propto P(\mathbf{x} | \mathbf{z}) P(\mathbf{z})$$

Bayes inferencia

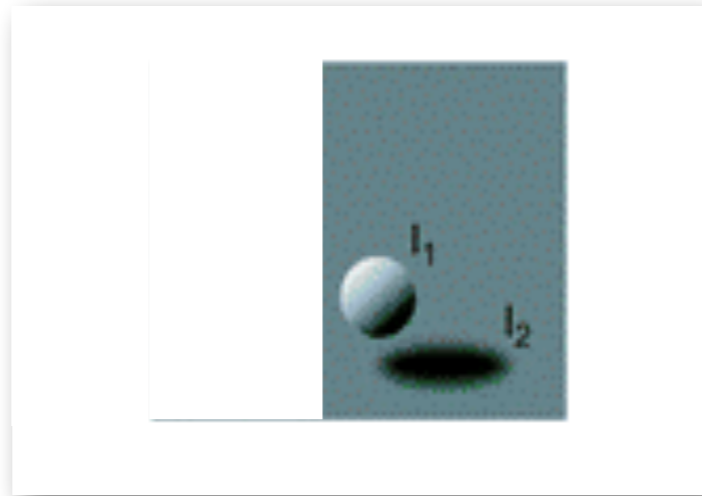
Miért érdekes a poszterior eloszlás?



Egyszerű generatív modellek

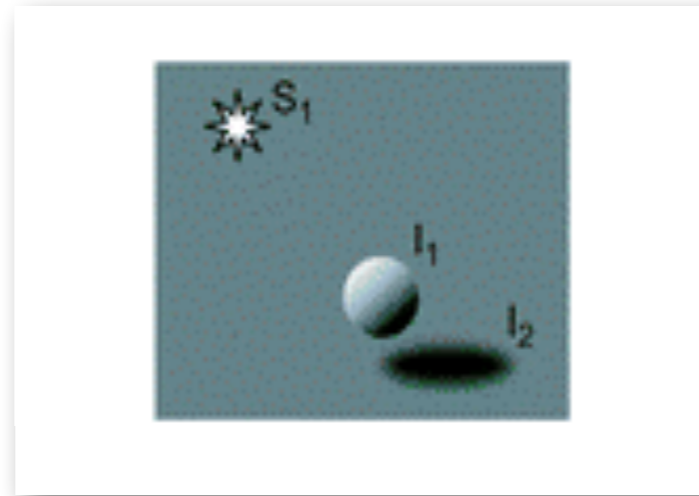
Kersten et al, Annu Rev Psychol (2004)

Egyszerű generatív modellek



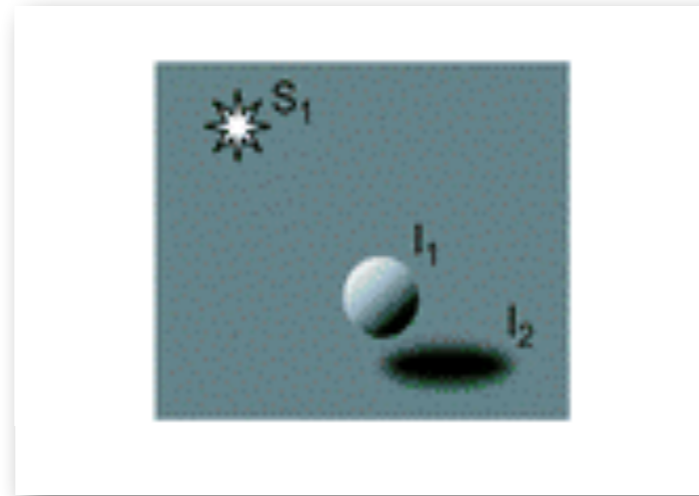
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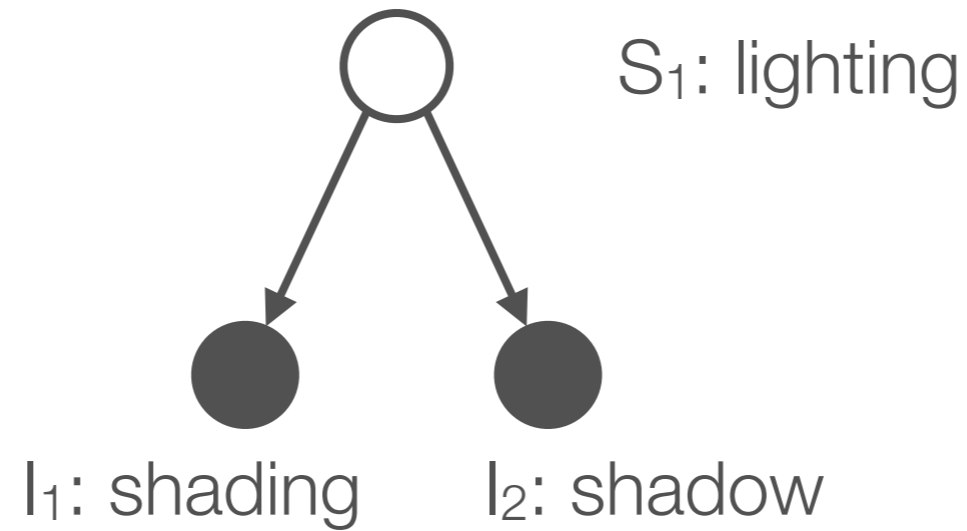


Kersten et al, Annu Rev Psychol (2004)

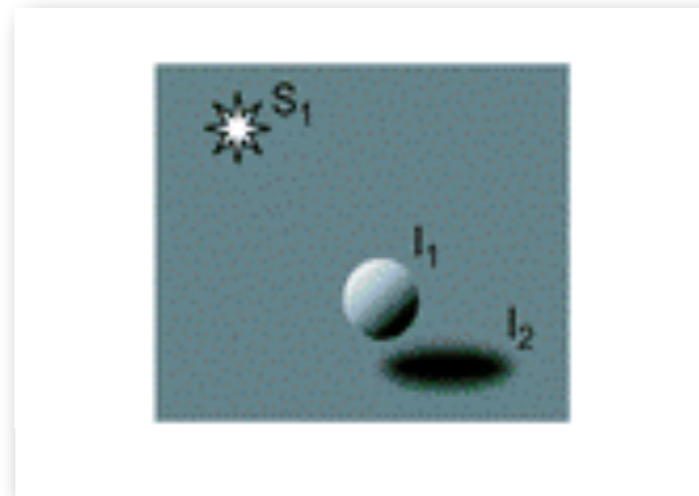
Egyszerű generatív modellek



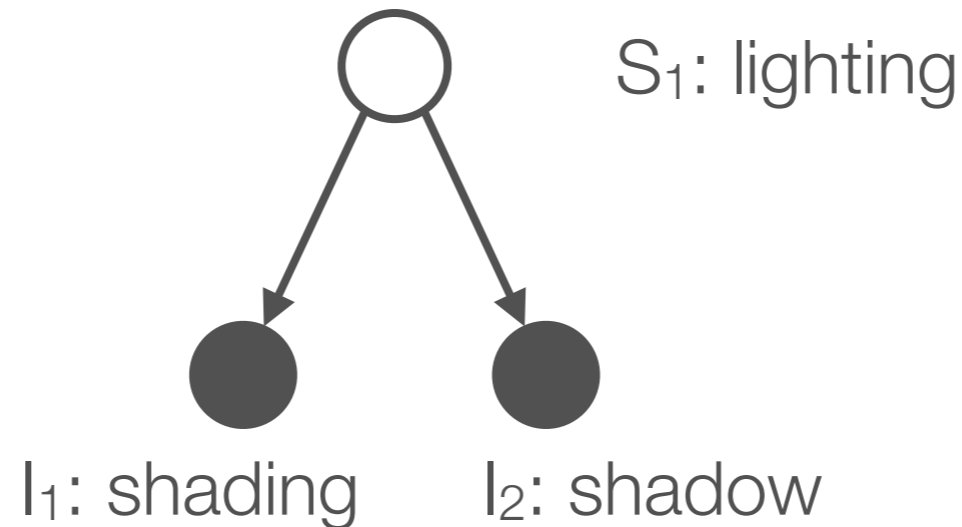
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Egyszerű generatív modellek



Kersten et al, Annu Rev Psychol (2004)

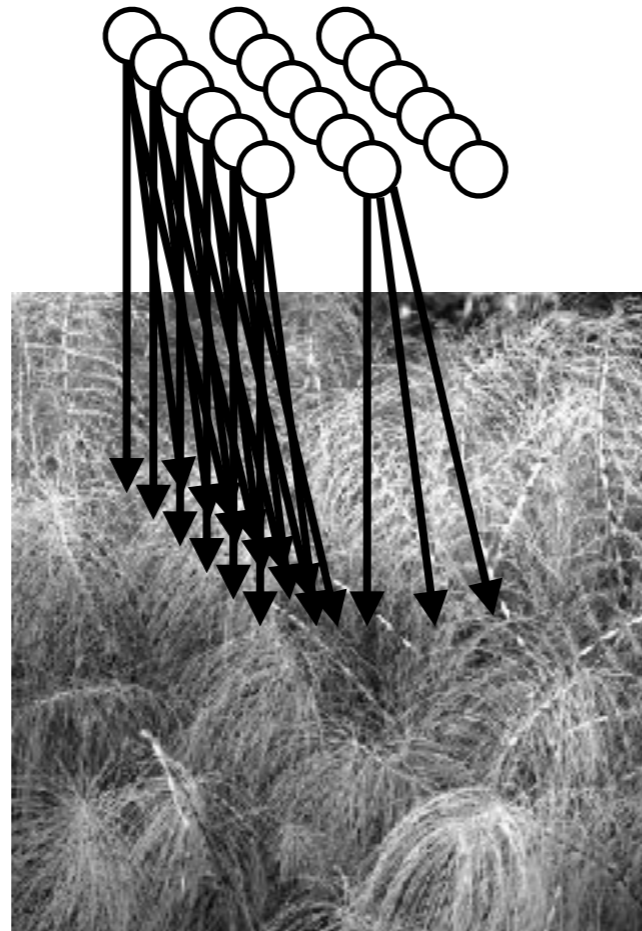


- Inferencia: feltételezve egy modellt, a változók értékét következtetni
- Tanulás: a modell paramétereit kikövetkeztetni a stimulusok statisztikájából

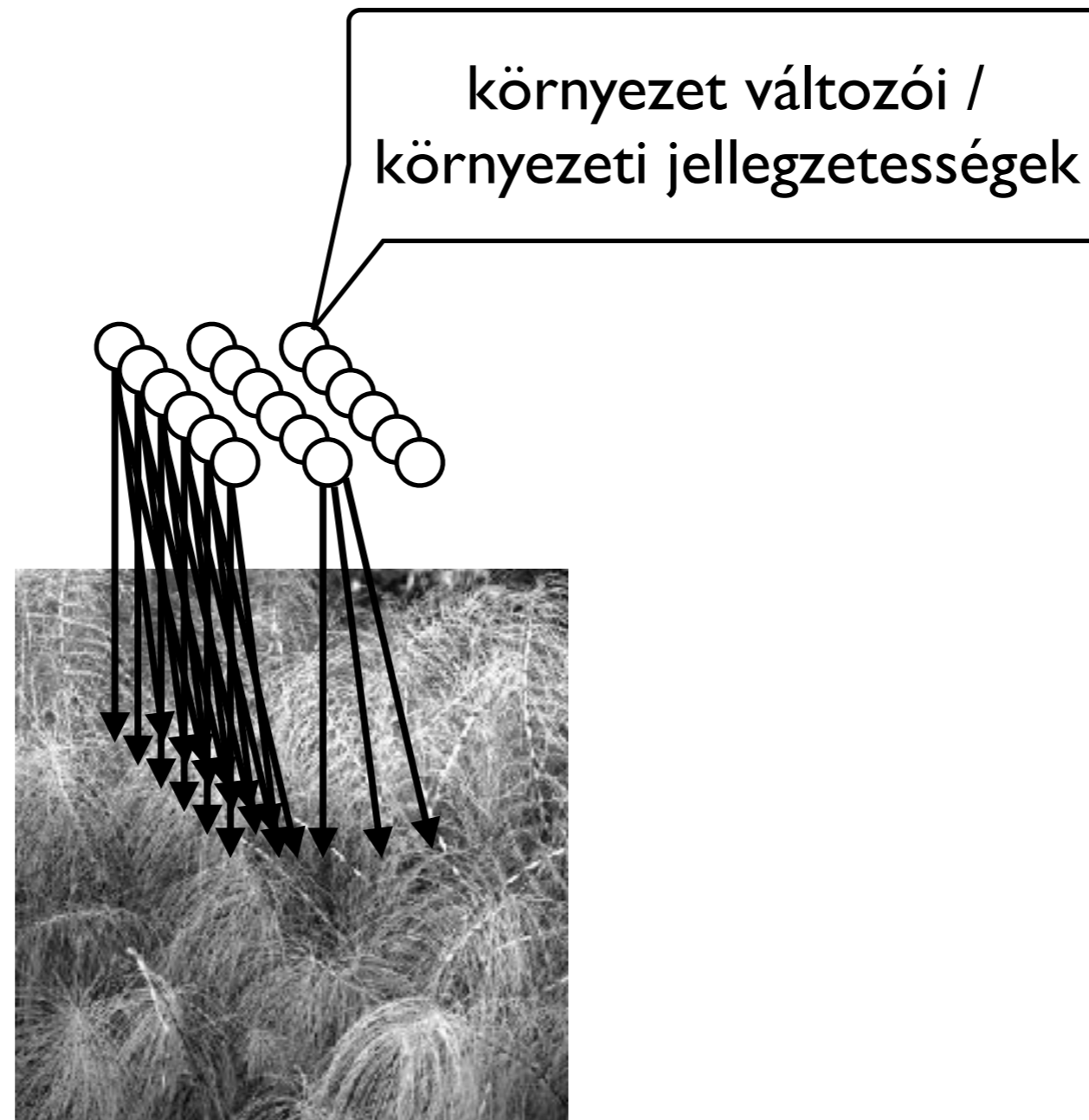
Generative model of natural images



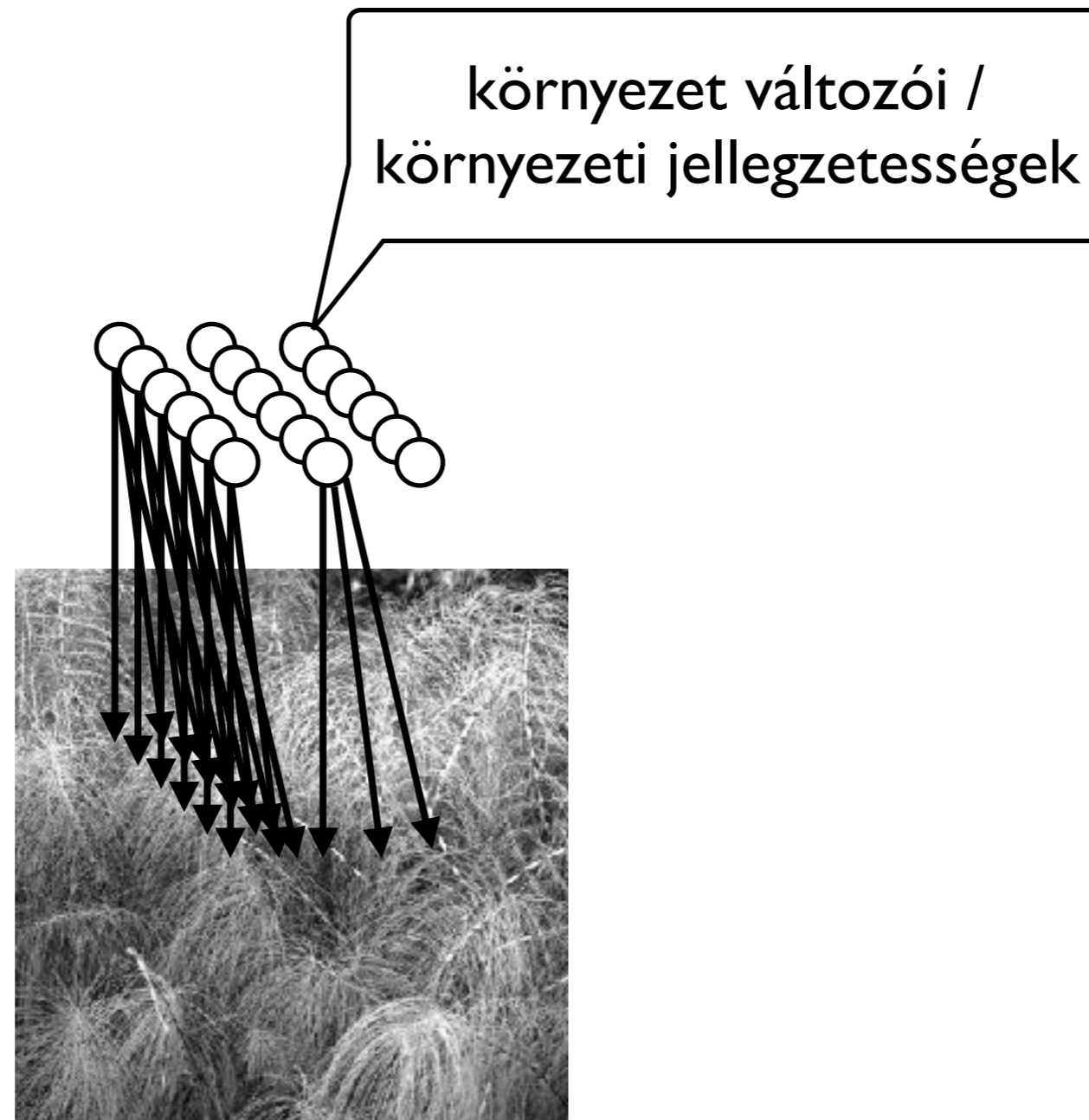
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Generative model of natural images

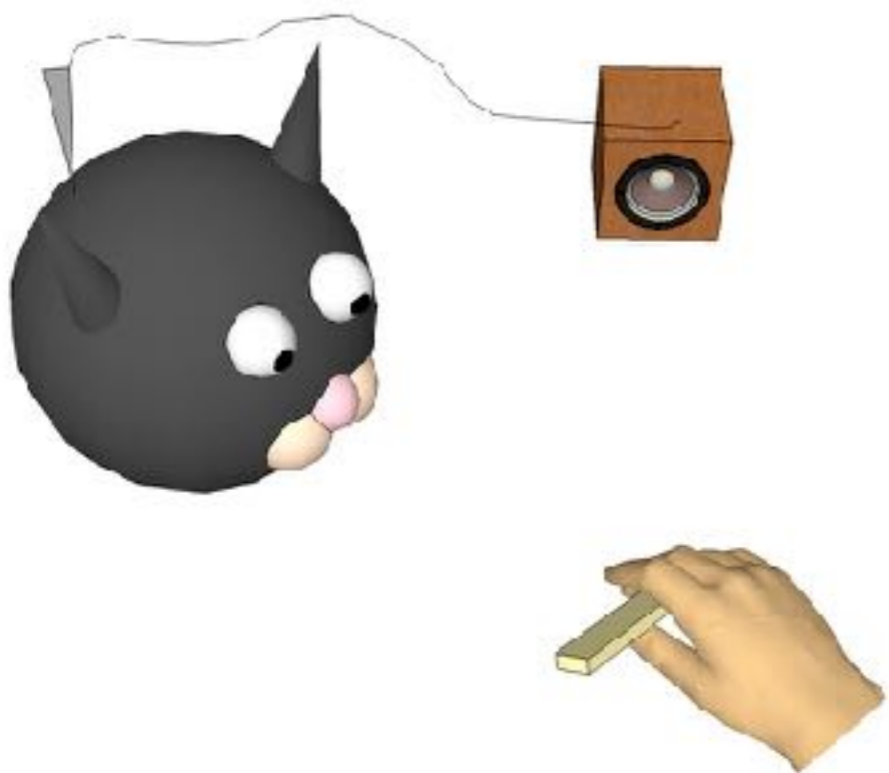


- Mit mondhatunk az agy által használt modellről?

Neurális válaszok



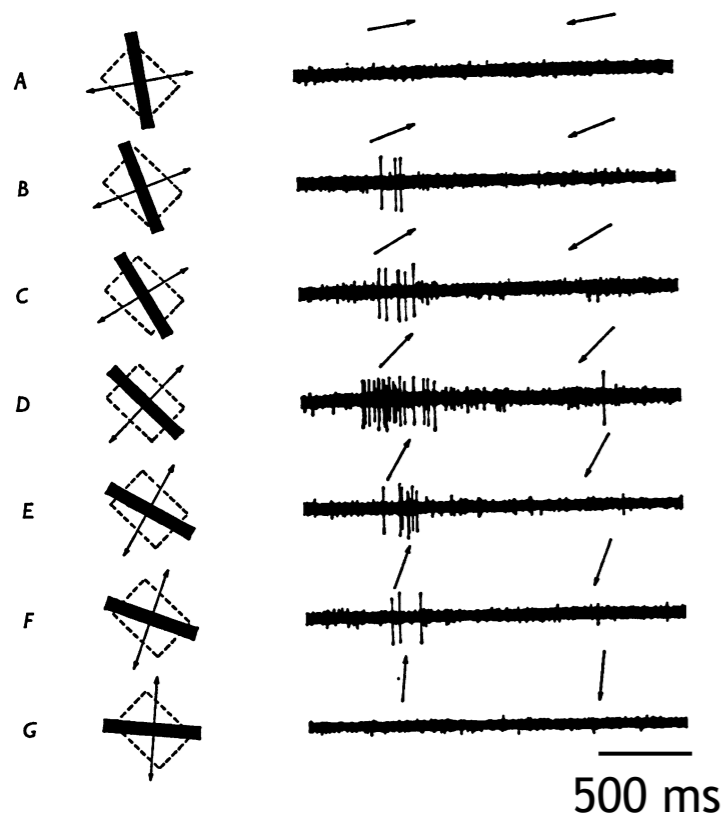
Neurális válaszok



Simple Cell

Neurális válaszok

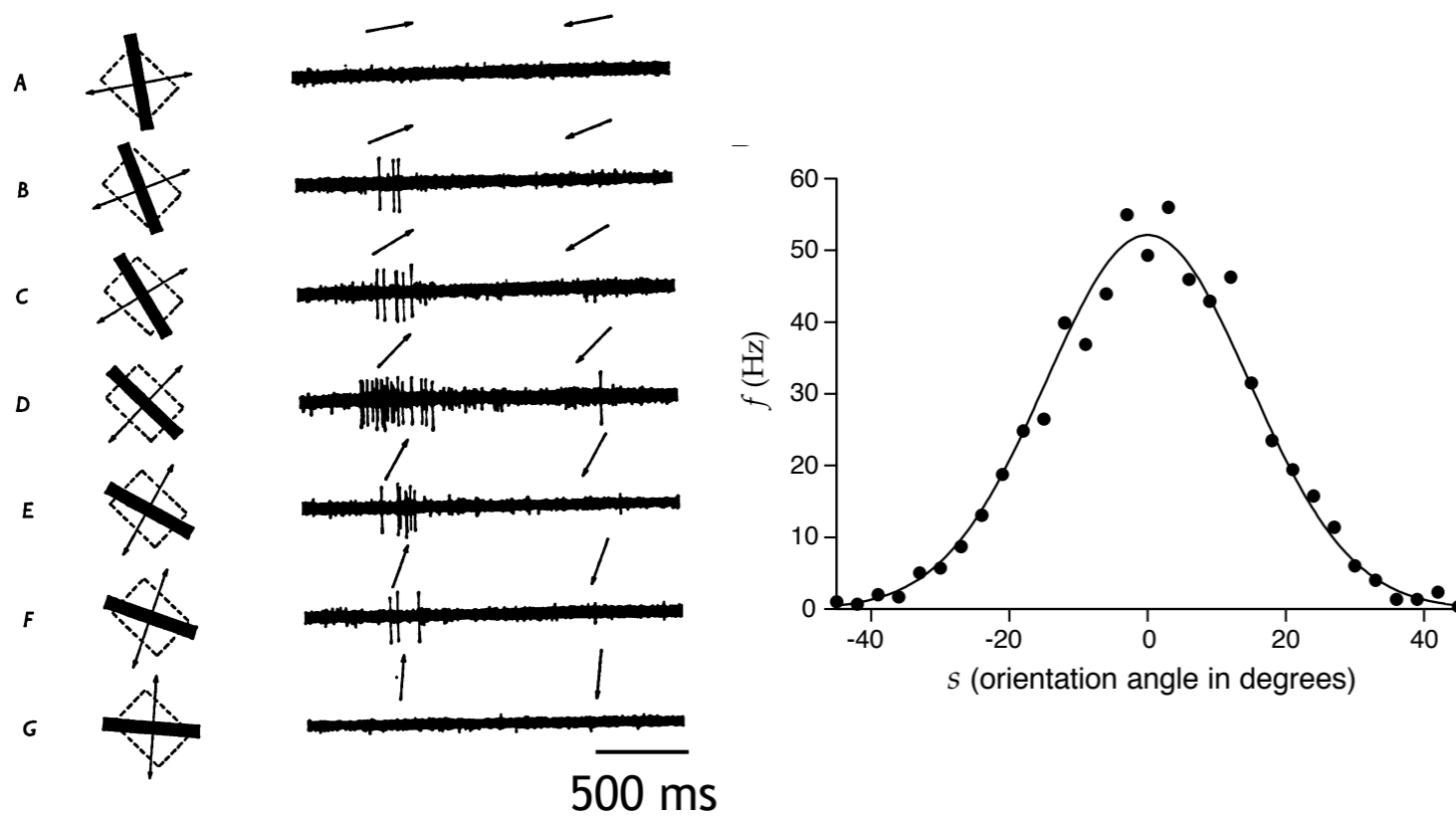
V1 characteristic response



Hubel & Wiesel, J Physiol 1968

Neurális válaszok

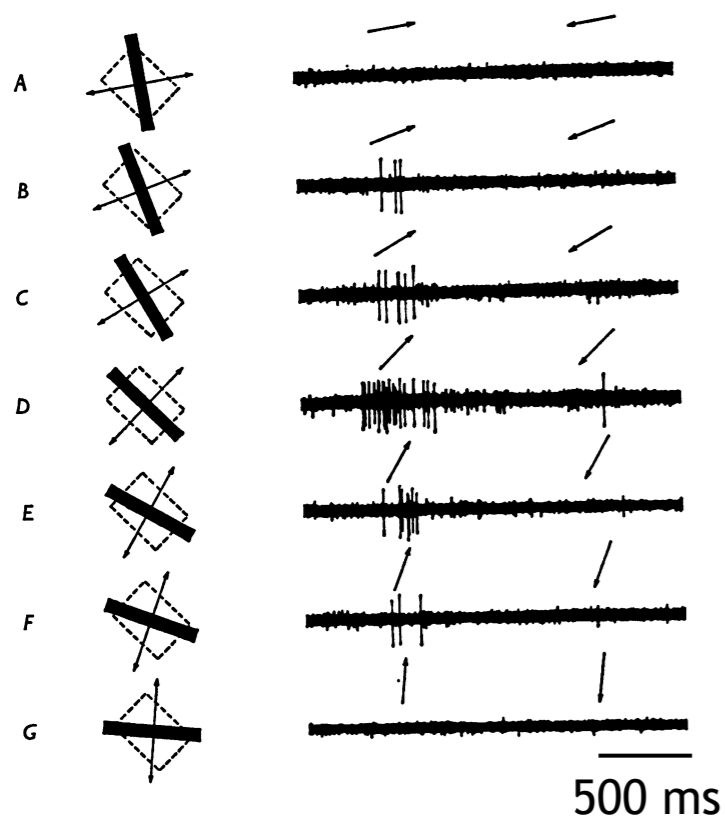
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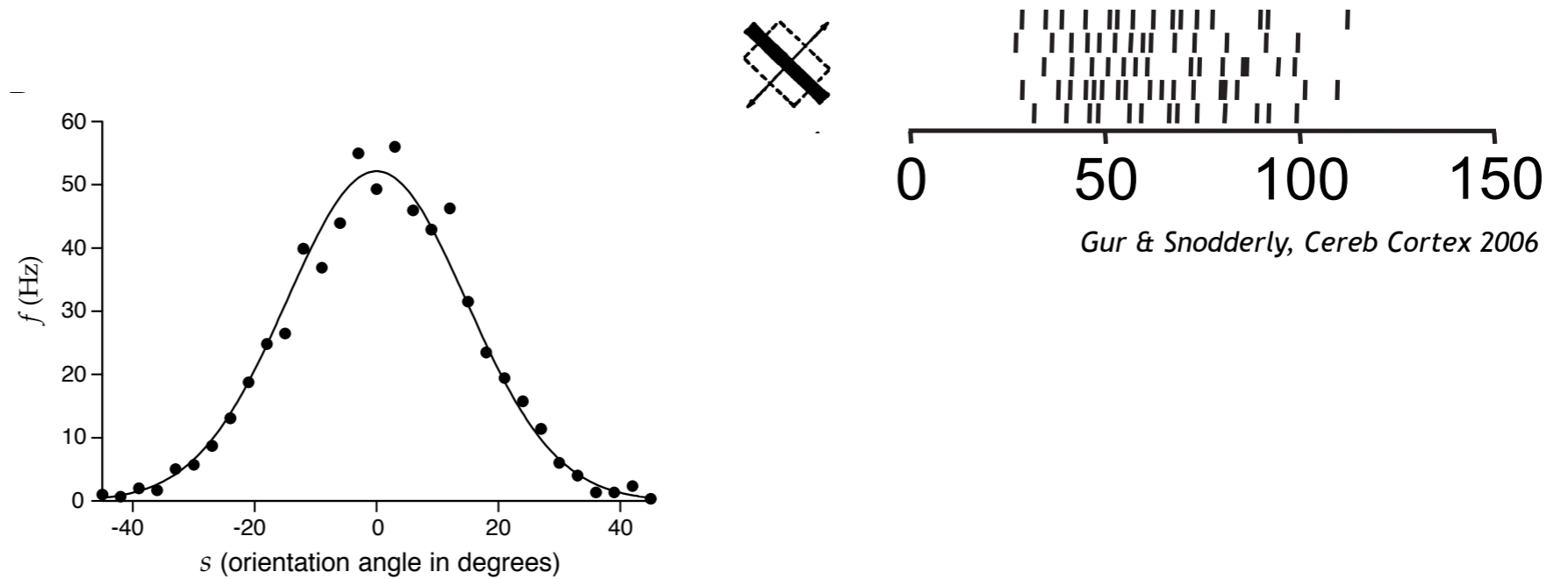
Neurális válaszok

V1 characteristic response



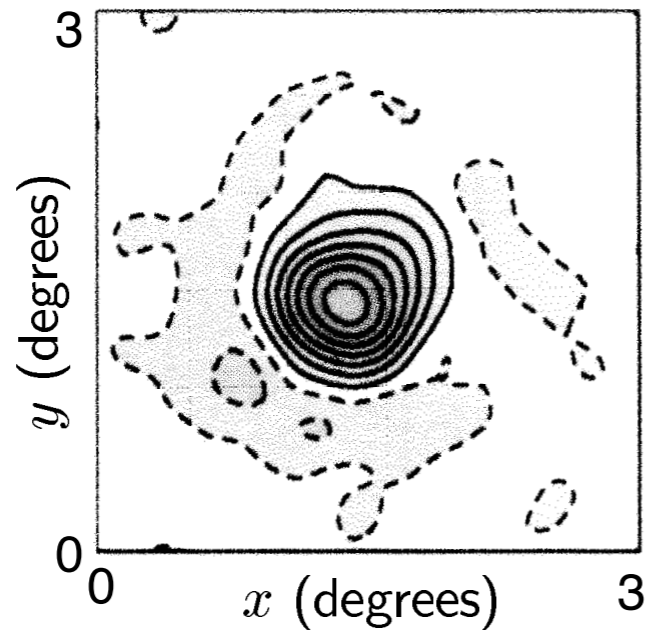
Hubel & Wiesel, *J Physiol* 1968

V1 spike train variability

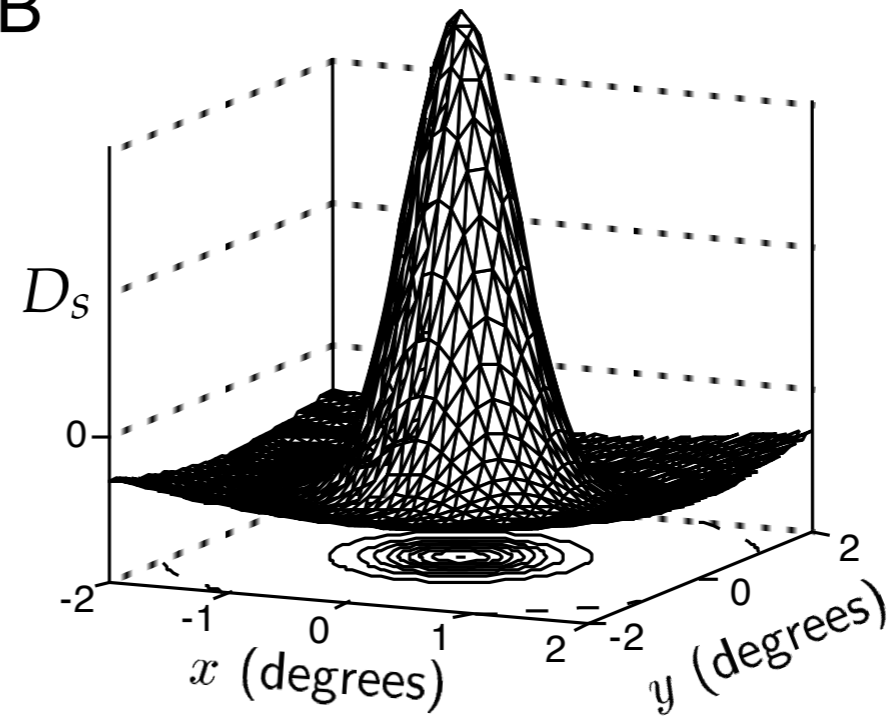


Receptív mező

A

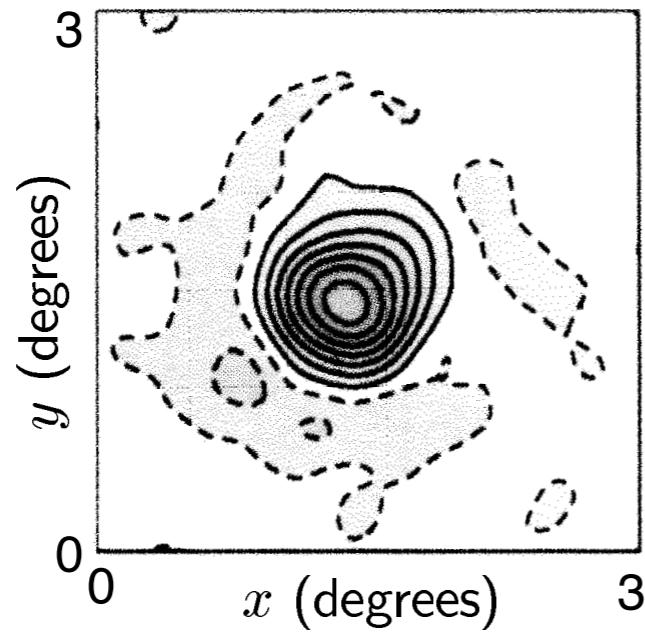


B

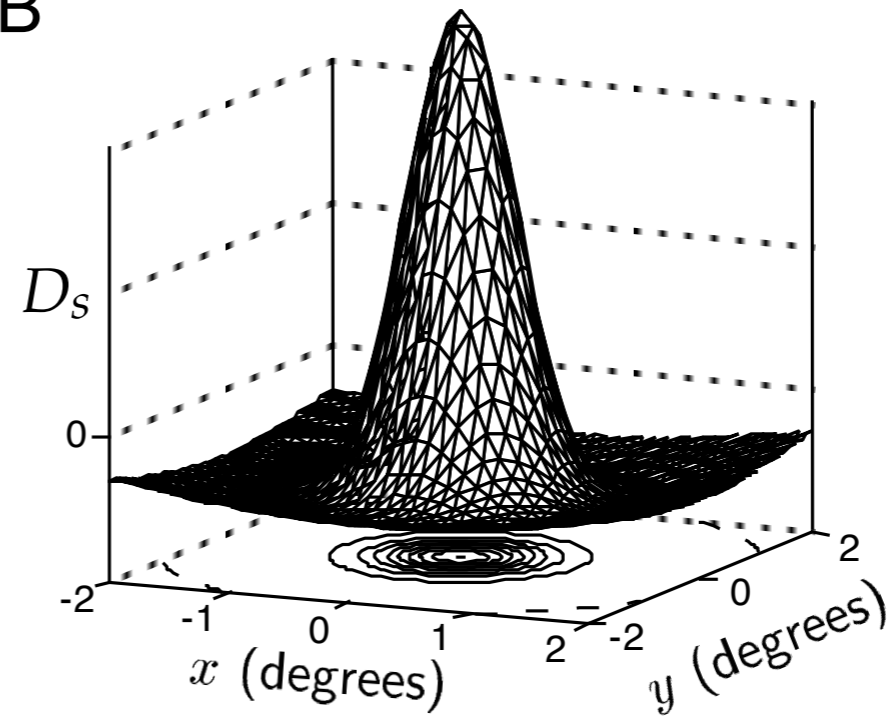


Receptív mező

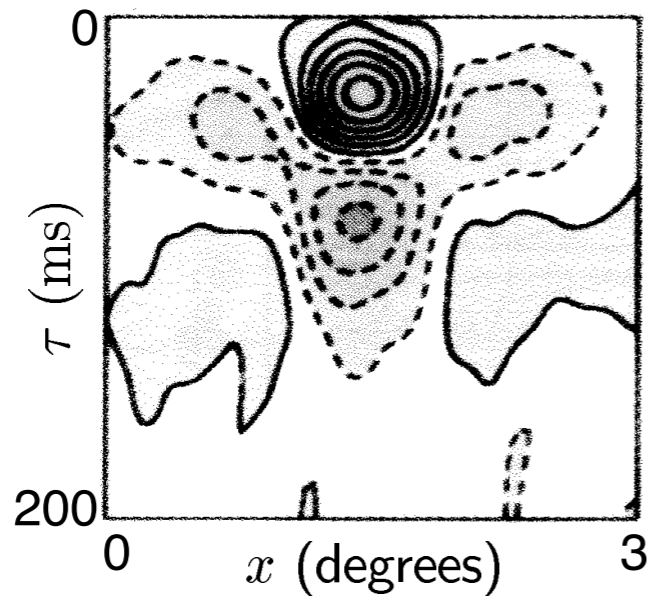
A



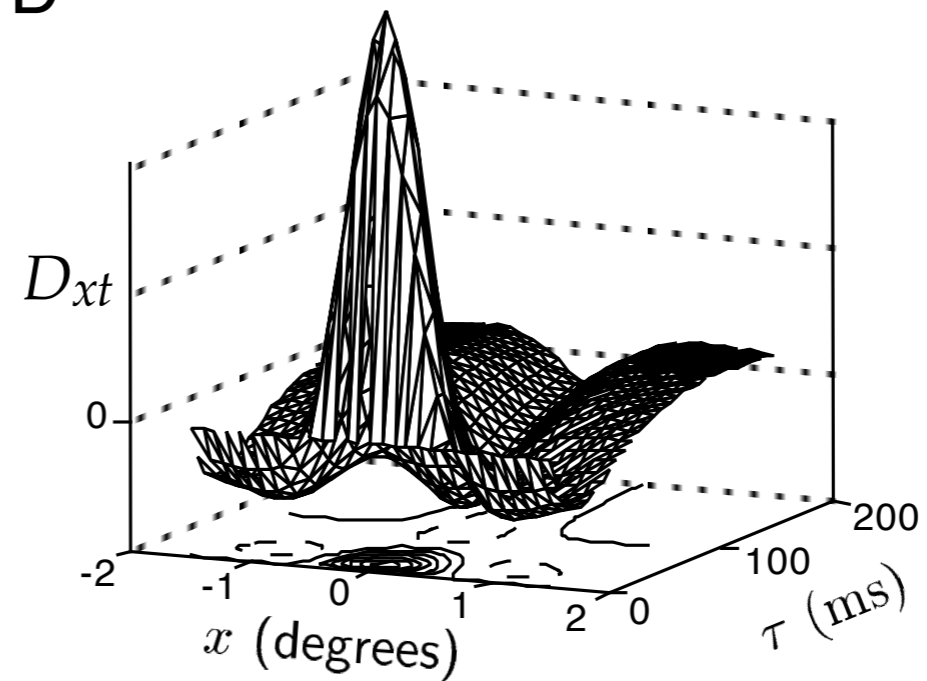
B



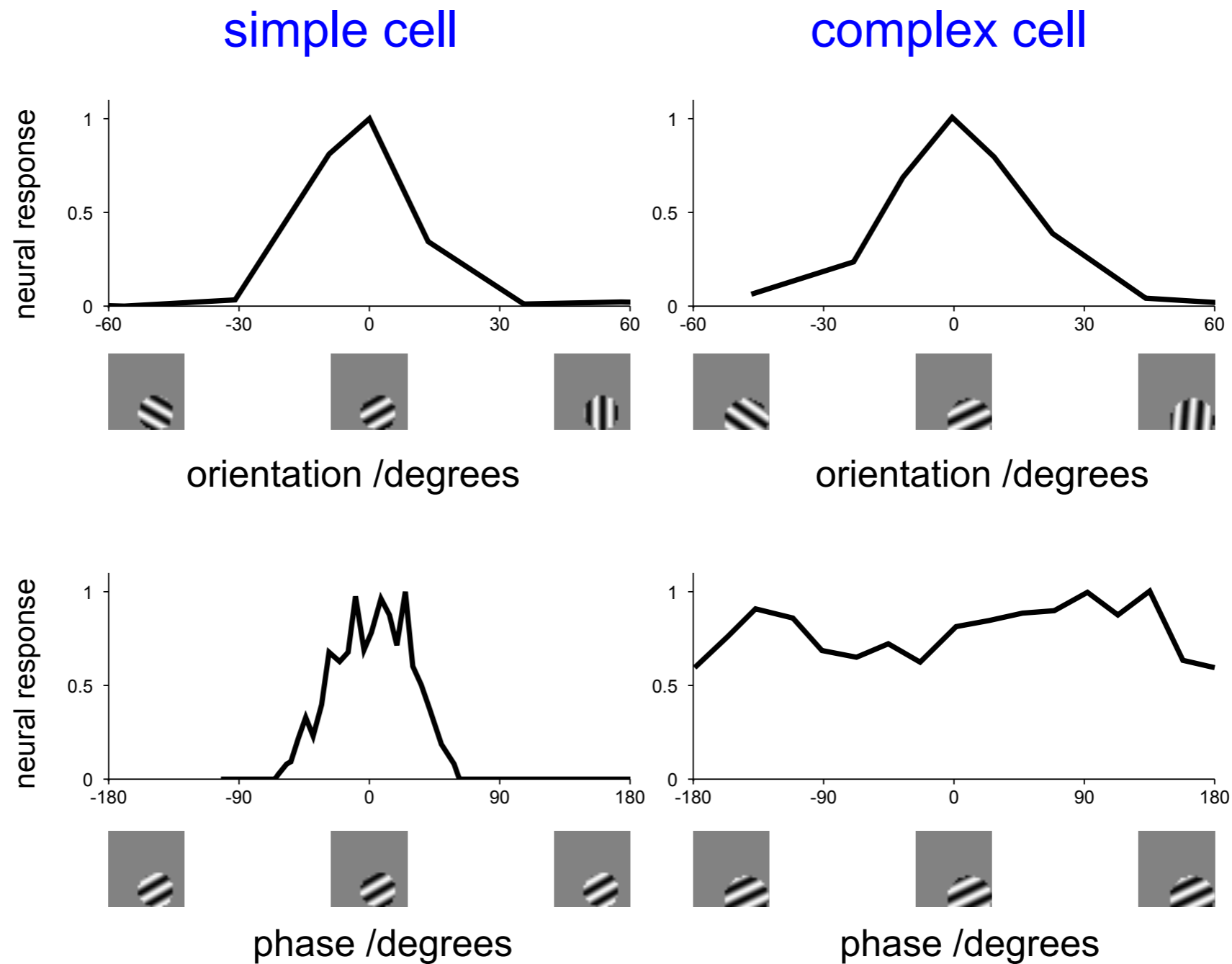
C



D



Receptív mező tulajdonságok

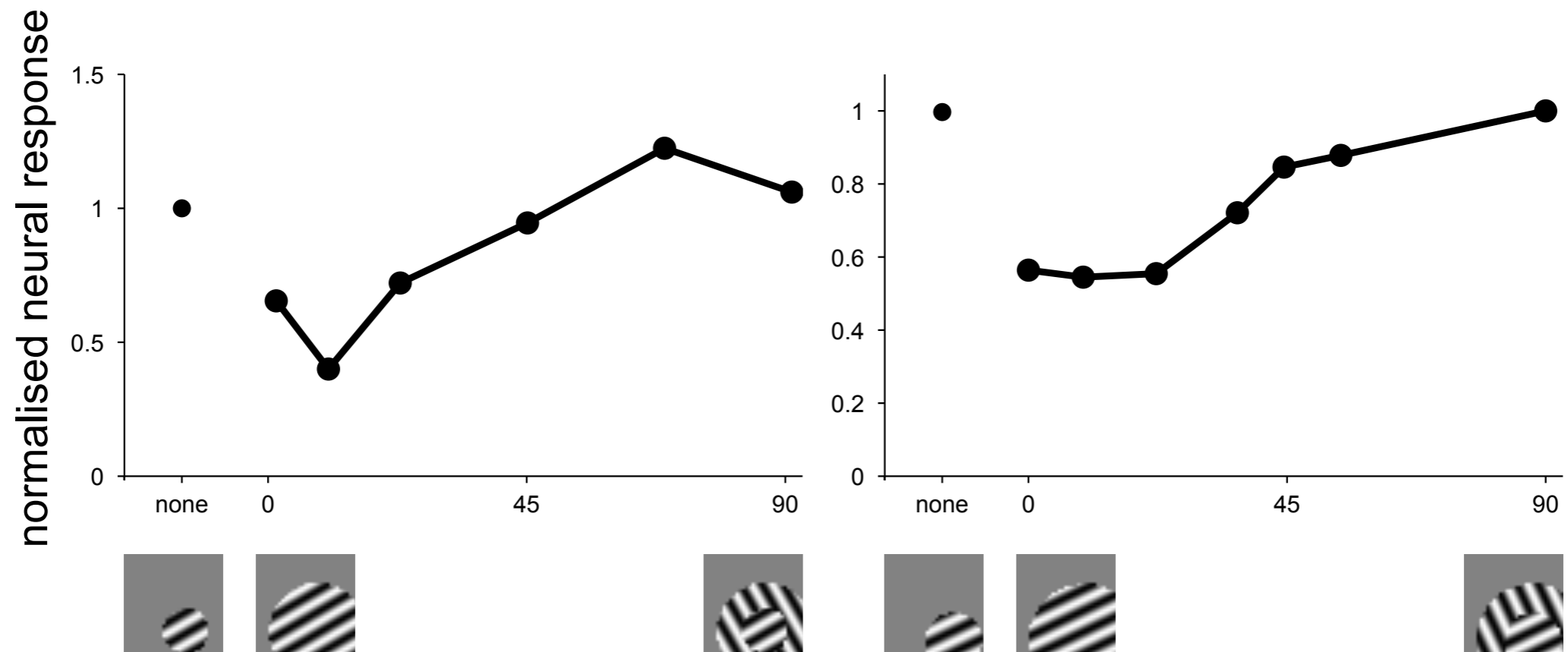


Movshon et al. 1978a, 1978b and Jones et al. 2001

Receptív mező tulajdonságok

simple cell

complex cell



surround orientation /degrees

surround orientation /degrees

Unsupervised learning

Input: x_1, x_2, \dots, x_t összefoglaló néven: adat -
vizuális, auditoros, szöveg

Gól: $P(\mathbf{x})$

Unsupervised learning

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$P(\mathbf{x})$ **Bonyolult!**
Miért is?

Unsupervised learning

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Miért is?

Egyszerűsítés: $P(\mathbf{x}) = P(\mathbf{x} | \mathbf{z}) P(\mathbf{z})$

Unsupervised learning

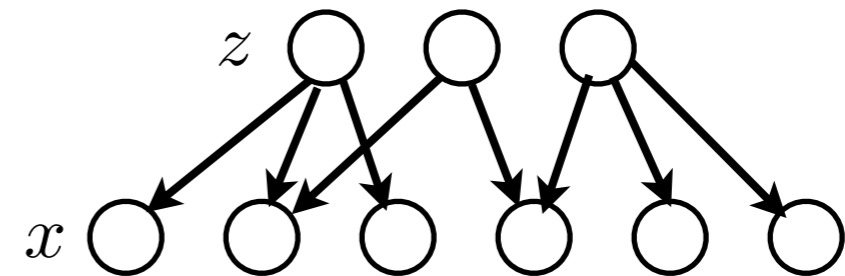
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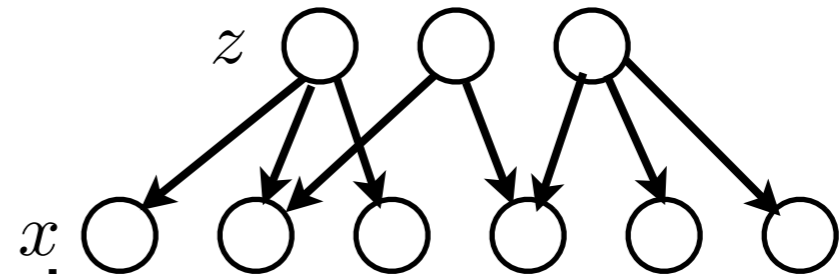
Gól: $P(\mathbf{x})$

$P(\mathbf{x})$ Bonyolult!

Miért is?

Egyszerűsítés: $P(\mathbf{x}) = P(\mathbf{x} | \mathbf{z}) P(\mathbf{z})$

- az adatot a “z”-k terében reprezentáljuk



Unsupervised learning

Input: x_1, x_2, \dots, x_t összefoglaló néven: adat -
vizuális, auditoros, szöveg

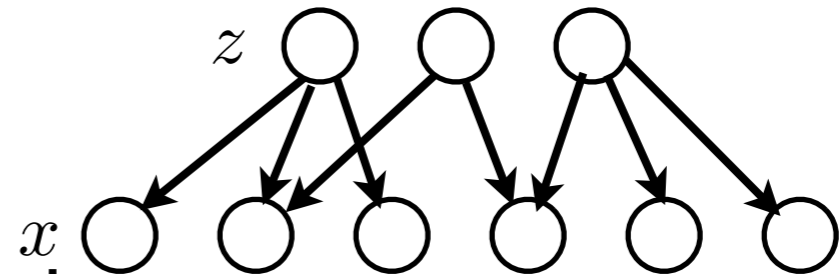
Gól: $P(\mathbf{x})$

$P(\mathbf{x})$ Bonyolult!

Miért is?

Egyszerűsítés: $P(\mathbf{x}) = P(\mathbf{x} | \mathbf{z}) P(\mathbf{z})$

- az adatot a “z”-k terében reprezentáljuk
- kategorizáció, dimenzió redukció



Unsupervised learning

Input: x_1, x_2, \dots, x_t összefoglaló néven: adat -
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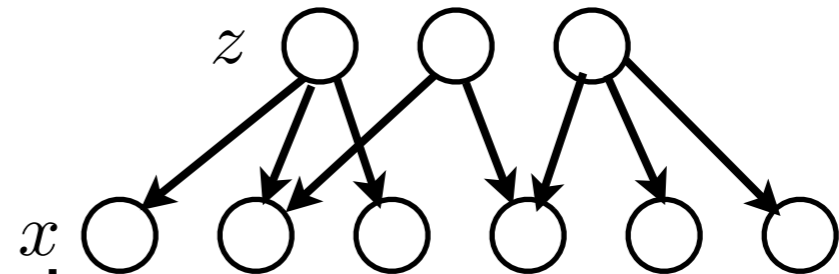
Gól: $P(\mathbf{x})$

$P(\mathbf{x})$ Bonyolult!

Miért is?

Egyszerűsítés: $P(\mathbf{x}) = P(\mathbf{x} | \mathbf{z}) P(\mathbf{z})$

- az adatot a “z”-k terében reprezentáljuk
- kategorizáció, dimenzió redukció
- általánosabban a feladat: predikció, döntéshozatal, kommunikáció



Lineáris modellek

$$P(x | z) = \text{Normal}(x; z, \theta) = C \exp \left((x - Az)^T \Sigma^{-1} (x - Az) \right)$$

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$$x = \mathbf{A} \cdot z + \epsilon$$

Lineáris modellek

$$P(x | z) = \text{Normal}(x; z, \theta) = C \exp\left(-\frac{1}{2}(x - Az)^T \Sigma^{-1} (x - Az)\right)$$

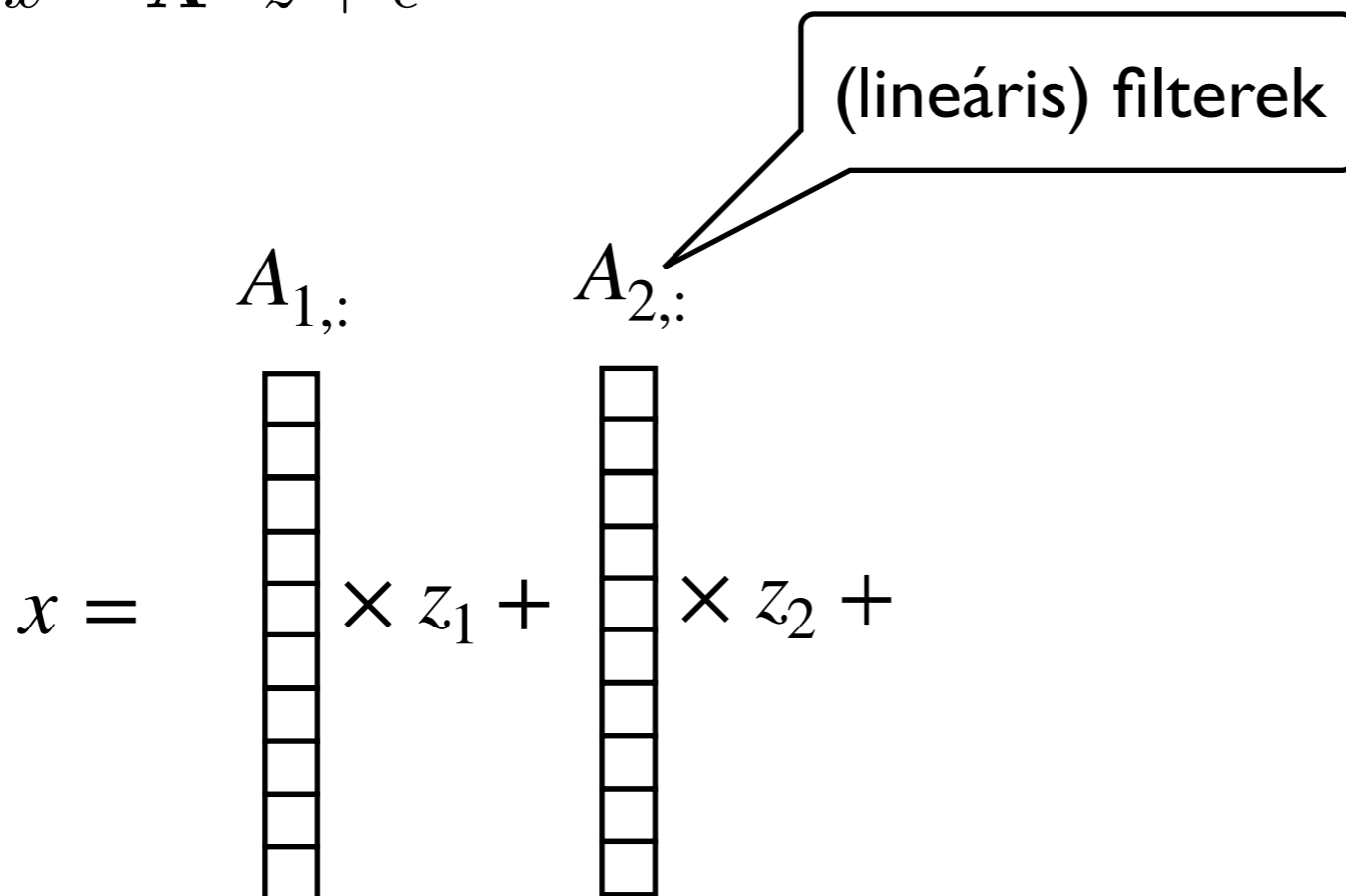
$$x = \mathbf{A} \cdot z + \epsilon$$

$$x = \begin{matrix} A_{1,:} \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \end{matrix} \times z_1 + \begin{matrix} A_{2,:} \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \end{matrix} \times z_2 +$$

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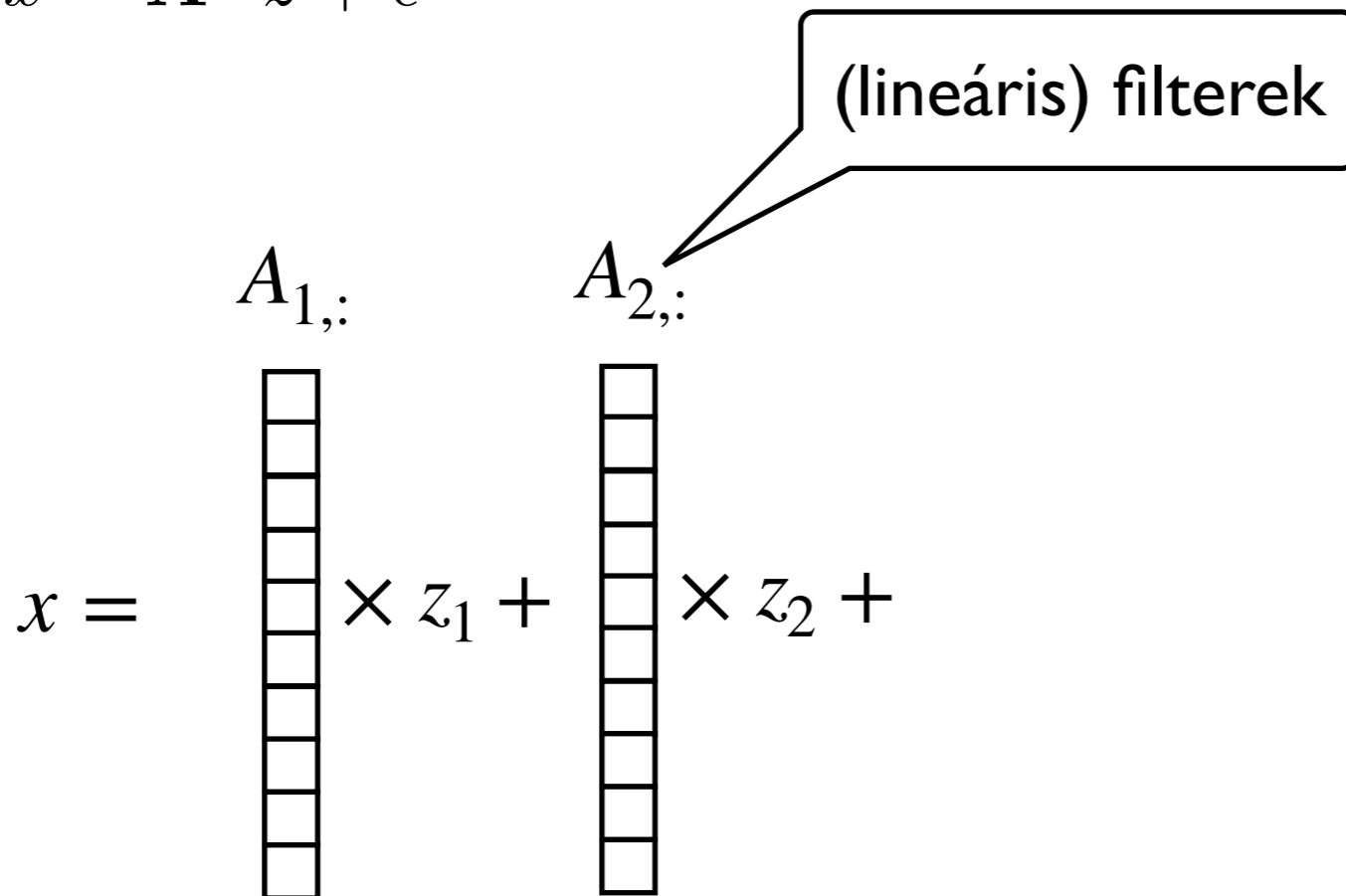
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$$x = \mathbf{A} \cdot z + \epsilon$$



PCA

- $\epsilon \rightarrow 0$
- \mathbf{A} oszlopvektorai ortogonálisak
- $D(\mathbf{x}) = D(\mathbf{z})$

Lineáris modellek

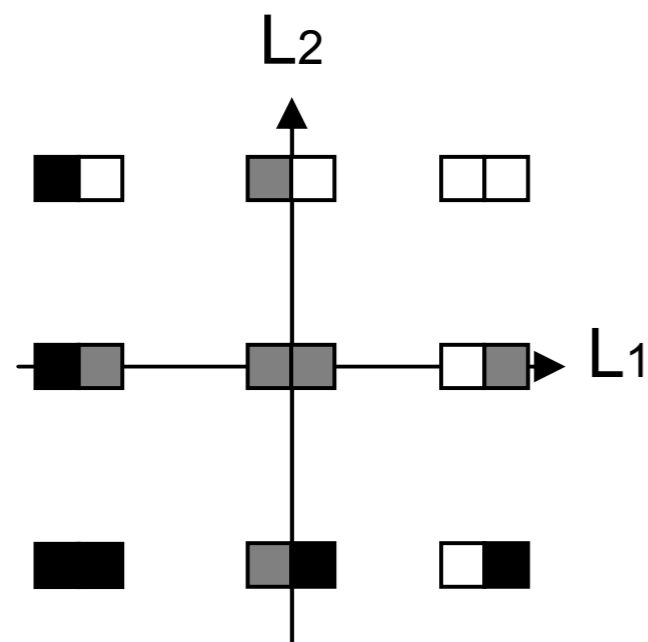
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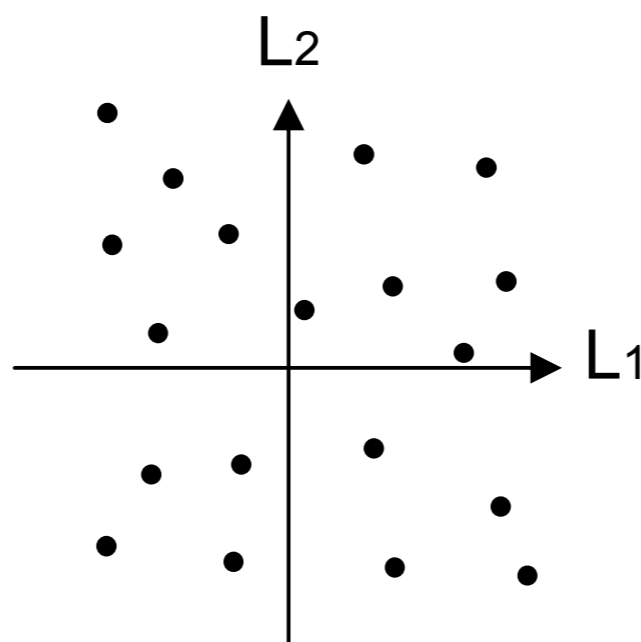
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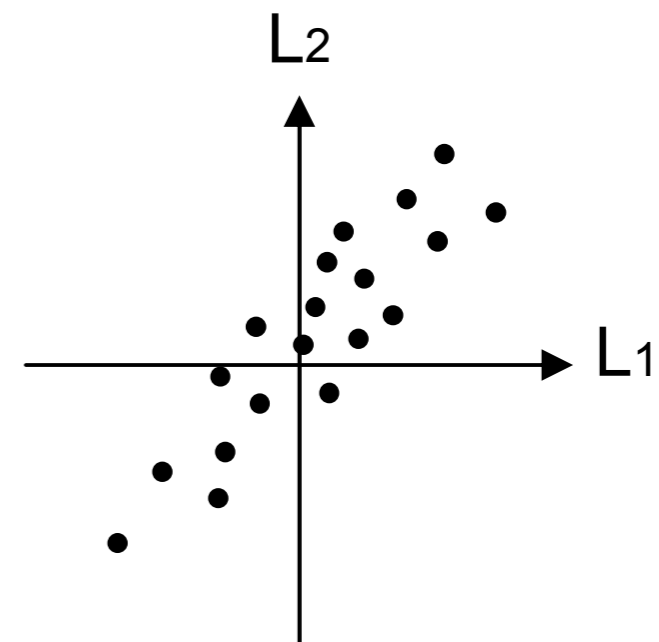
$$x = \mathbf{A} \cdot z + \epsilon$$



State space of two pixel images



Random images



Structured images

Principal component analysis

- tömörítési célfüggvény: találjuk meg azokat az irányokat az adatban, melyek mentén az adatban a legnagyobb variancia van

Principal component analysis

- tömörítési célfüggvény: találjuk meg azokat az irányokat az adatban, melyek mentén az adatban a legnagyobb variancia van

$$1/N \sum_{n=1}^N (A_1^T x_n - A_1^T \bar{x})^2 = A_1^T S A_1$$

Principal component analysis

- tömörítési célfüggvény: találjuk meg azokat az irányokat az adatban, melyek mentén az adatban a legnagyobb variancia van

$$\frac{1}{N} \sum_{n=1}^N (A_1^T x_n - A_1^T \bar{x})^2 = A_1^T S A_1$$

$$S = \frac{1}{N} \sum_{n=1}^N (x_n - \bar{x})(x_n - \bar{x})^T$$

Principal component analysis

- tömörítési célfüggvény: találjuk meg azokat az irányokat az adatban, melyek mentén az adatban a legnagyobb variancia van

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adat kovariancia struktúrája

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Principal component analysis

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Optimalizálás egy peremfeltétellel: Lagrange multiplikátor

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Kielégíthető akkor, ha A_1 S sajátvektora

Lineáris modellek

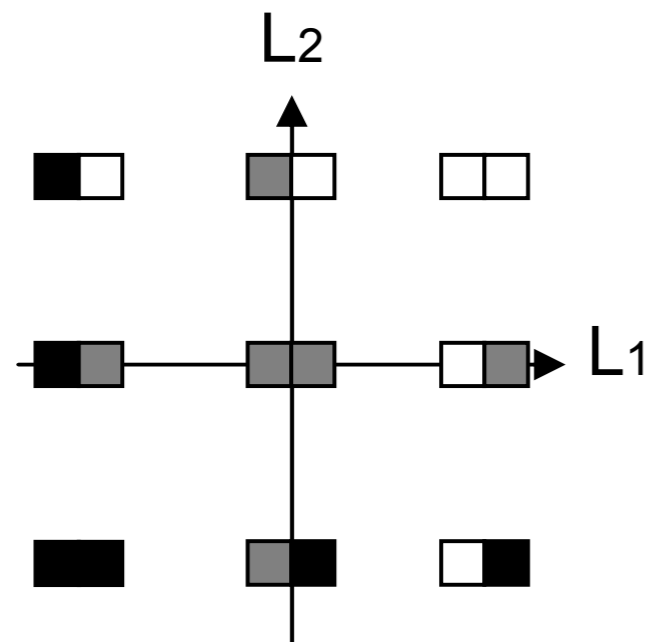
$$P(x | z) = \text{Normal}(x; z, \theta) = C \exp\left(-\frac{1}{2}(x - Az)^T \Sigma^{-1} (x - Az)\right)$$

$$x = \mathbf{A} \cdot z + \epsilon$$

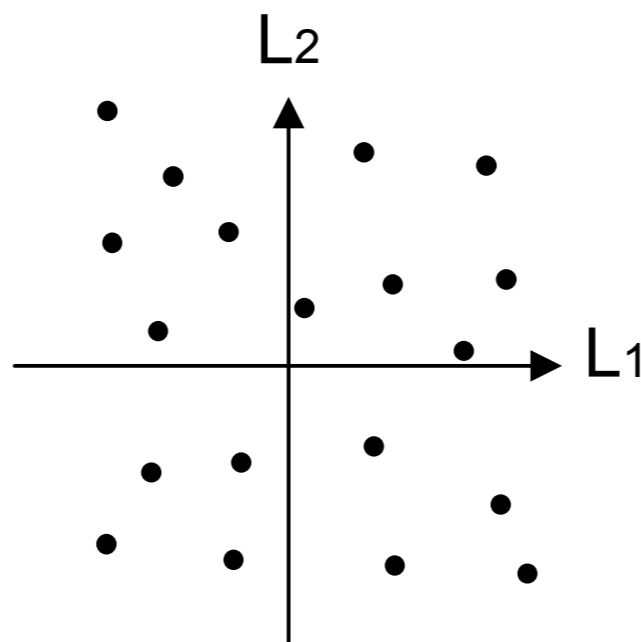
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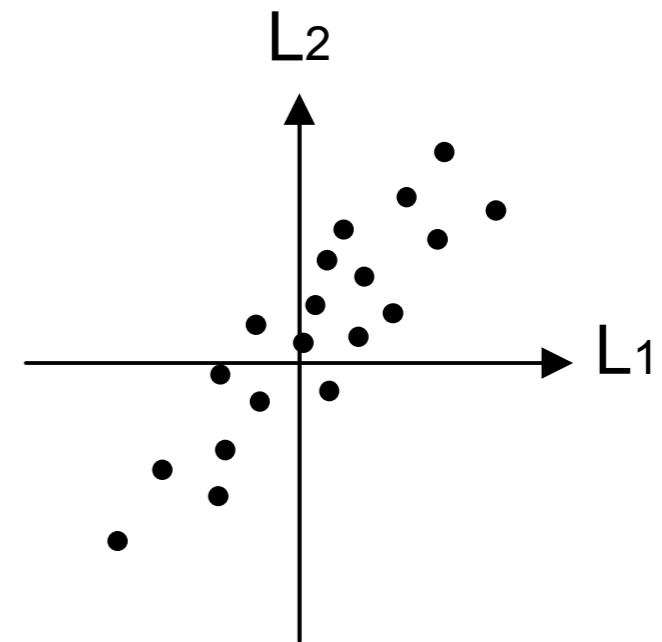
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State space of two pixel images



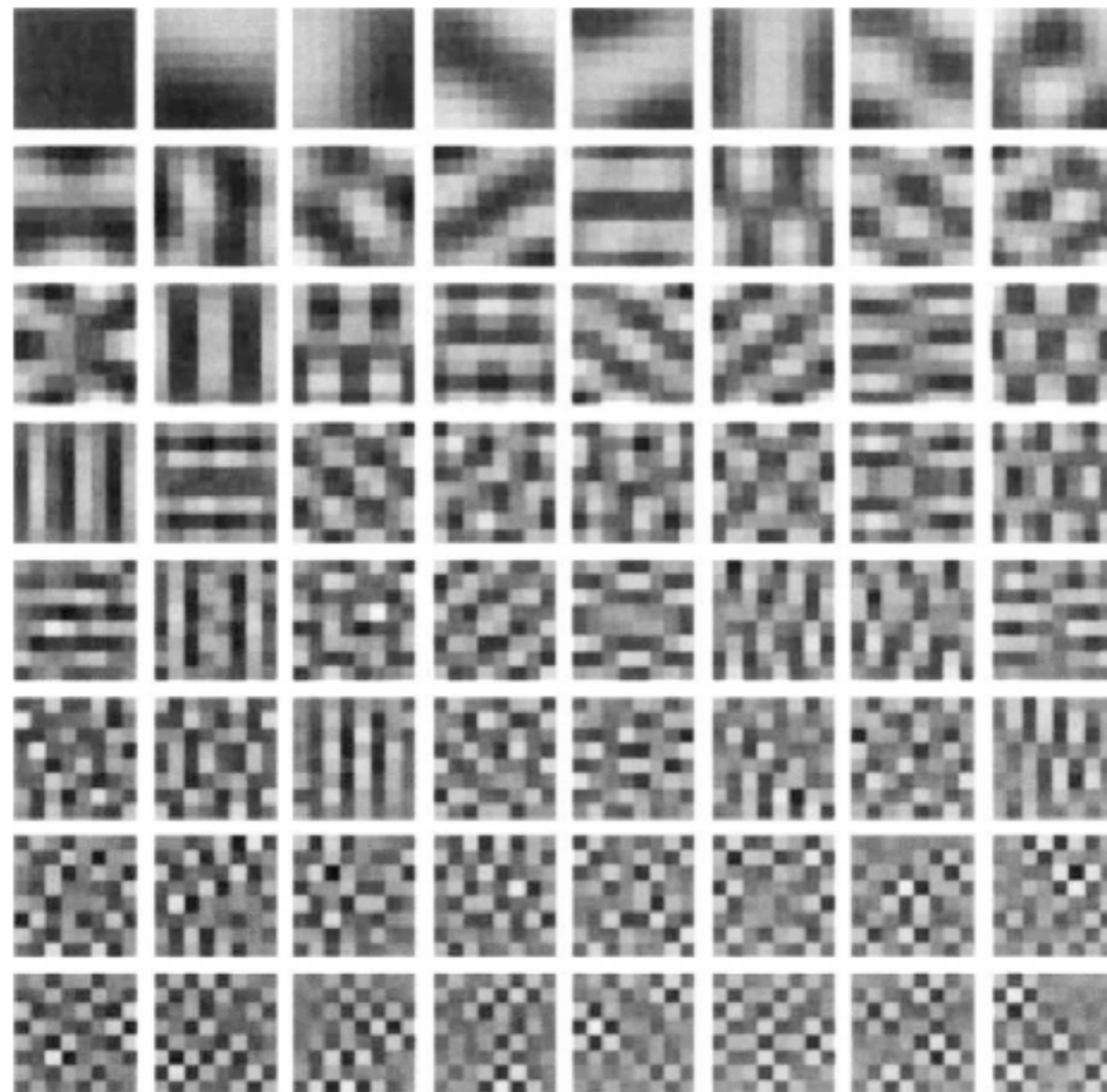
Random images



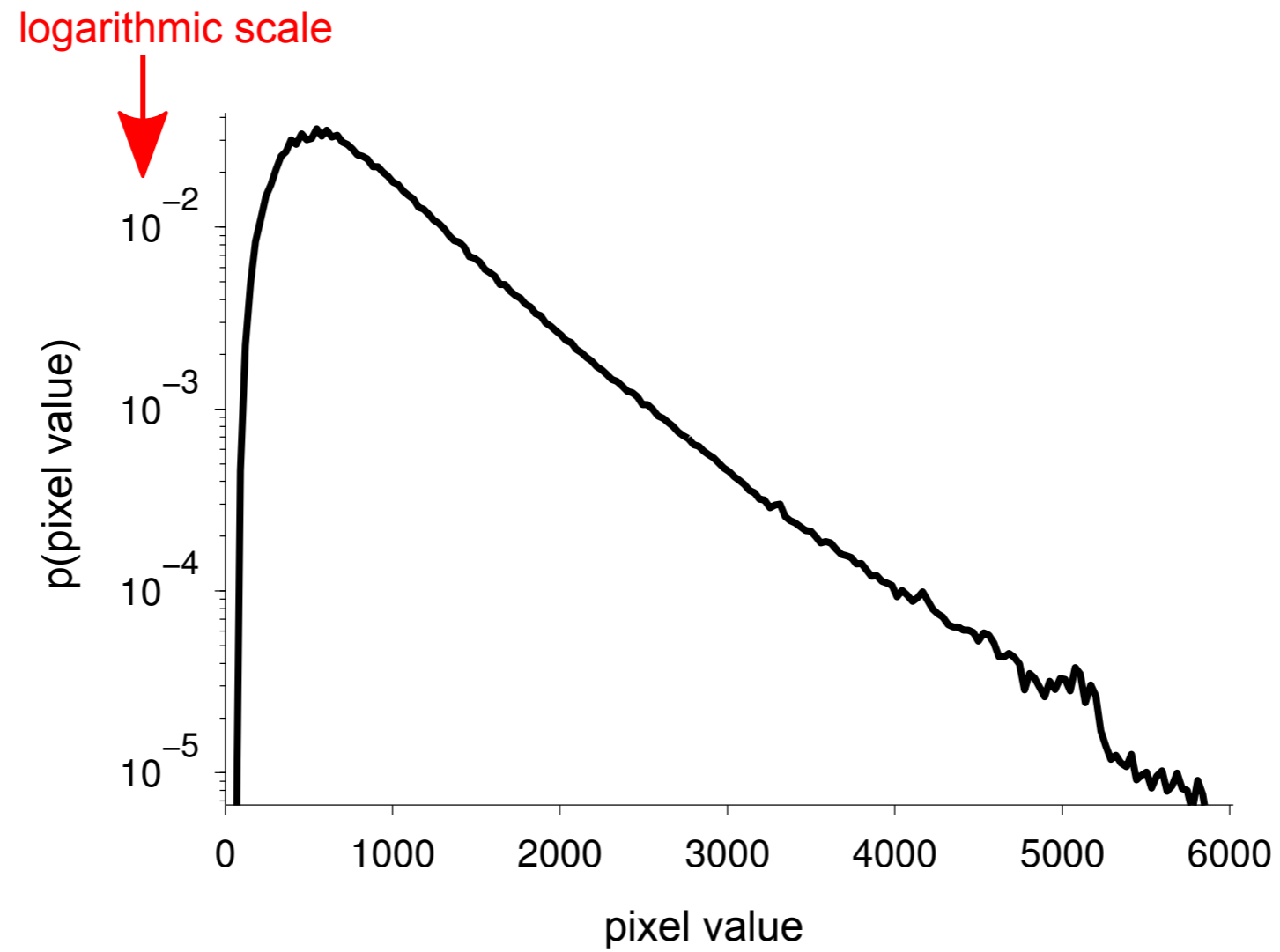
Structured images

PCA tulajdonságok

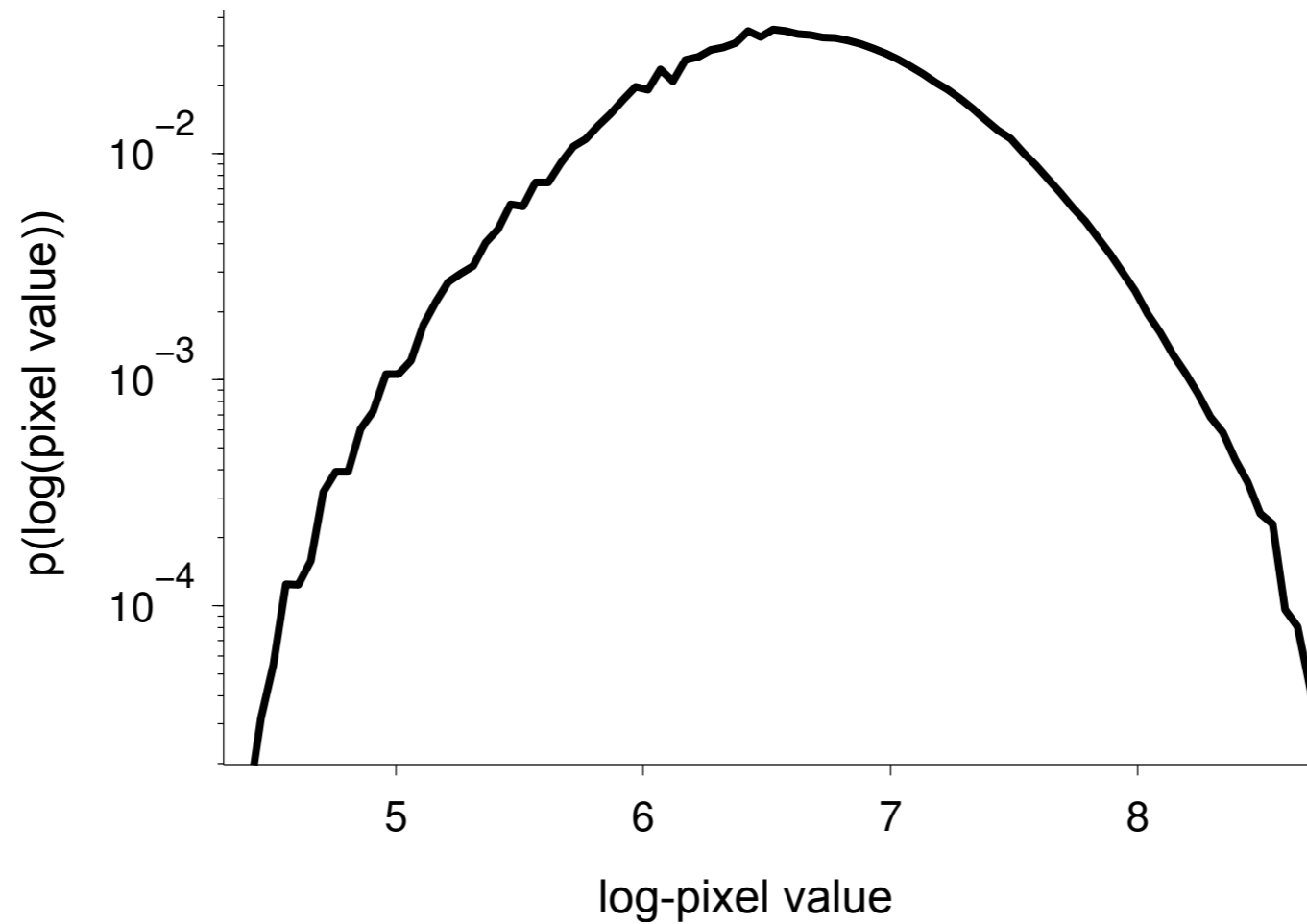
- Kompakt kódot eredményez
- Egy adatpont (kép) leírásáért általában a teljes hálózat felel



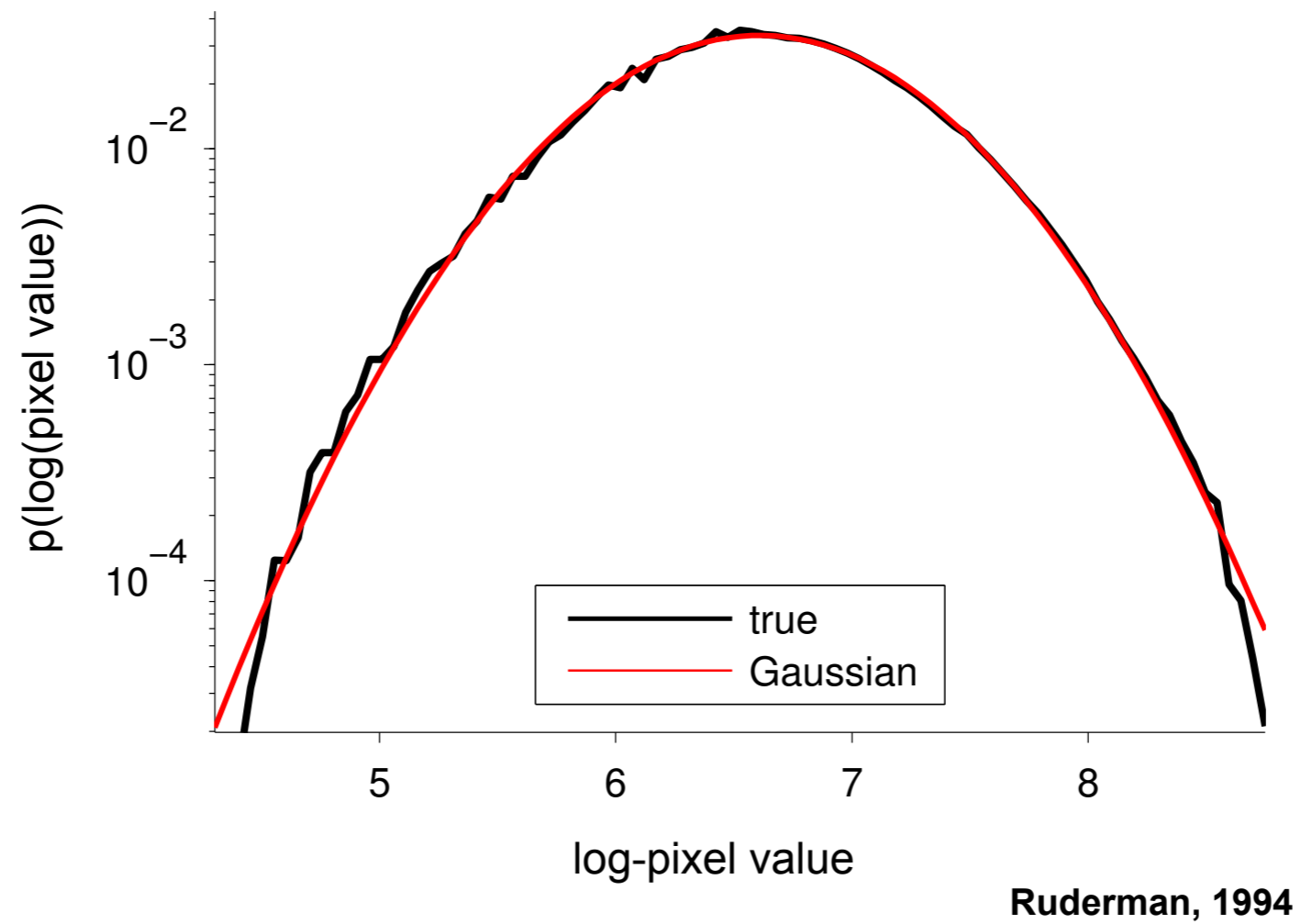
Marginális statisztika



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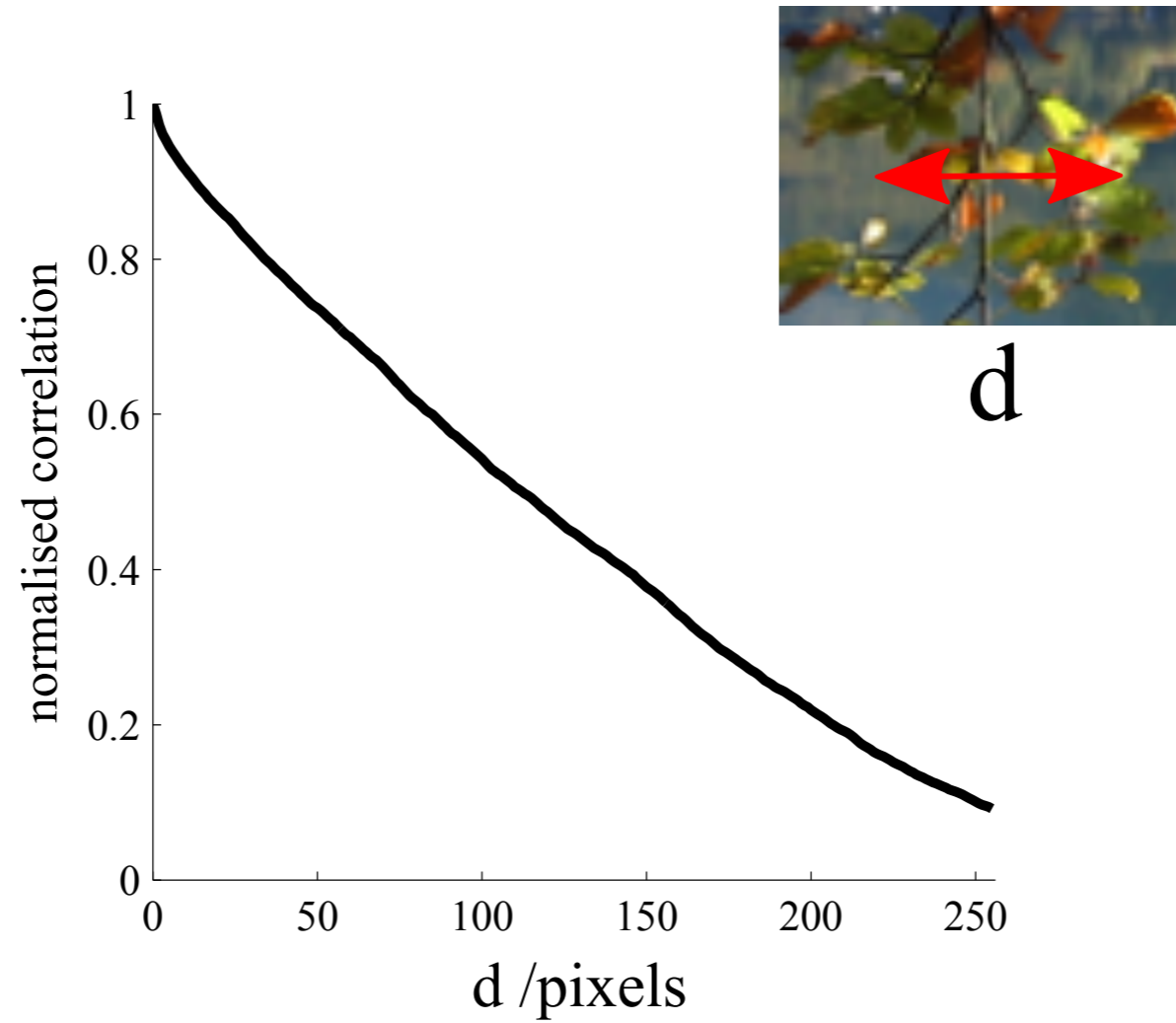
Ruderman, 1994

Pixel korrelációk

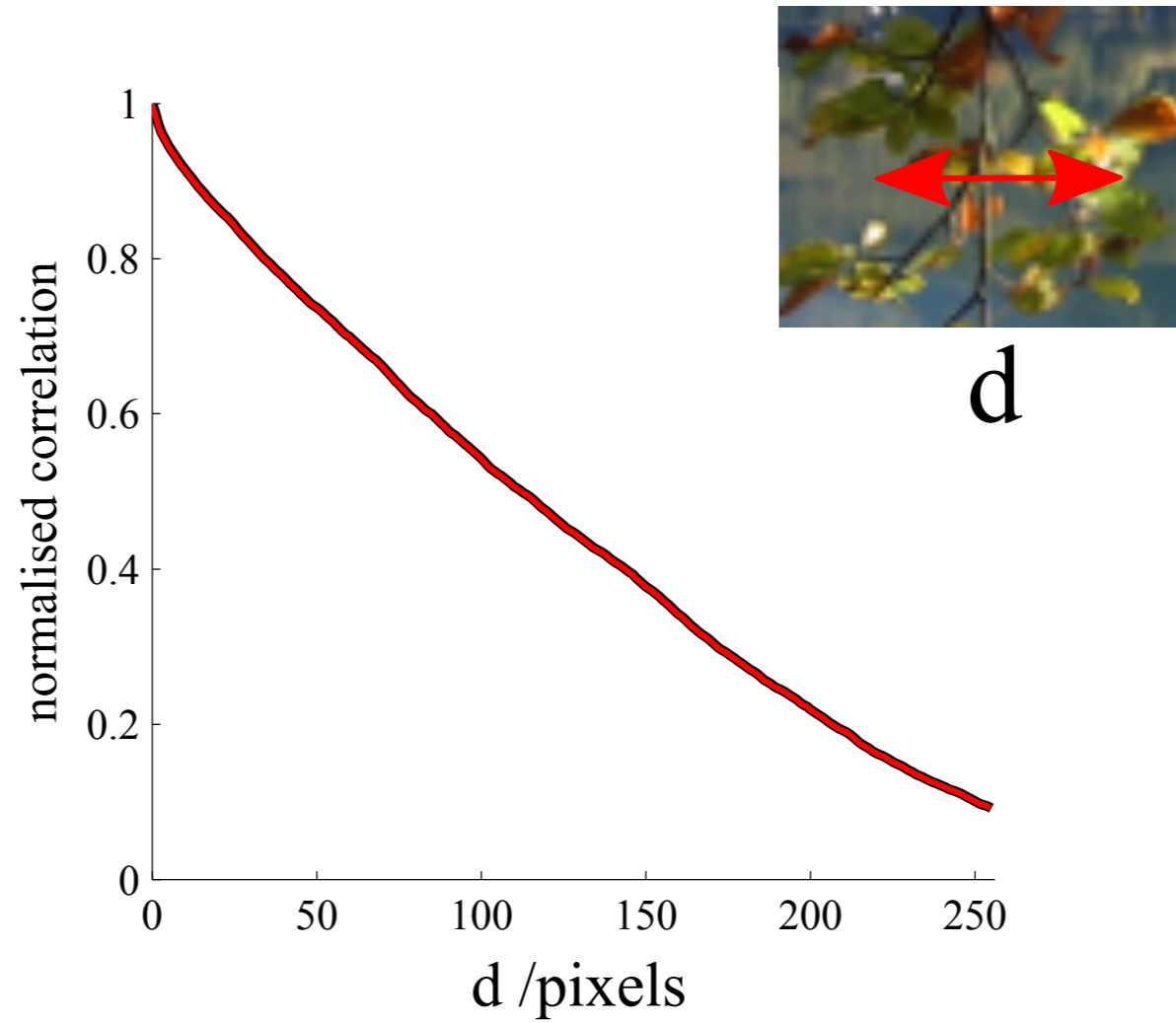


d

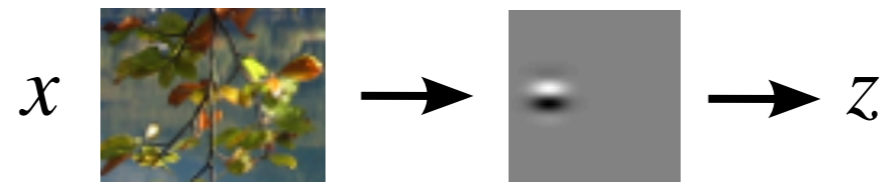
Pixel korrelációk



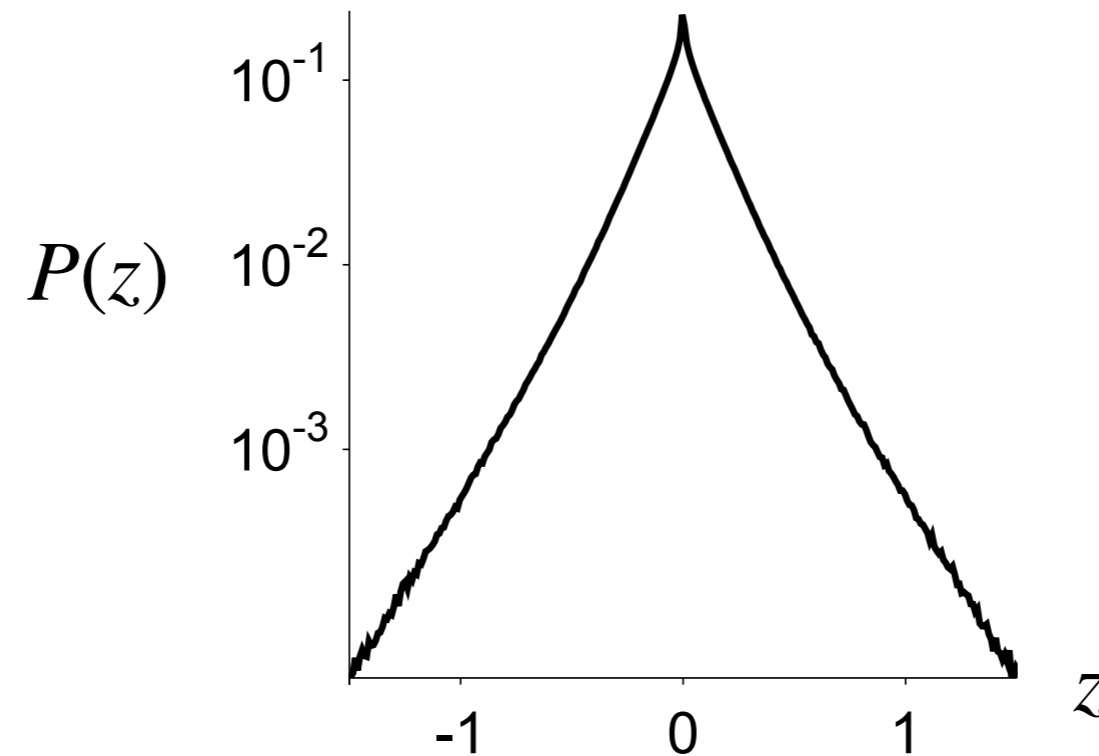
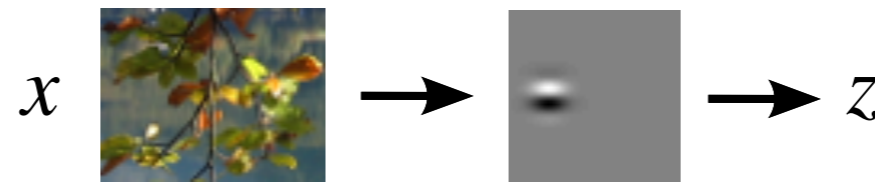
Pixel korrelációk



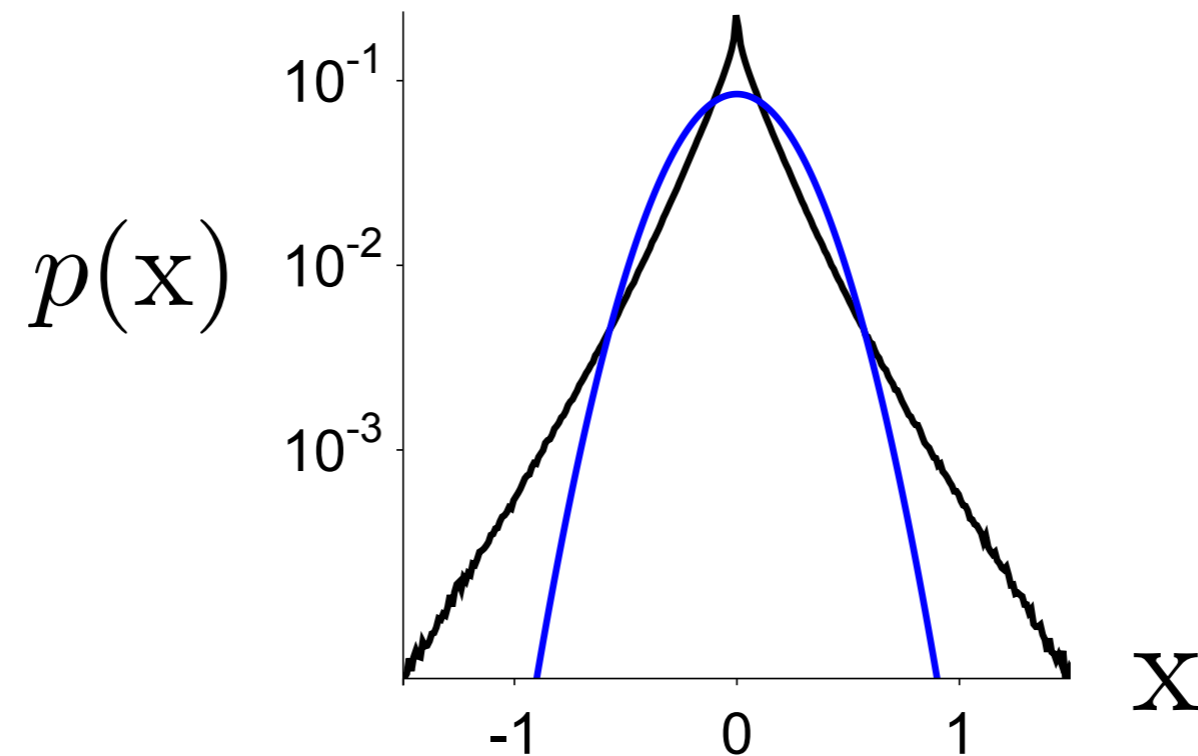
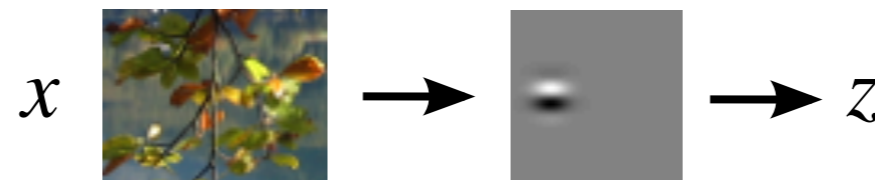
Magasabb rendű statisztika: ritkaság



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Sparse kódolás, ICA

- “z”-k függetlenek
- y priorja “ritka” ($P(z)$)

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Komputációs kritériumok:

- Hiteles rekonstrukció
költség egy adatpontra (képre):

$$\text{cost}_1 = \left(x - \sum_i A'_i \cdot z_i \right)^2$$

- Kis “energiafelhasználás (kevés szimultán aktív neuron)
további költség a kód “ritkasága”:

$$\text{cost}_2 = - \sum_i S \left(\frac{z_i}{\sigma} \right)$$

S a Gauss-nál nagyobb kurtózissal bíró eloszlás

- teljes költség (~energia):

$$E = -\text{cost}_1 - \lambda \text{cost}_2$$

Sparse kód tanulása: E-M

Algoritmus:

- Itáráció EM lépésekkel
- Random kezdeti feltételek
- Adott konnektivitási mátrixnál az aktiviások segítségével a költség minimalizálása
- Adott aktivitásokkal a költség minimalizálása a súlyok adaptálásával

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$$\dot{z}_i = \mathbf{A}_i x_t - \sum_j \mathbf{A}'_i \mathbf{A}_j z_j - \frac{\lambda}{\sigma} S' \left(\frac{z_i}{\sigma} \right)$$

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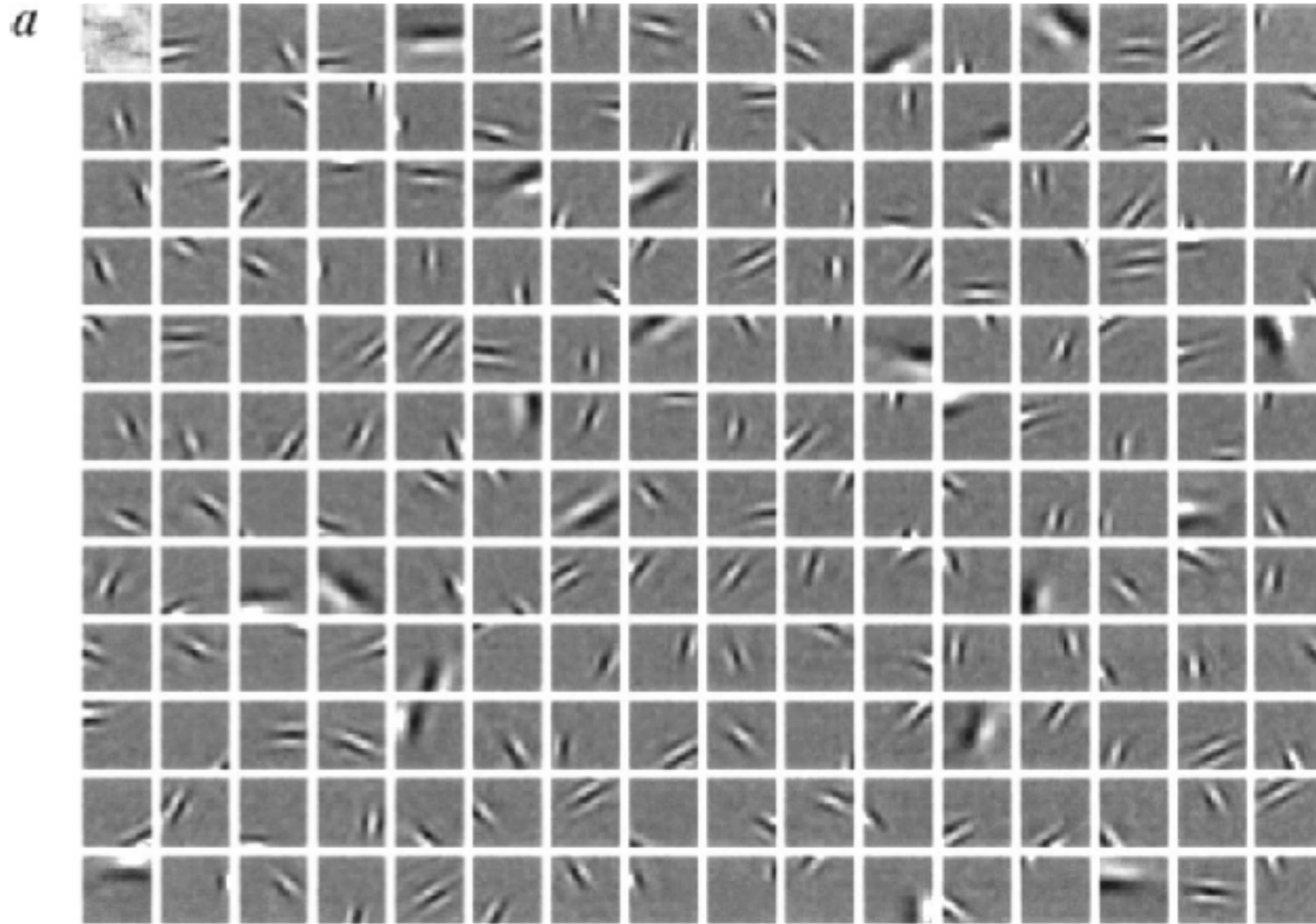
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$$\Delta A_i = \eta \langle a_i [x - \hat{x}] \rangle_t$$

Sparse kódolás: eredmény

tréningezés természetes képekkel

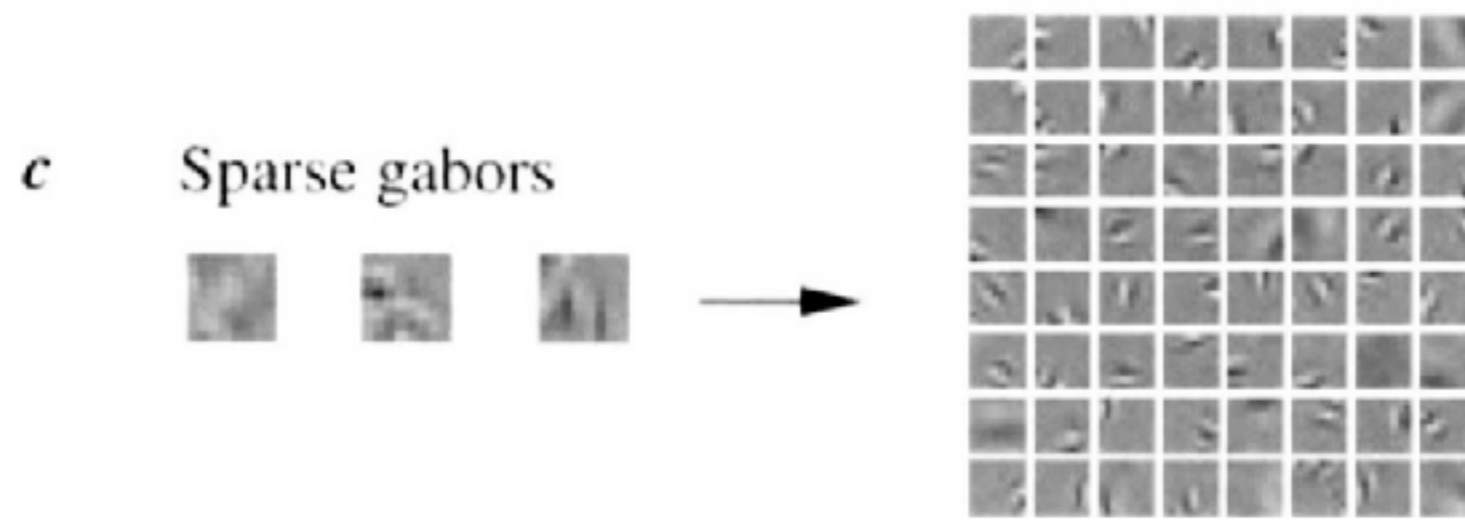


Olshausen & Field '96

A kialakult bázis:

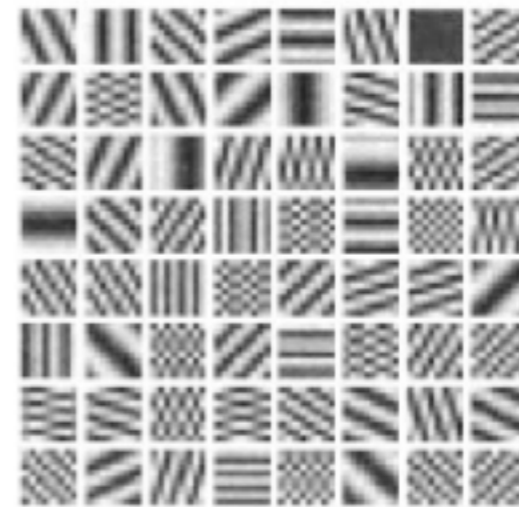
- irányított
- térbeli sávszűrést valósít meg
- lokalizált

Tanulás és stimulus statisztika

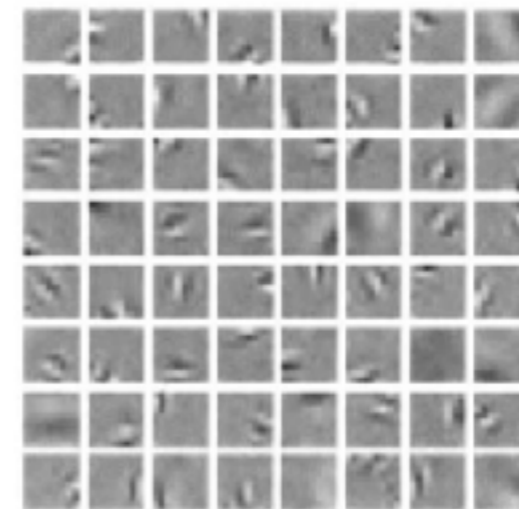


Tanulás és stimulus statisztika

b Sparse gratings

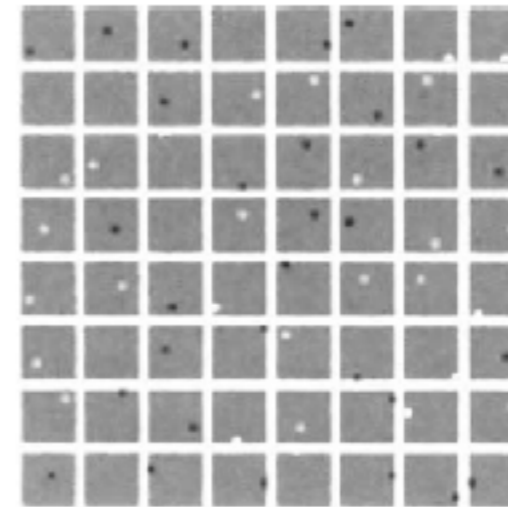


c Sparse gabors

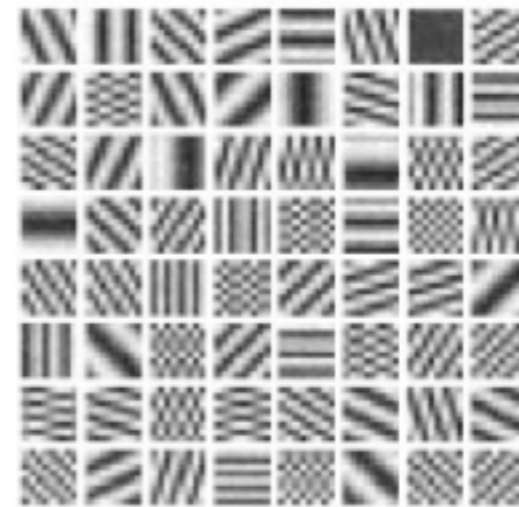


Tanulás és stimulus statisztika

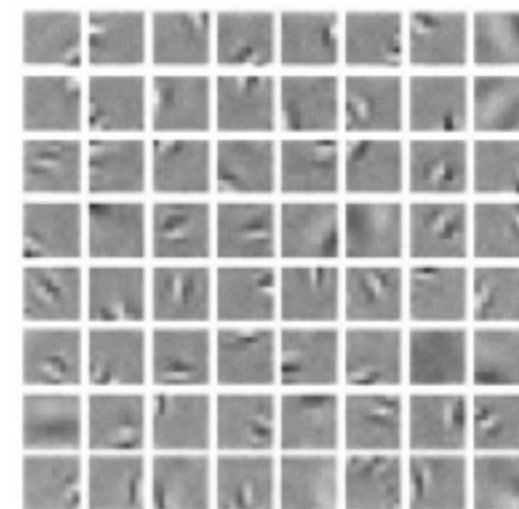
a Sparse pixels



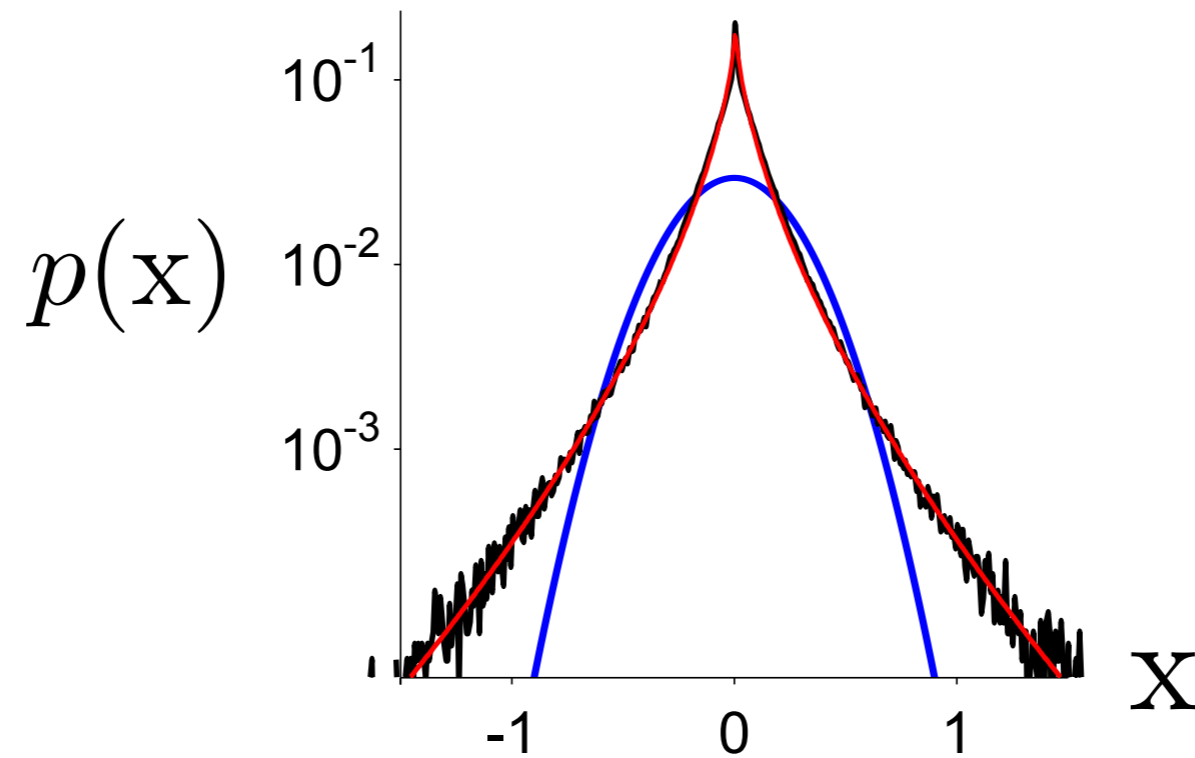
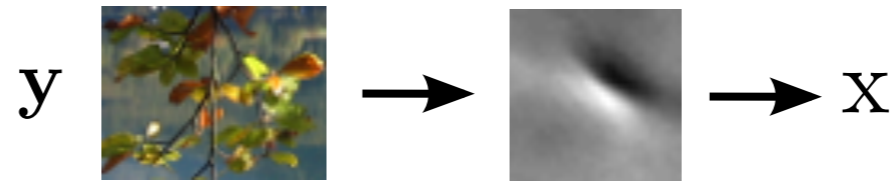
b Sparse gratings



c Sparse gabors



Magasabb rendű korrelációk: ritkaság

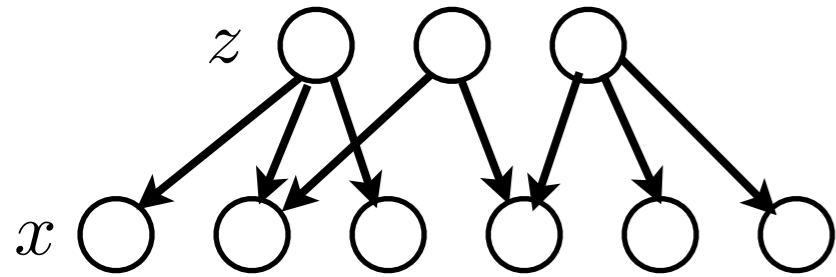


Generatív/rekogníciós modell

$$P(\mathbf{x}) = P(\mathbf{x} | \mathbf{z}) P(\mathbf{z})$$

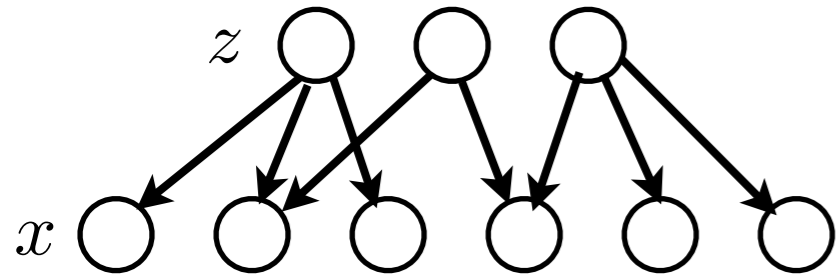
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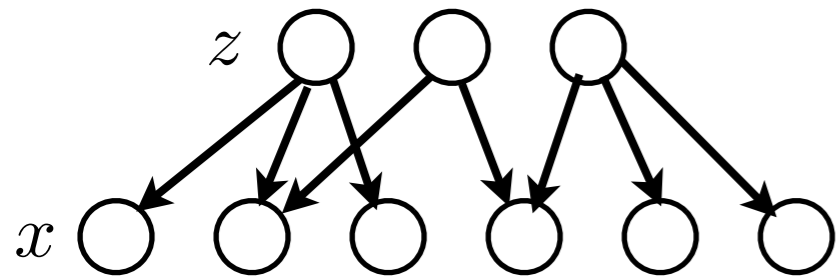
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szituáció / környezet

Generatív/rekogníciós modell

$$P(\mathbf{x}) = P(\mathbf{x} | \mathbf{z}) P(\mathbf{z})$$

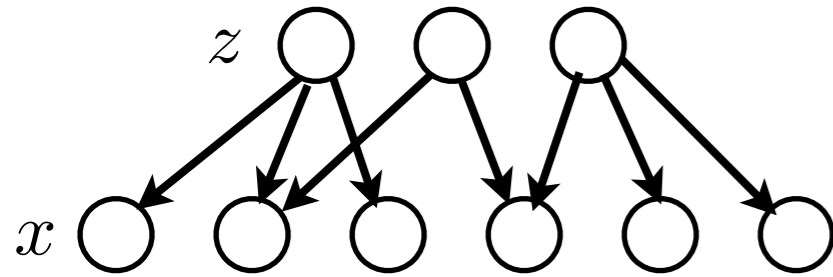


szituáció / környezet

objektumok

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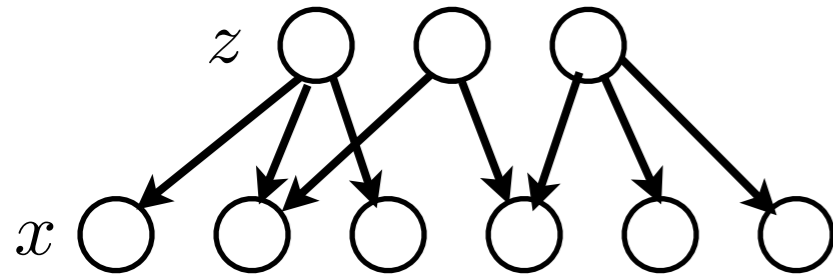
szituáció / környezet

objektumok

objektum elhelyezkedése |
méret, hely, helyzet, világítás

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szituáció / környezet

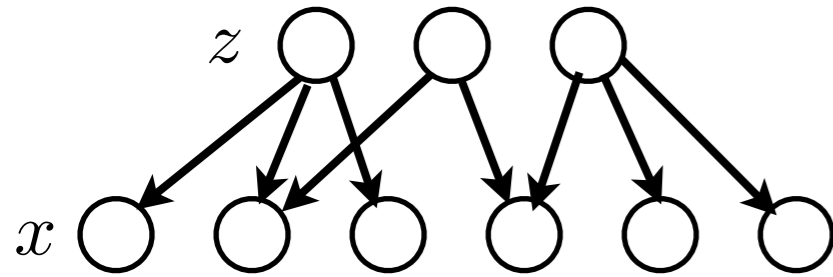
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objektum elhelyezkedése |
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objektum tulajdonságai |
élek, felületi mintázatok

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szituáció / környezet

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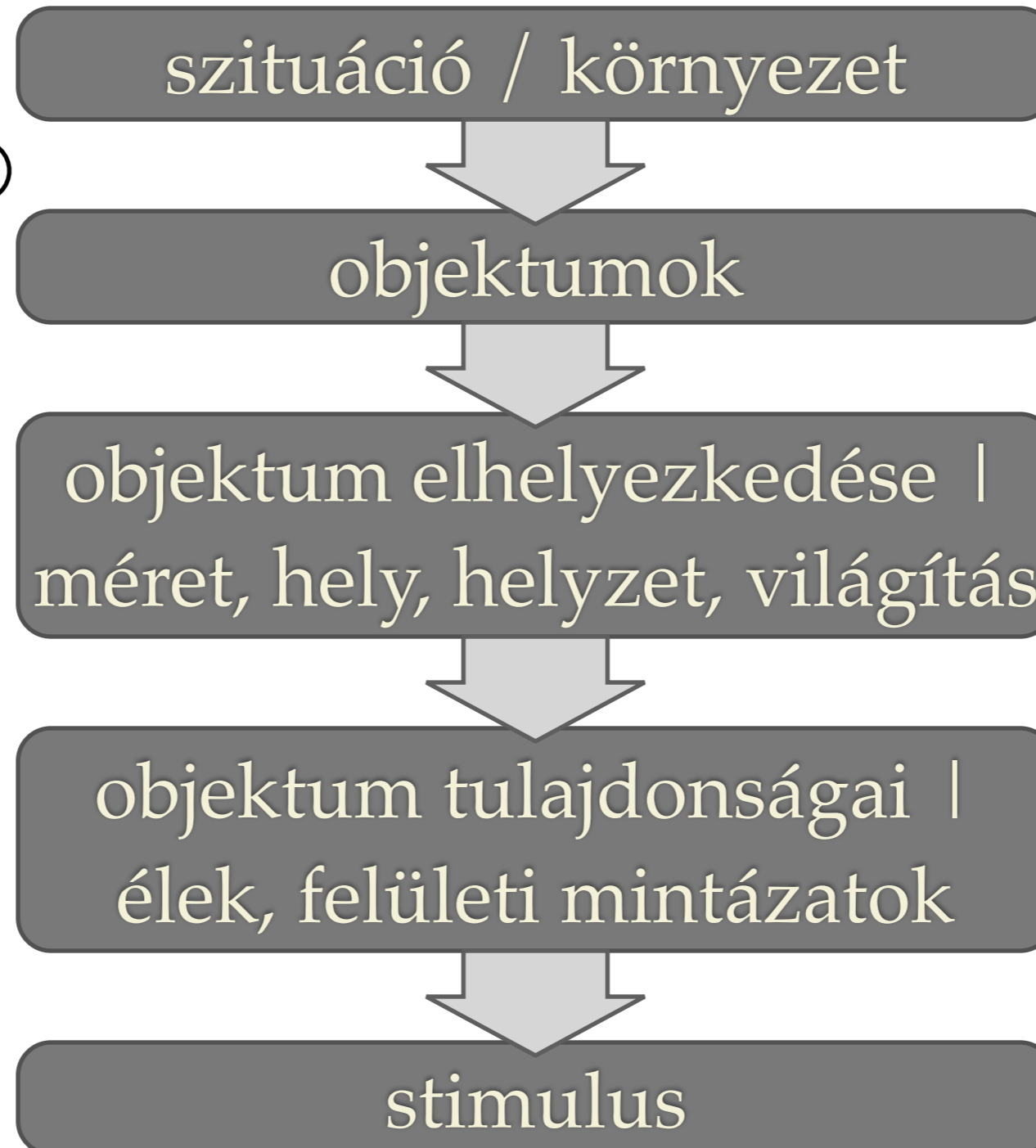
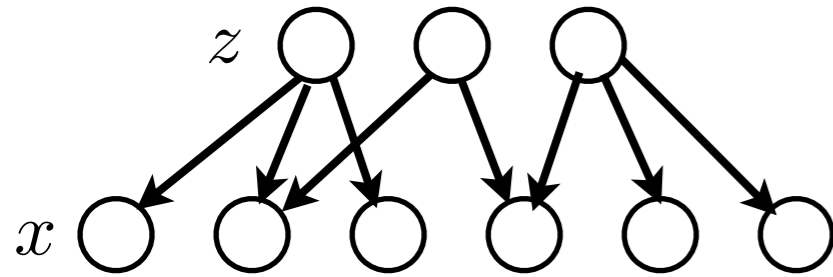
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stimulus

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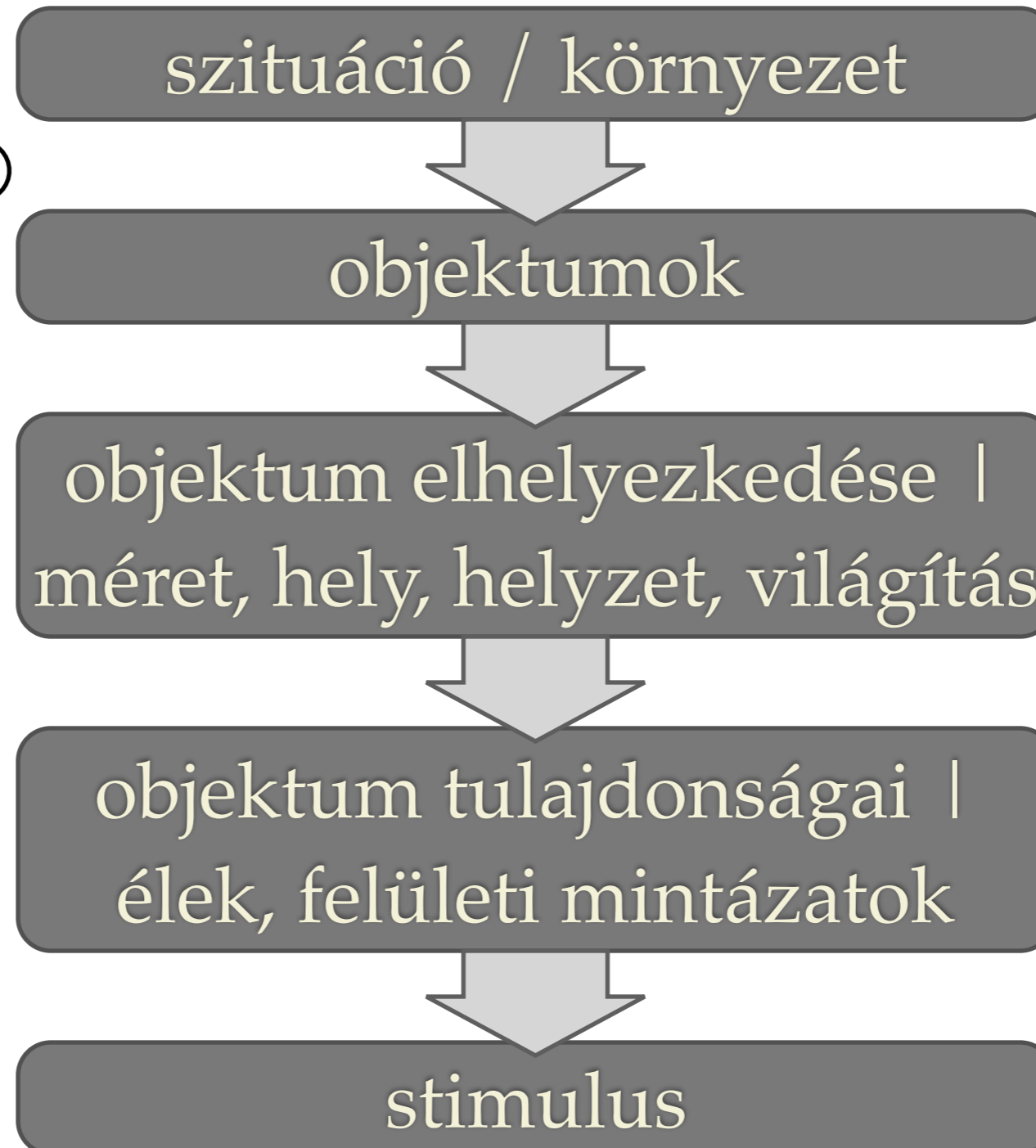
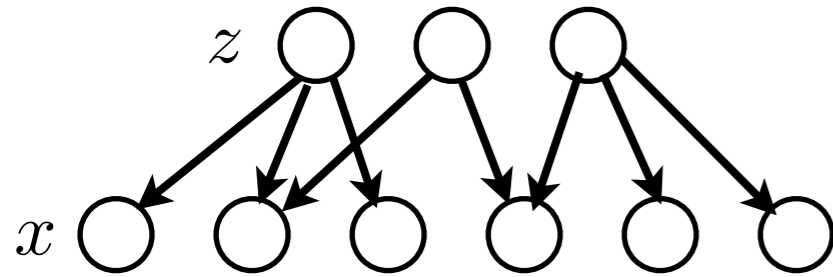
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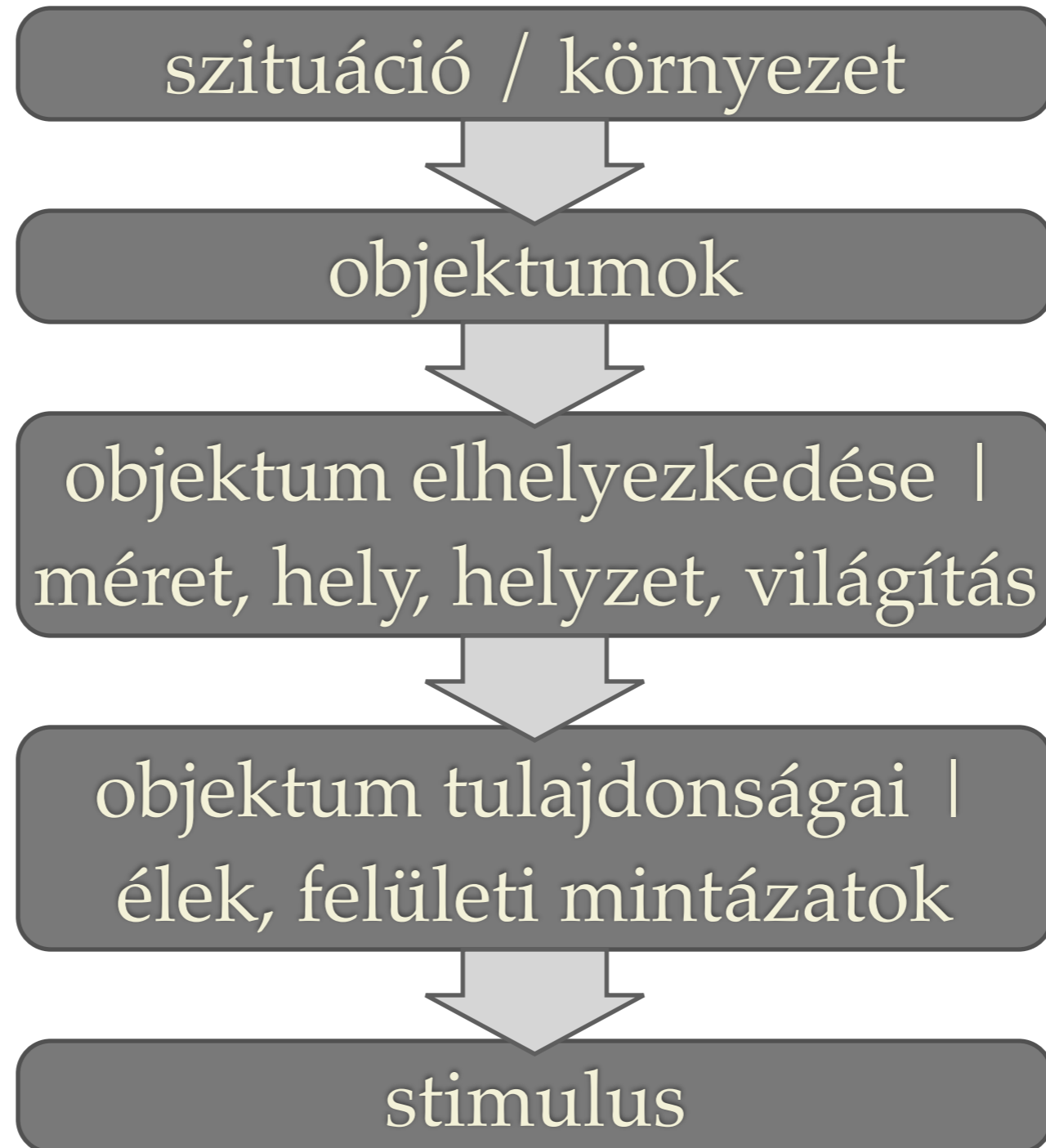
generatív modell

Generatív/rekogníciós modell

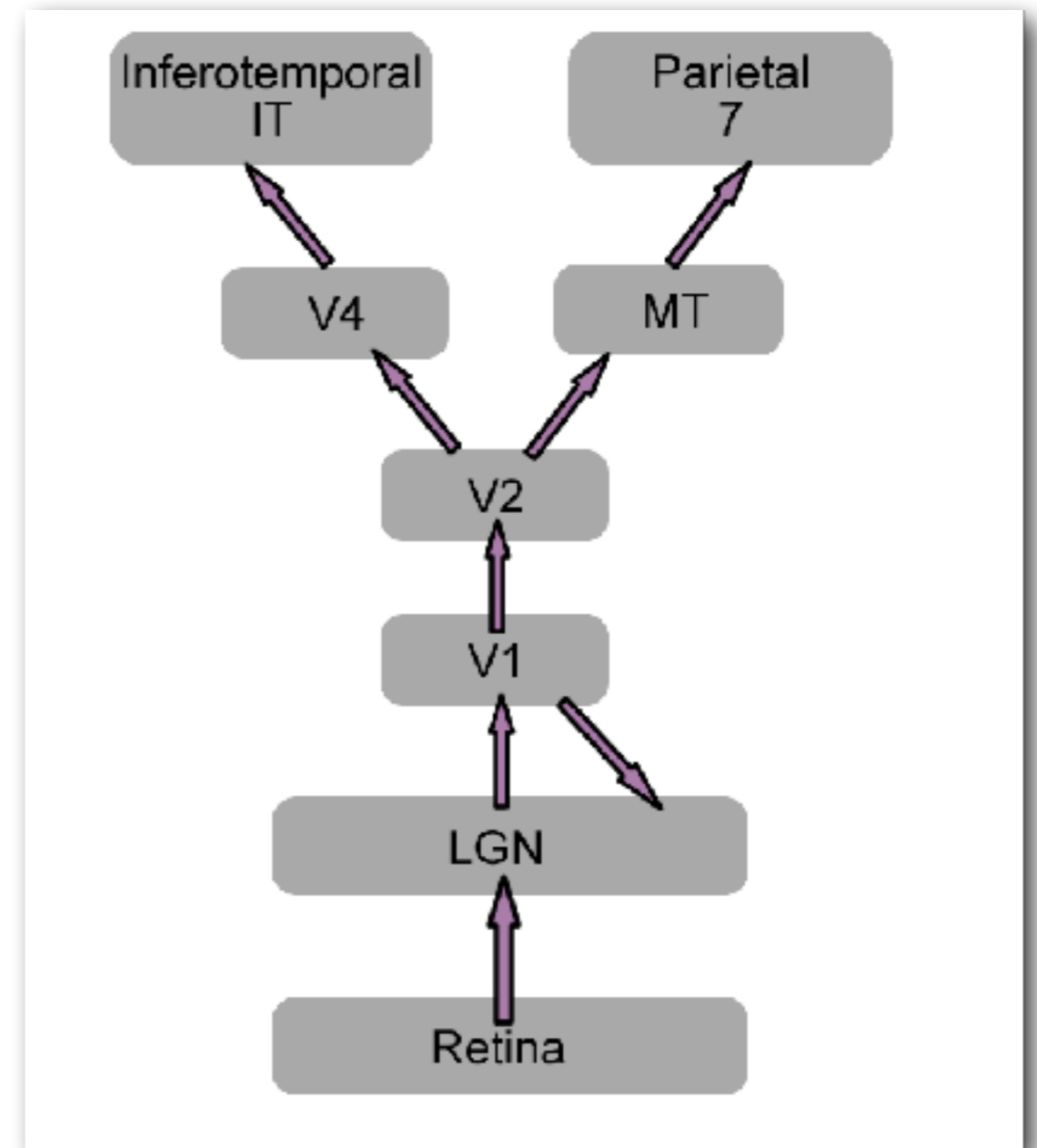
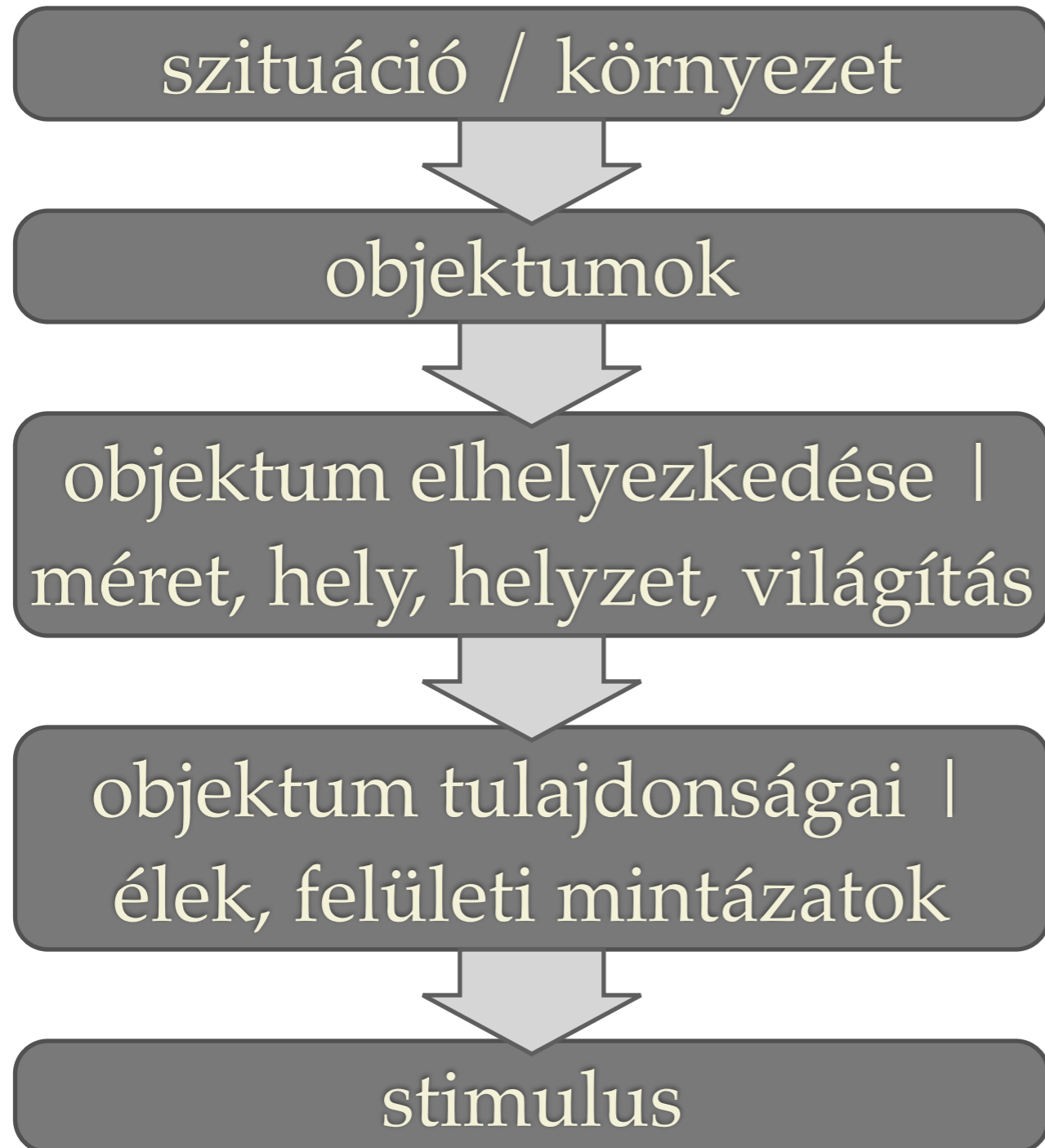
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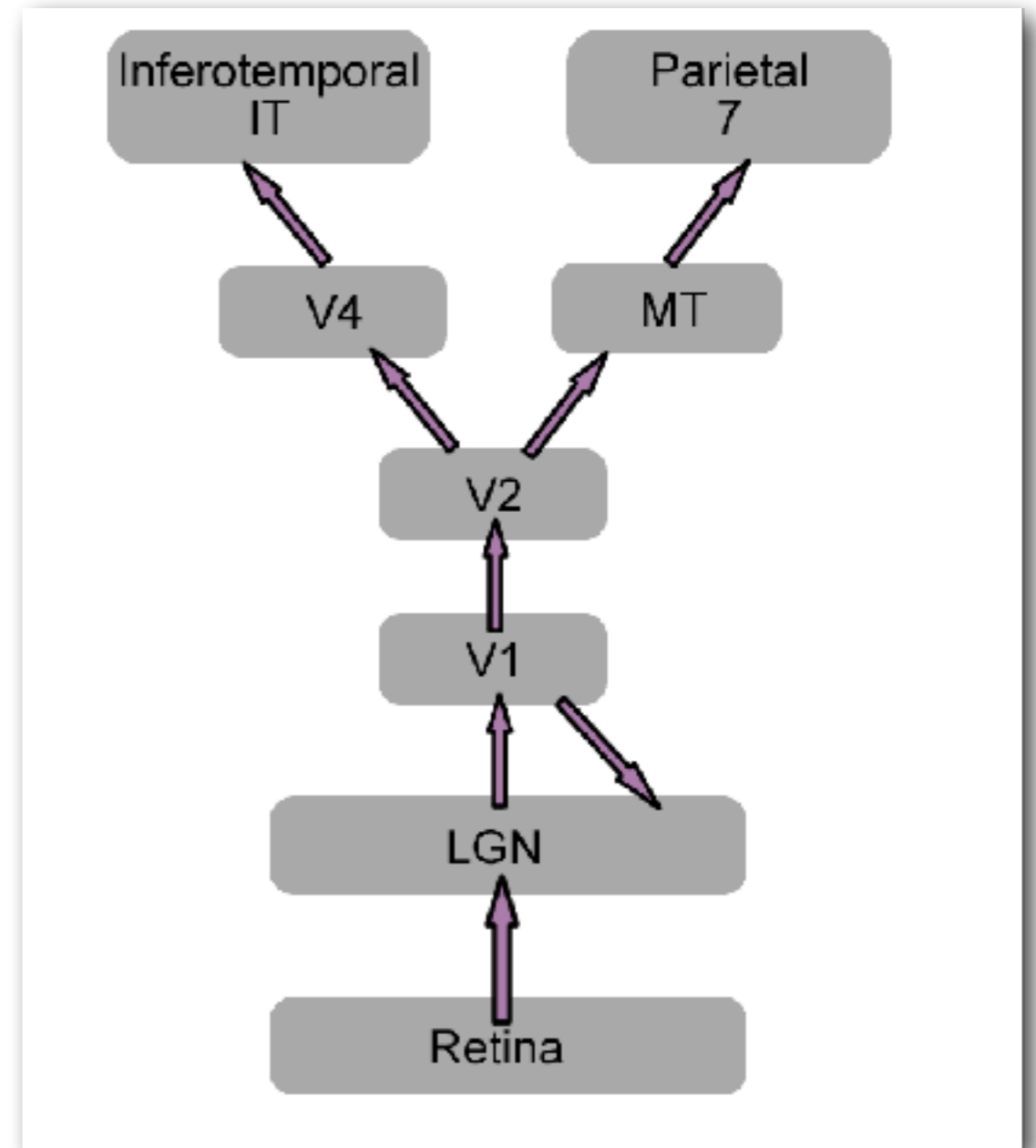
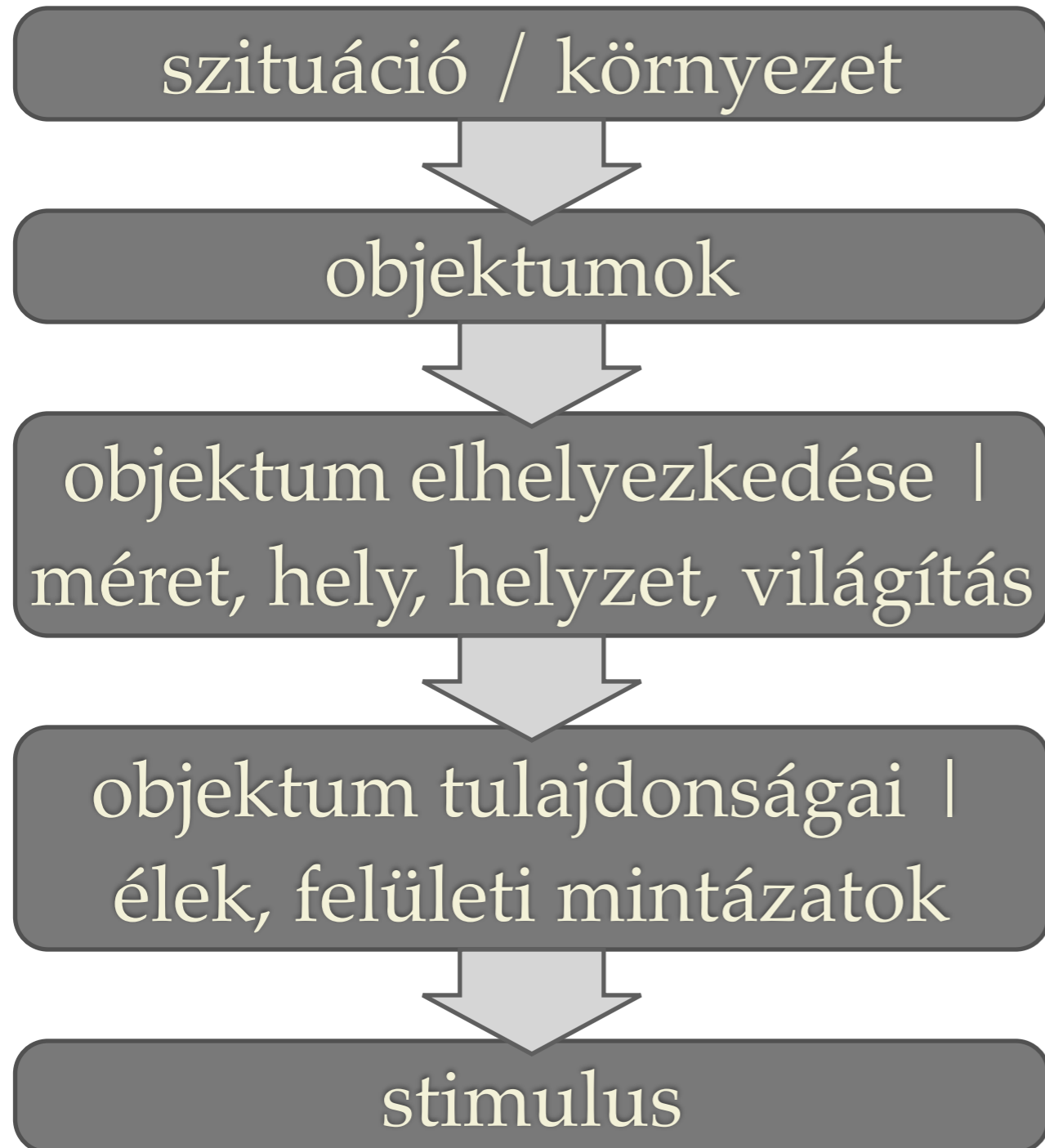
Generatív/rekogníciós modell



Generatív/rekogníciós modell



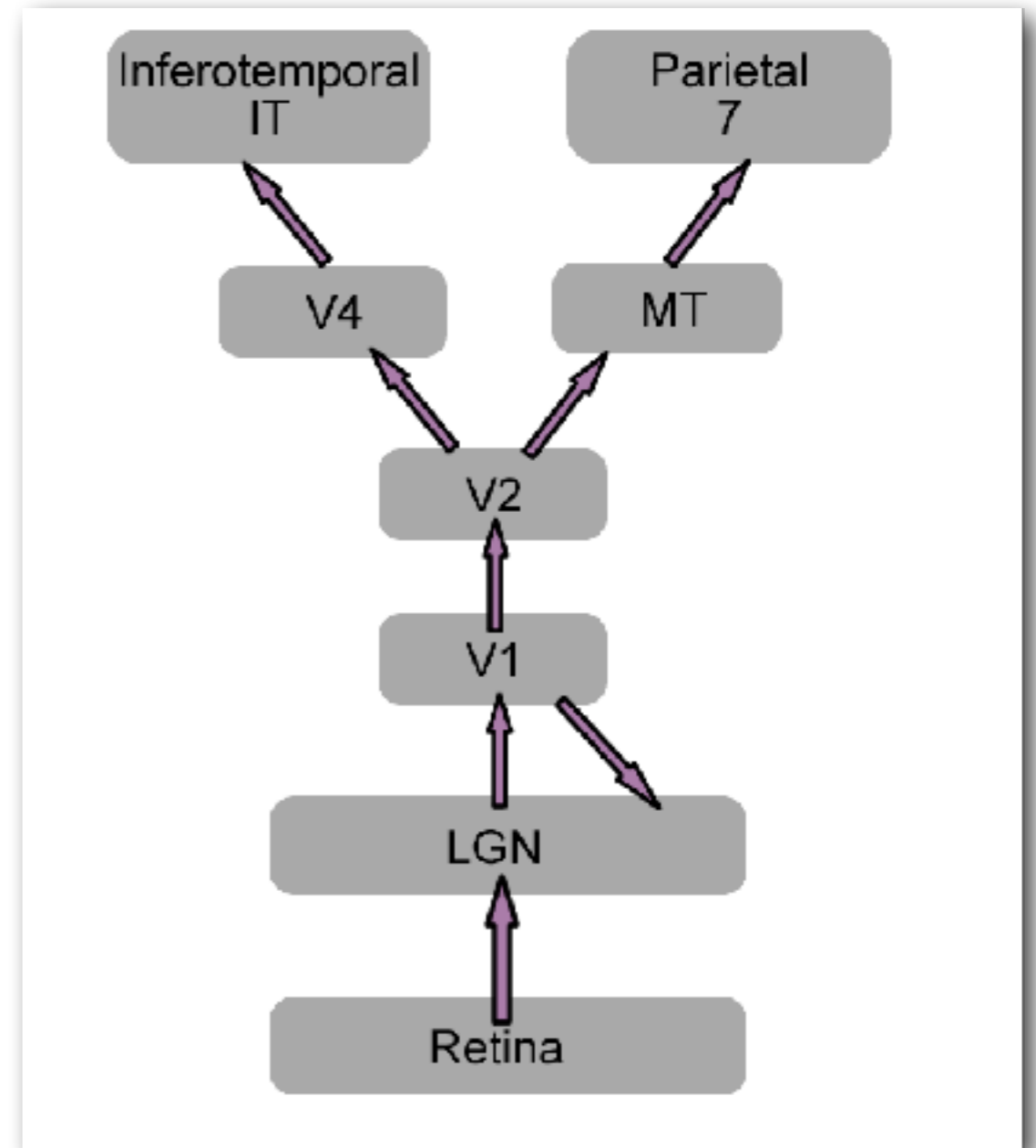
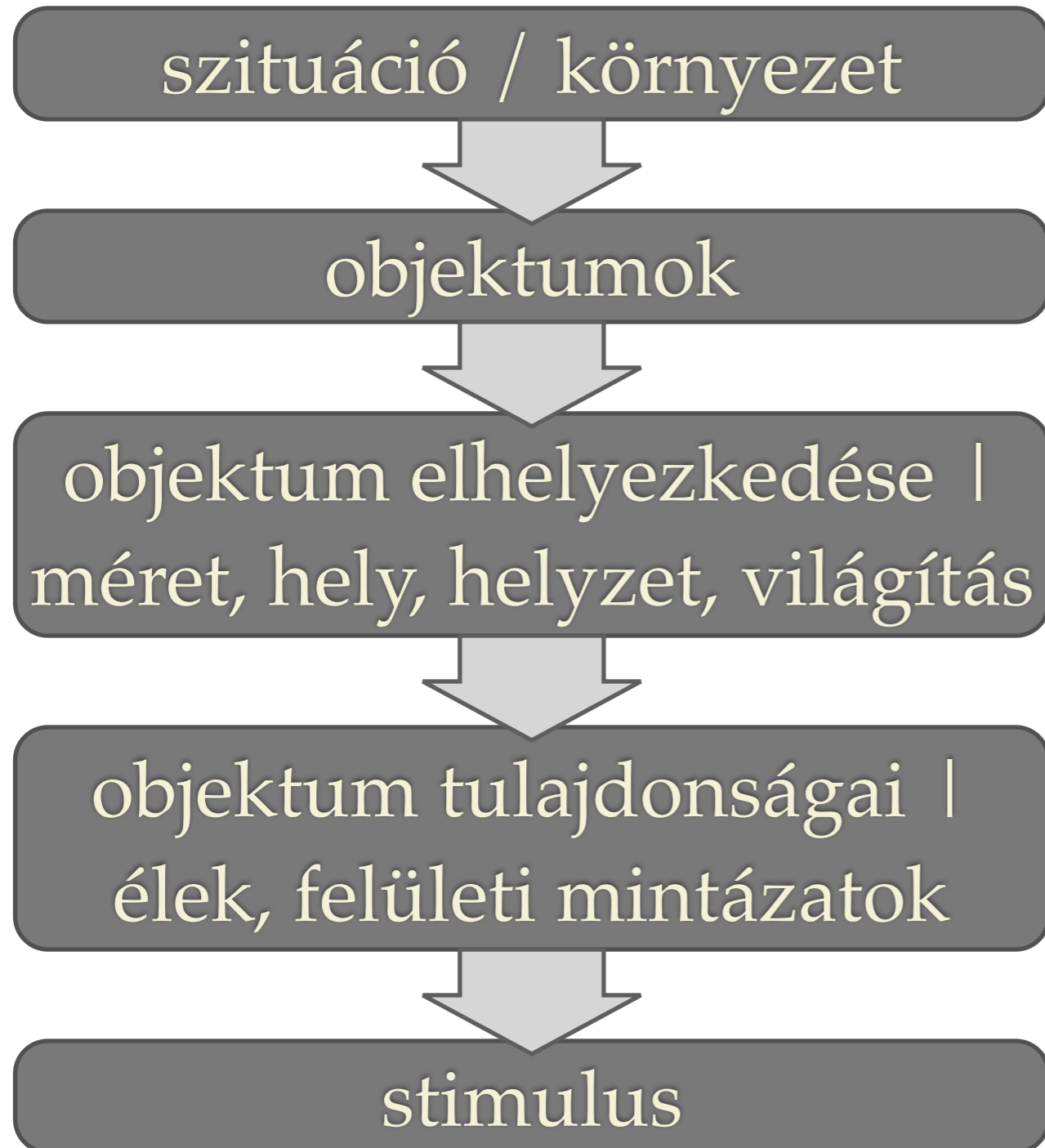
Generatív/rekogníciós modell



Modell definíció -> rekogníció:

$$P(x|z)$$

Generatív/rekogníciós modell

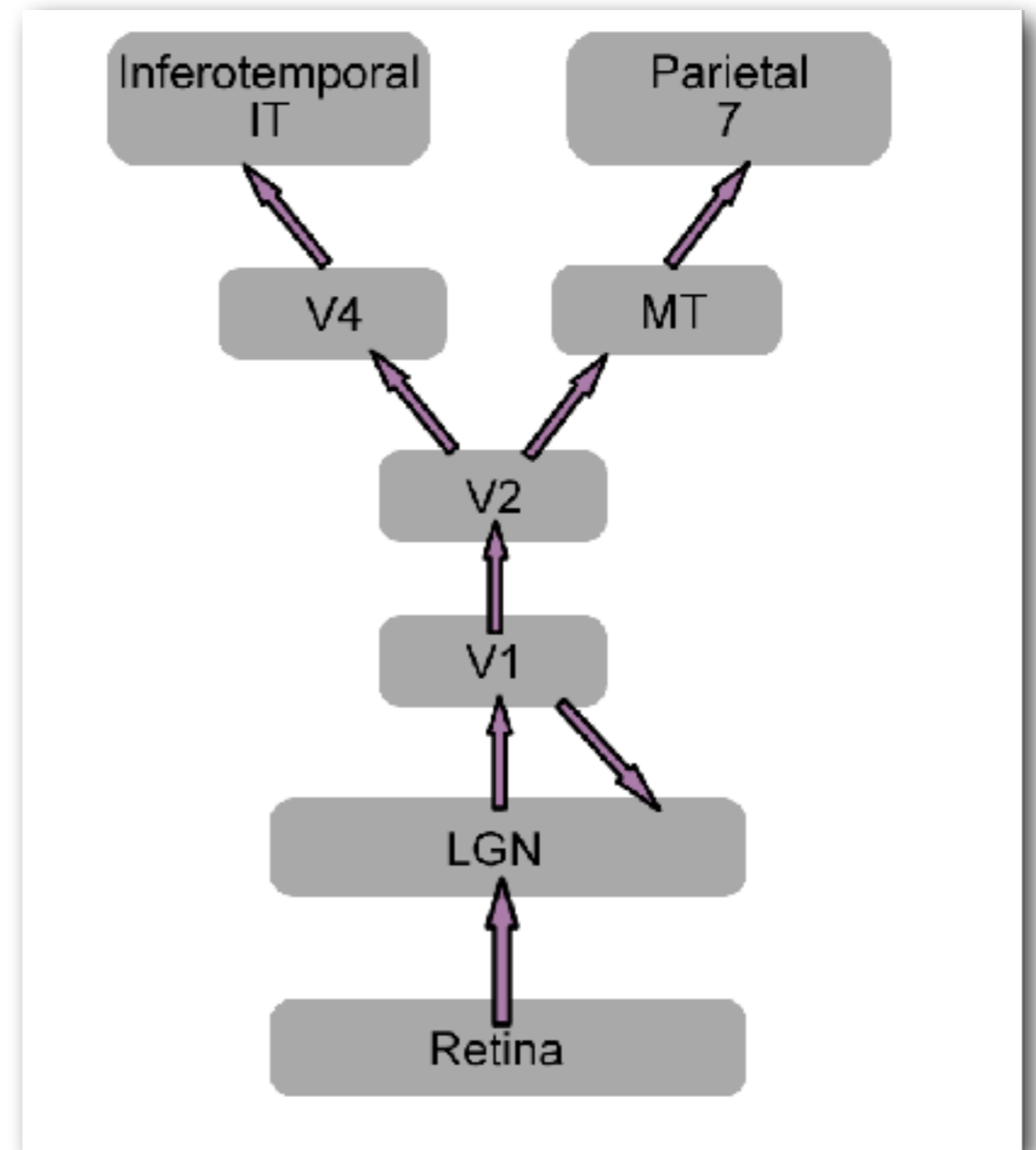
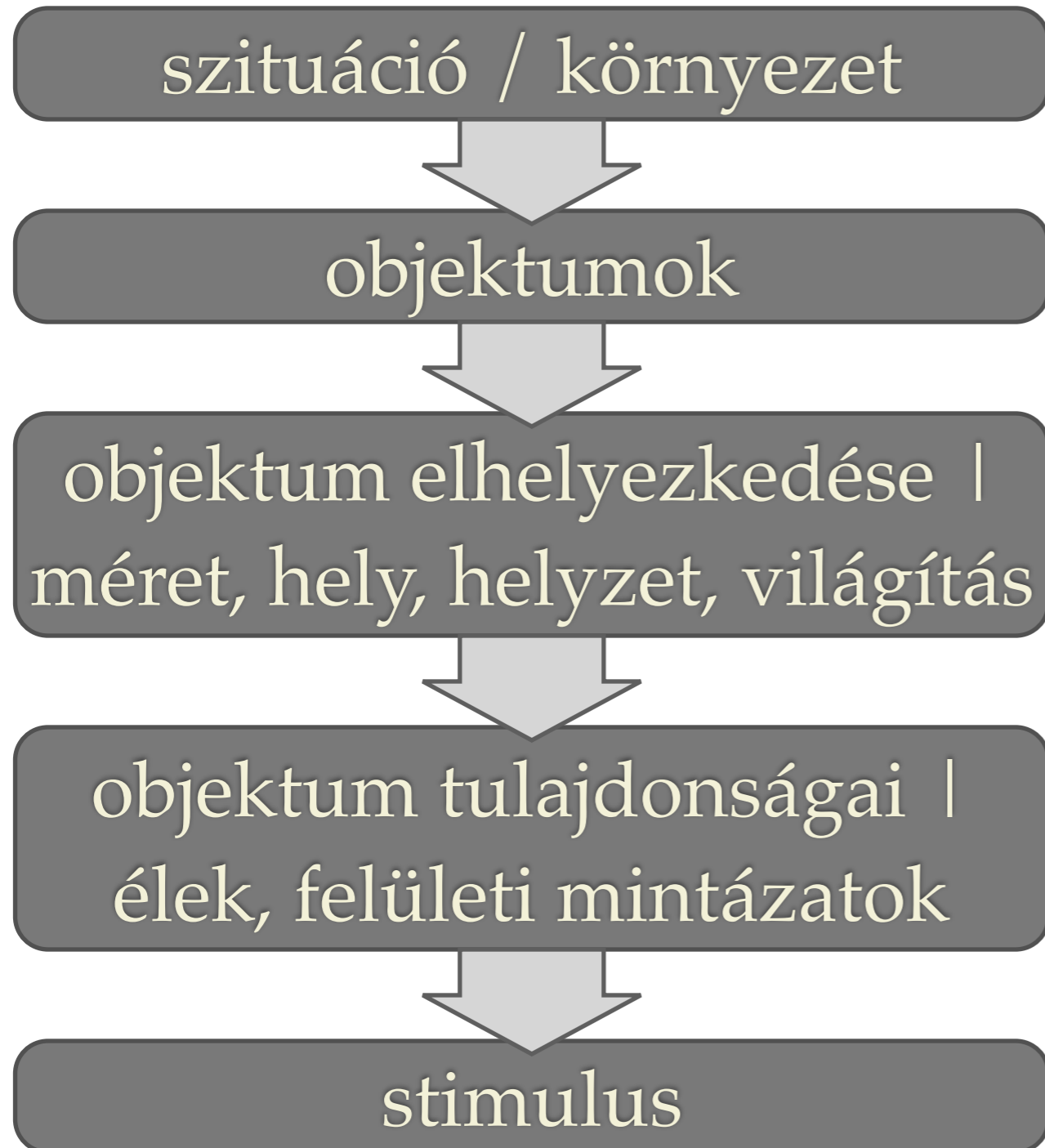


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$$P(x|z)$$

Inferencia igénye -> rekogníció:

Generatív/rekogníciós modell



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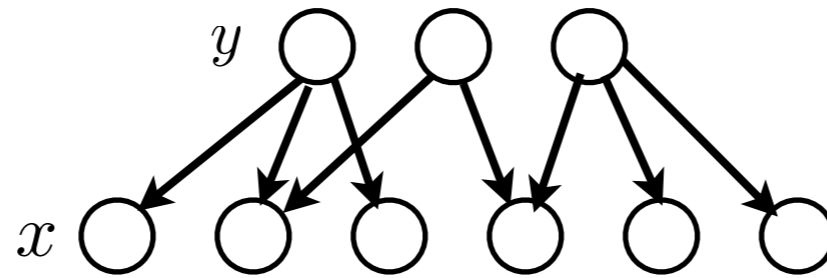
$$P(x|z)$$

Inferencia igénye -> rekogníció:

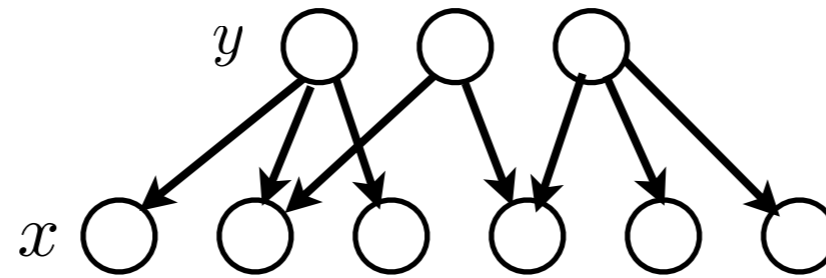
$$P(z|x)$$

Independens komponensek

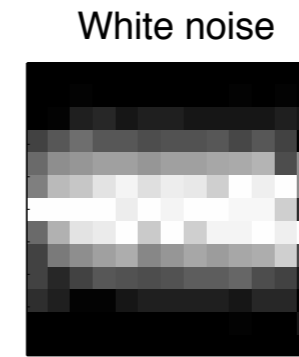
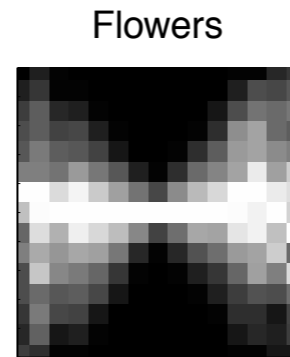
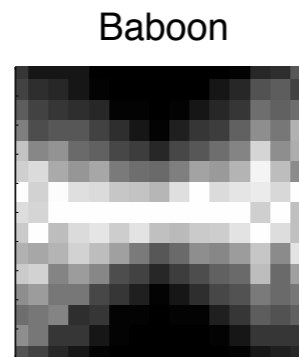
Independens komponensek



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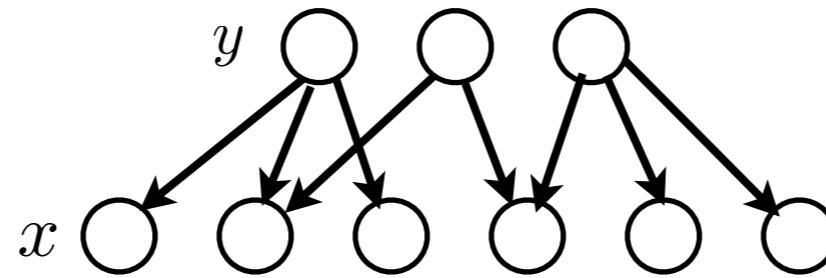


a

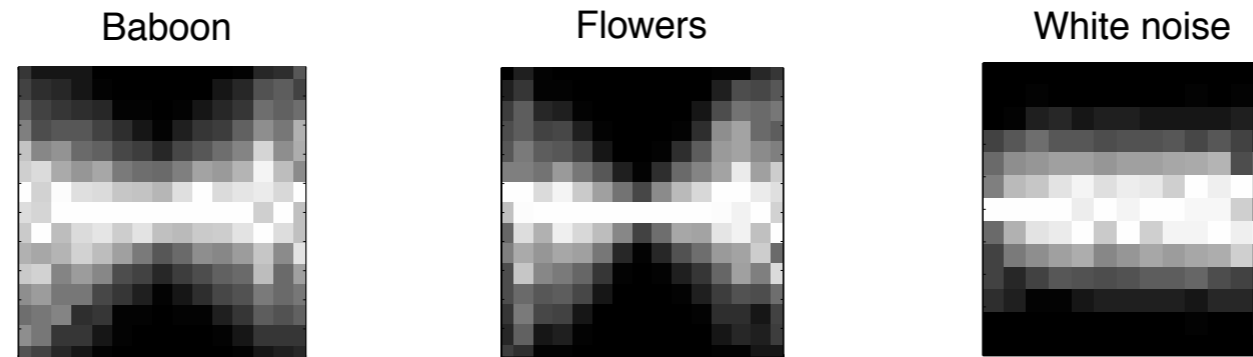


Schwartz & Simoncelli, 2001

Independens komponensek



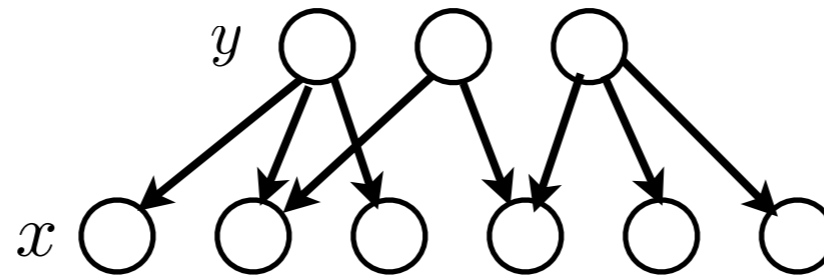
a



Schwartz & Simoncelli, 2001

$$\text{var}(y_1 | y_2) = w y_2^2 + \sigma^2$$

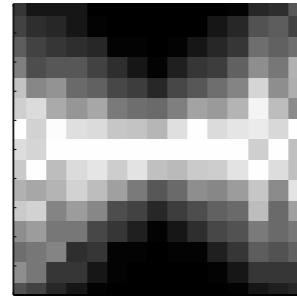
Independens komponensek



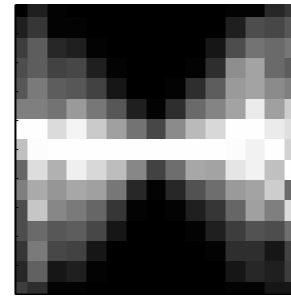
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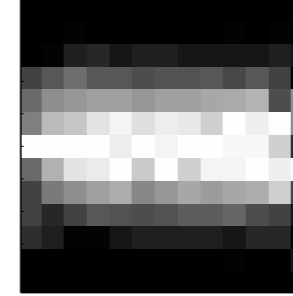
Baboon



Flowers



White noise



Schwartz & Simoncelli, 2001

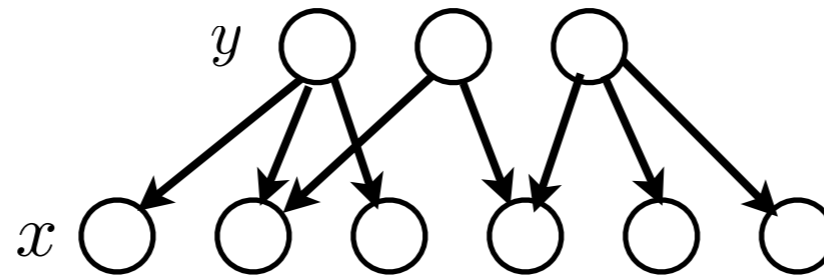
$$\text{var}(y_1 | y_2) = w y_2^2 + \sigma^2$$

amennyiben y_1 aktivitását normalizáljuk

$$R_1 = \frac{y_1^2}{w y_2^2 + \sigma^2}$$

akkor a függés eliminálható

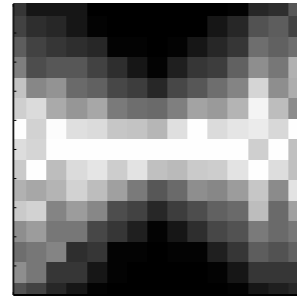
Independens komponensek



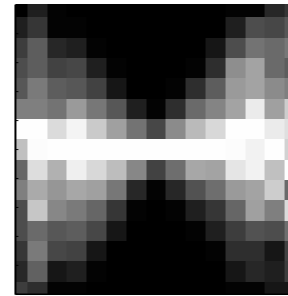
a



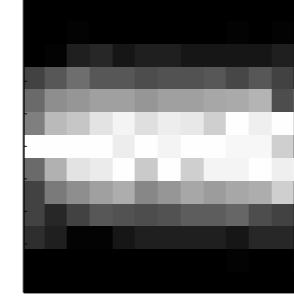
Baboon



Flowers



White noise



Schwartz & Simoncelli, 2001

$$\text{var}(y_1 | y_2) = w y_2^2 + \sigma^2$$

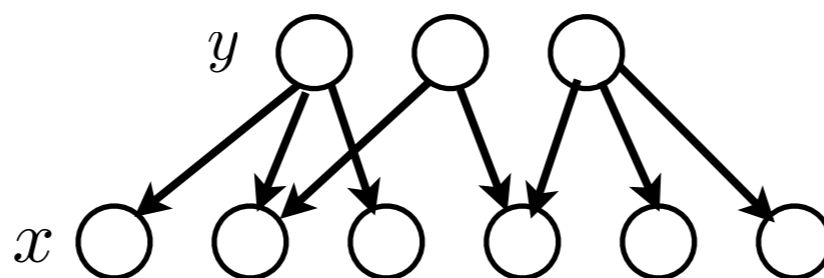
amennyiben y_1 aktivitását normalizáljuk

$$R_1 = \frac{y_1^2}{w y_2^2 + \sigma^2}$$

akkor a függés eliminálható

feltételezés: analóg módon több másik sejtre: $R_i = \frac{y_i^2}{\sum_j w_{ji} y_j^2 + \sigma^2}$

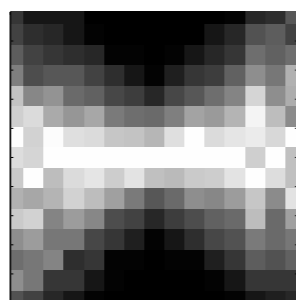
Independens komponensek



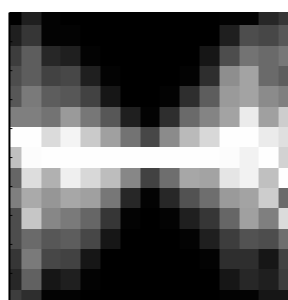
a



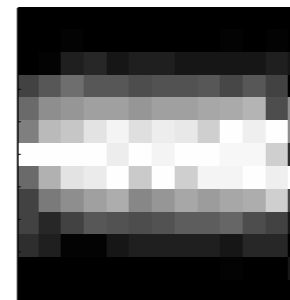
Baboon



Flowers



White noise



Schwartz & Simoncelli, 2001

$$\text{var}(y_1 | y_2) = w y_2^2 + \sigma^2$$

amennyiben y_1 aktivitását normalizáljuk

$$R_1 = \frac{y_1^2}{w y_2^2 + \sigma^2}$$

akkor a függés eliminálható

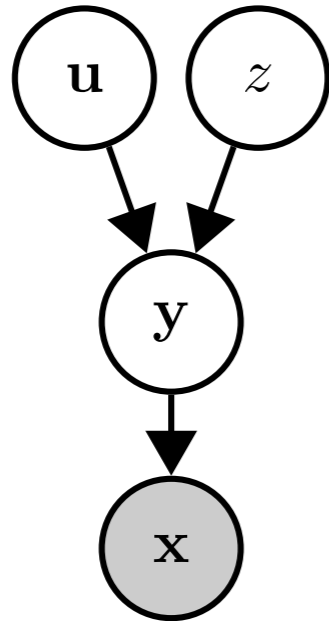
feltételezés: analóg módon több másik sejtre: $R_i = \frac{y_i^2}{\sum_j w_{ji} y_j^2 + \sigma^2}$

divizív normalizáció

y_i^2

$$\sum_j w_{ji} y_j^2 + \sigma^2$$

Gaussian Scale Mixtures



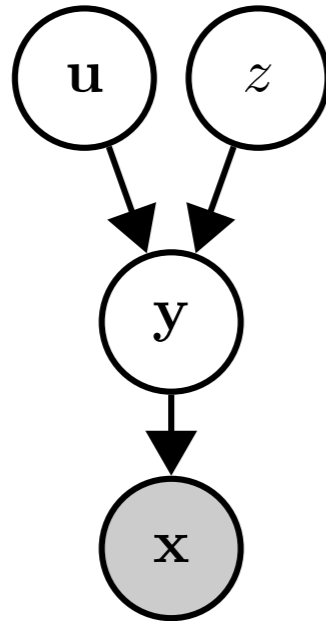
$$P(\mathbf{x}|\mathbf{y}) = \mathcal{N}(\mathbf{x}; \mathbf{A}\mathbf{y}, \sigma_x^2 \mathbf{I})$$

$$\mathbf{y} = z \mathbf{u}$$

$$P(\mathbf{u}) = \mathcal{N}(\mathbf{u}; \mathbf{0}, \mathbf{C})$$

$$P(z) = \text{Gamma}(z; k, \theta)$$

Gaussian Scale Mixtures



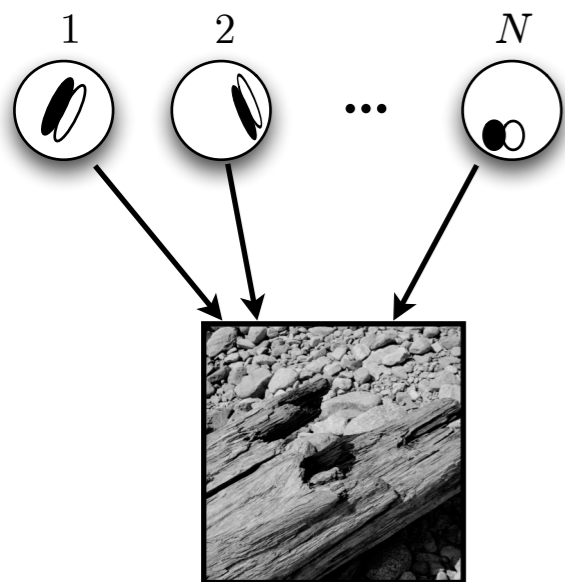
$$P(\mathbf{x}|\mathbf{y}) = \mathcal{N}(\mathbf{x}; \mathbf{A}\mathbf{y}, \sigma_x^2 \mathbf{I})$$

$$\mathbf{y} = z \mathbf{u}$$

$$P(\mathbf{u}) = \mathcal{N}(\mathbf{u}; \mathbf{0}, \mathbf{C})$$

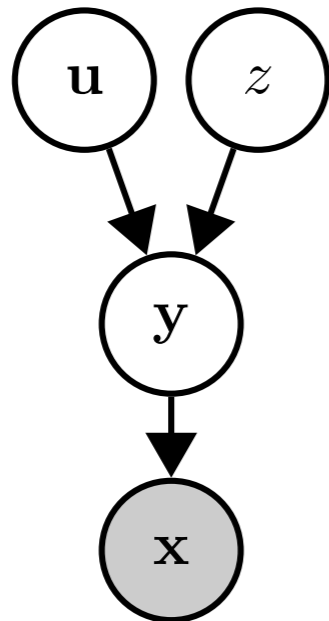
$$P(z) = \text{Gamma}(z; k, \theta)$$

linear features



image

Gaussian Scale Mixtures



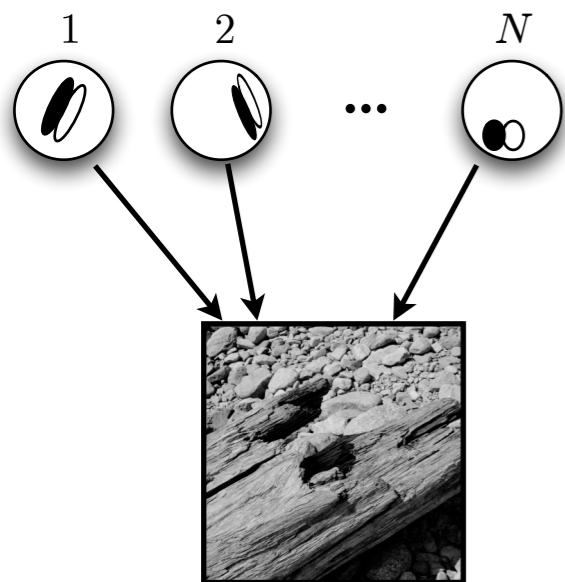
$$P(\mathbf{x}|\mathbf{y}) = \mathcal{N}(\mathbf{x}; \mathbf{A}\mathbf{y}, \sigma_x^2 \mathbf{I})$$

$$\mathbf{y} = z \mathbf{u}$$

$$P(\mathbf{u}) = \mathcal{N}(\mathbf{u}; \mathbf{0}, \mathbf{C})$$

$$P(z) = \text{Gamma}(z; k, \theta)$$

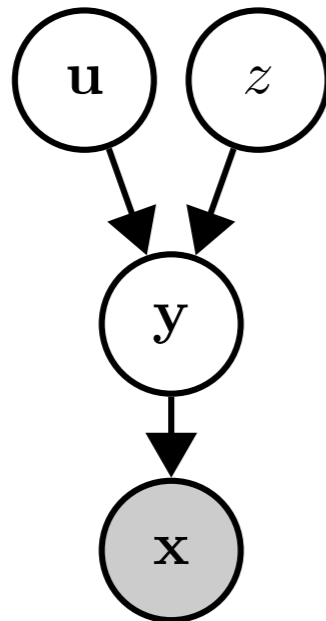
linear features



$$\text{image} = a_1 \text{feature}_1 + a_2 \text{feature}_2 + \dots + a_N \text{feature}_N + \text{noise}$$

image

Gaussian Scale Mixtures



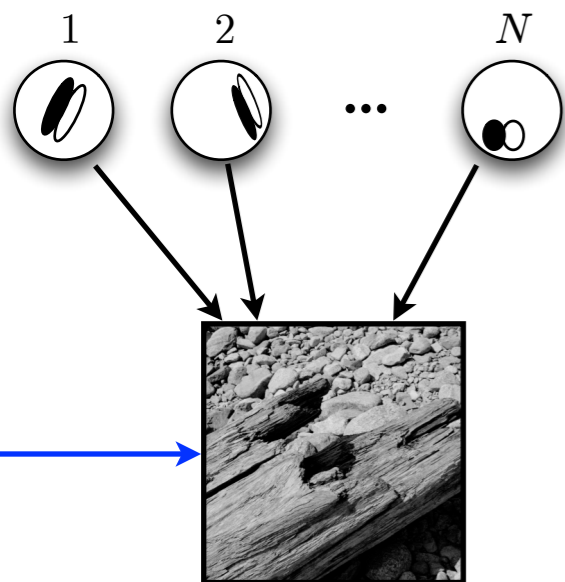
$$P(\mathbf{x}|\mathbf{y}) = \mathcal{N}(\mathbf{x}; \mathbf{A}\mathbf{y}, \sigma_x^2 \mathbf{I})$$

$$\mathbf{y} = z \mathbf{u}$$

$$P(\mathbf{u}) = \mathcal{N}(\mathbf{u}; \mathbf{0}, \mathbf{C})$$

$$P(z) = \text{Gamma}(z; k, \theta)$$

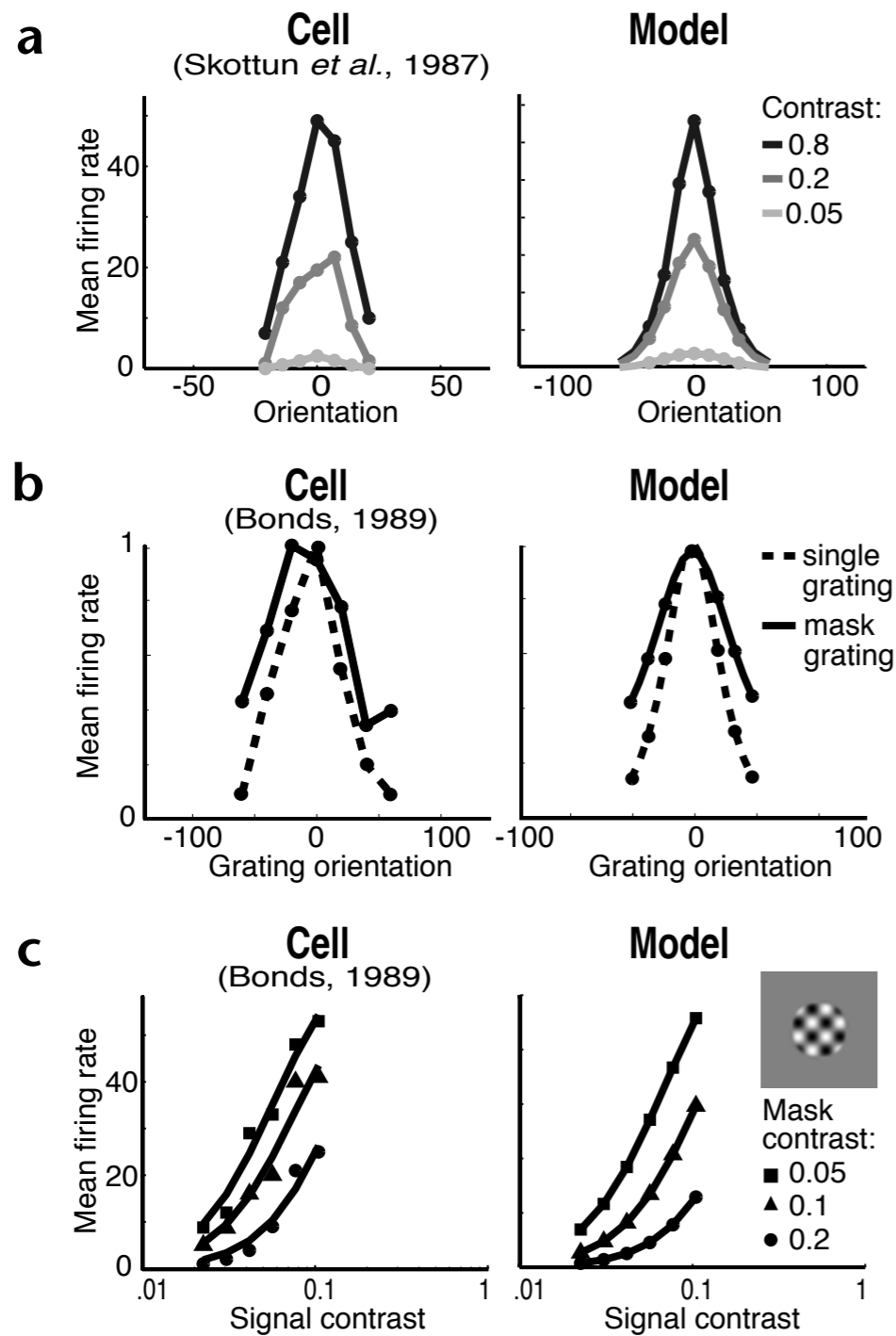
linear features



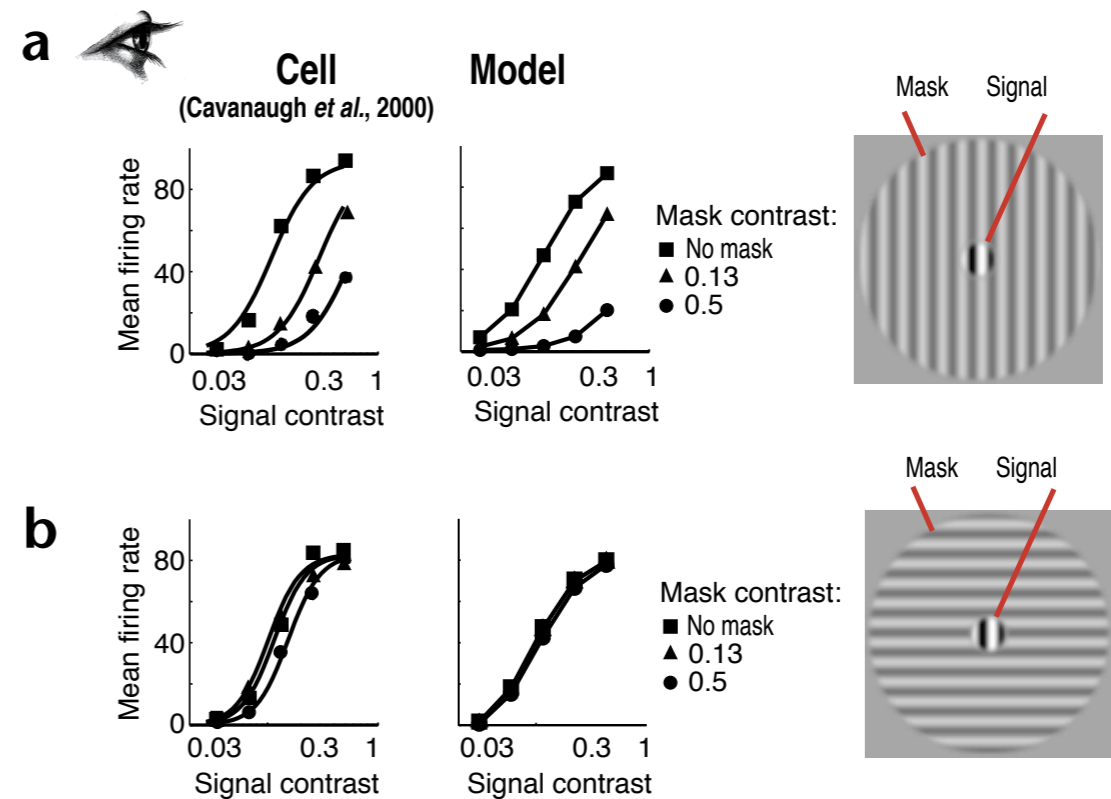
image

$$\text{image} = \text{contrast} \times (a_1 \text{feature}_1 + a_2 \text{feature}_2 + \dots + a_N \text{feature}_N + \text{noise})$$

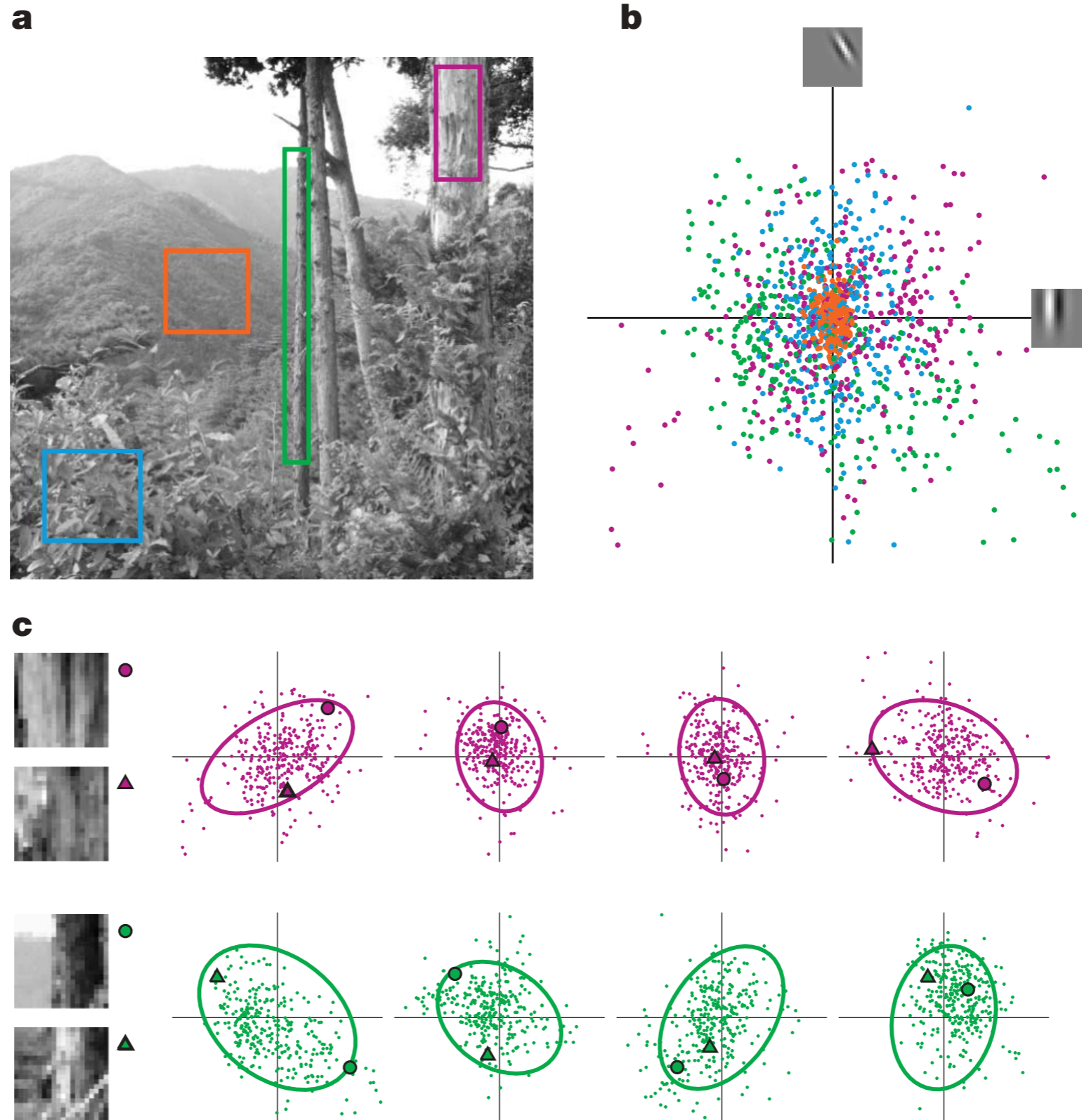
Neurális adatok és GSM



Schwartz & Simoncelli, 2001

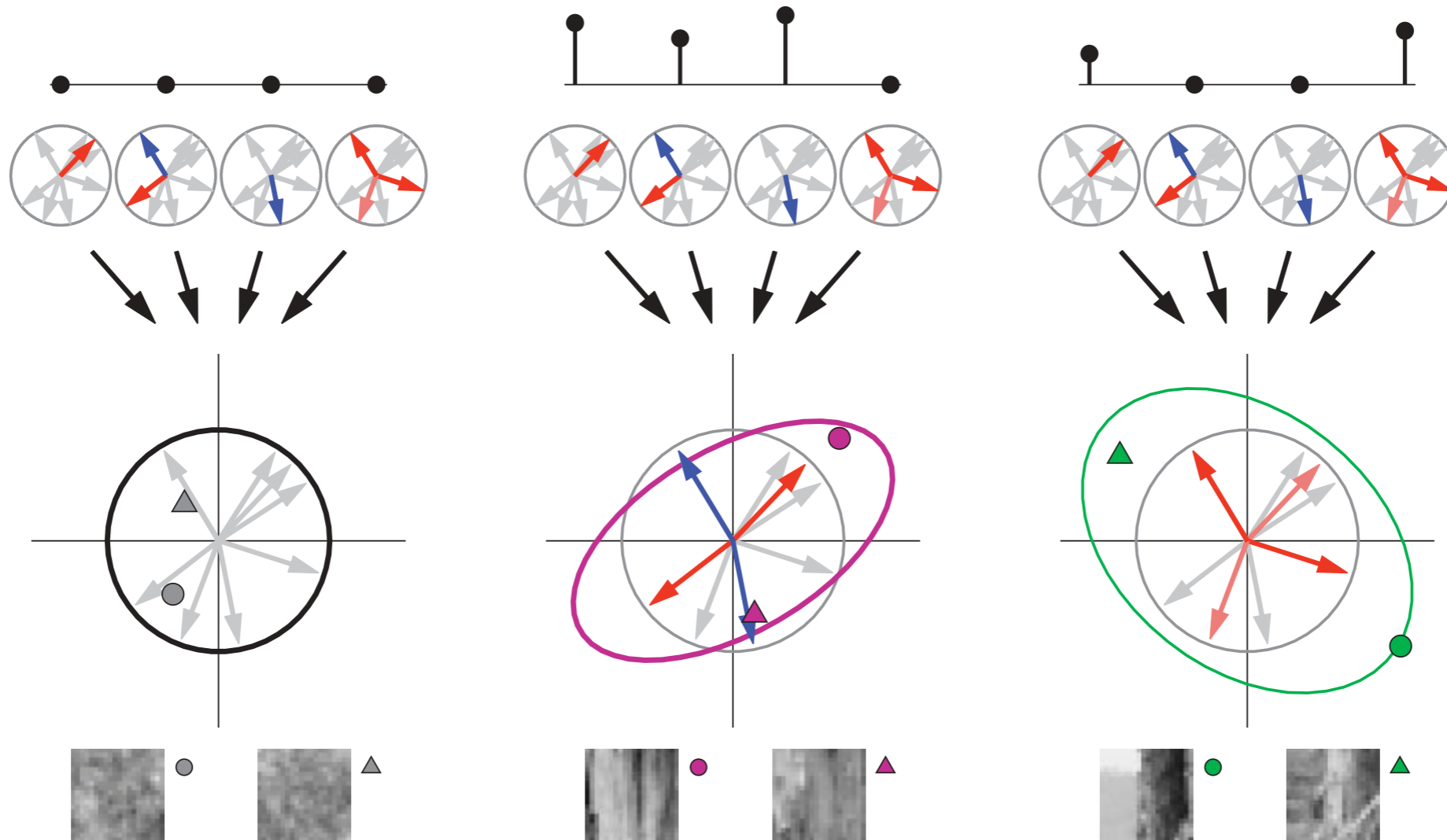


Egyrétegű modellek: filter korrelációk

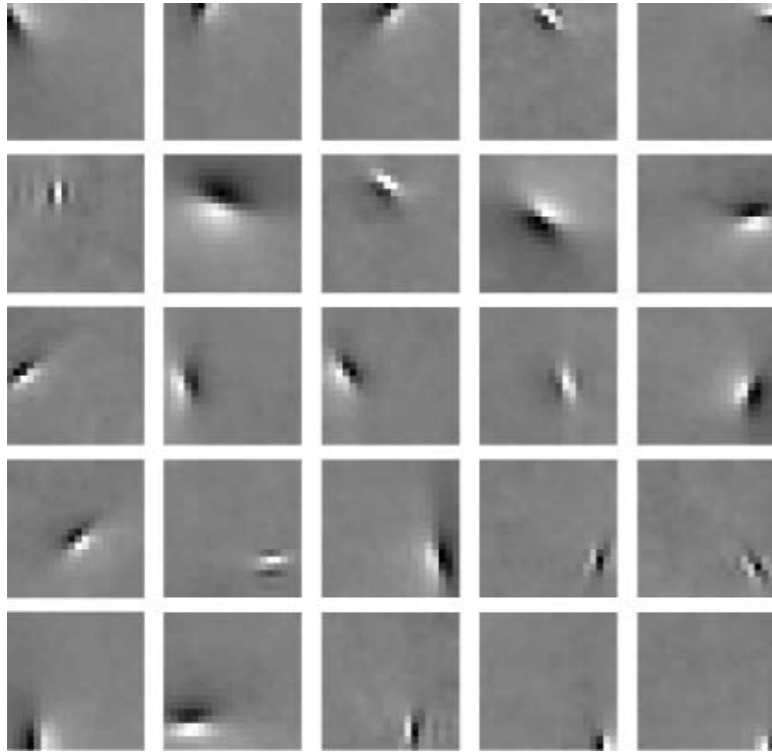


Egyrétegű modellek: filter korrelációk

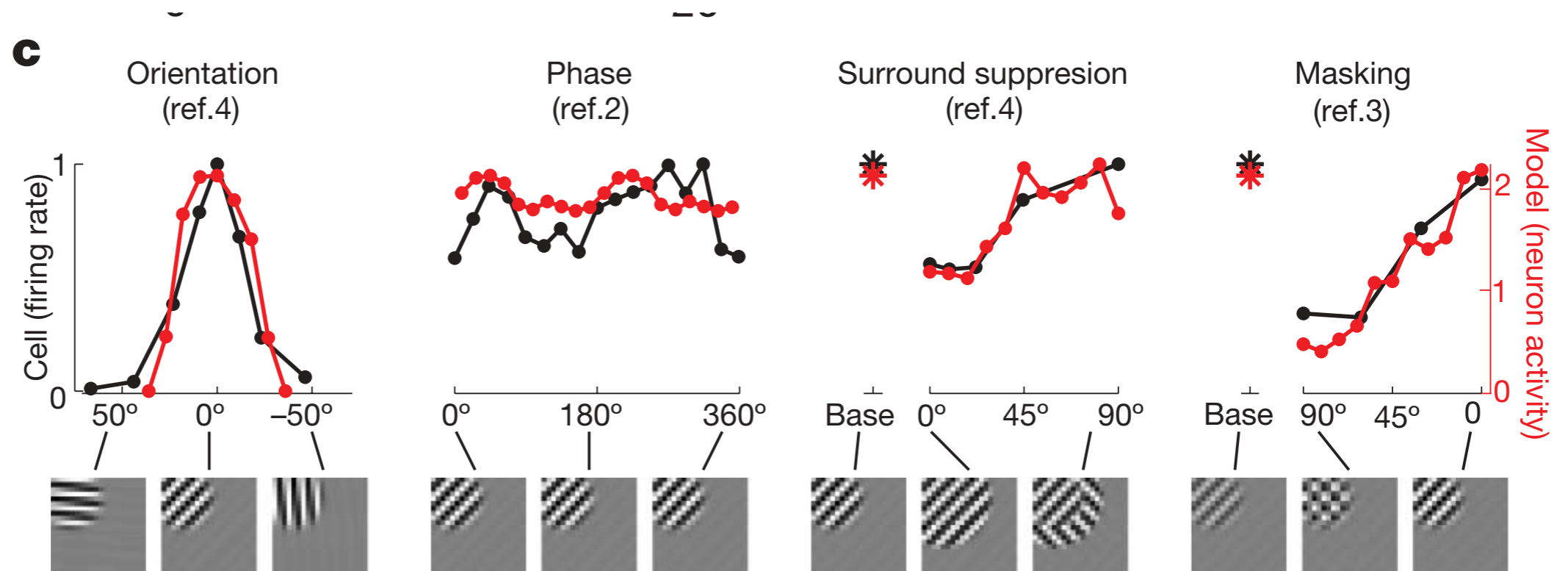
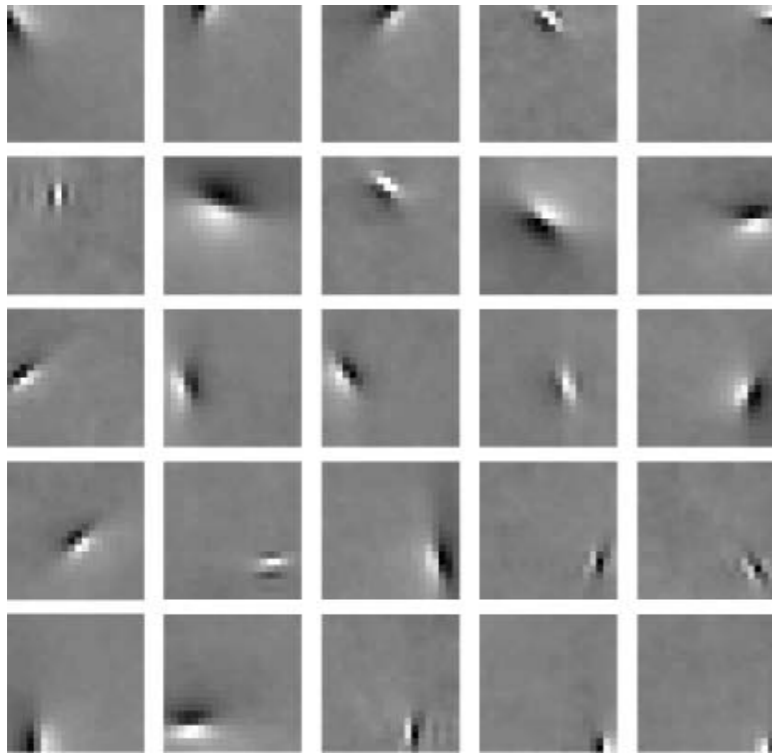
Változók a korrelációs struktúrát kódolják



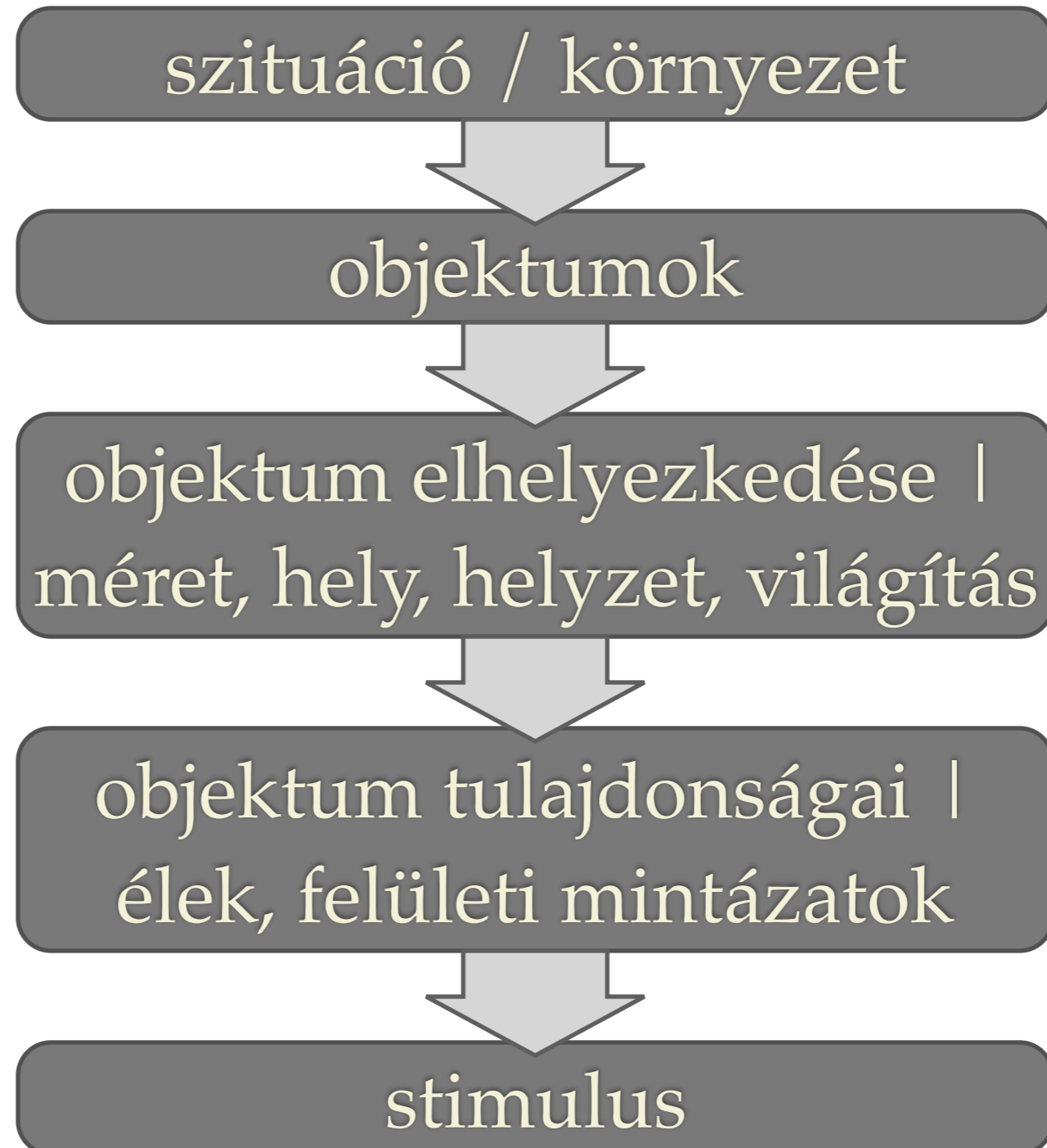
Egyrétegű modellek: filter korrelációk



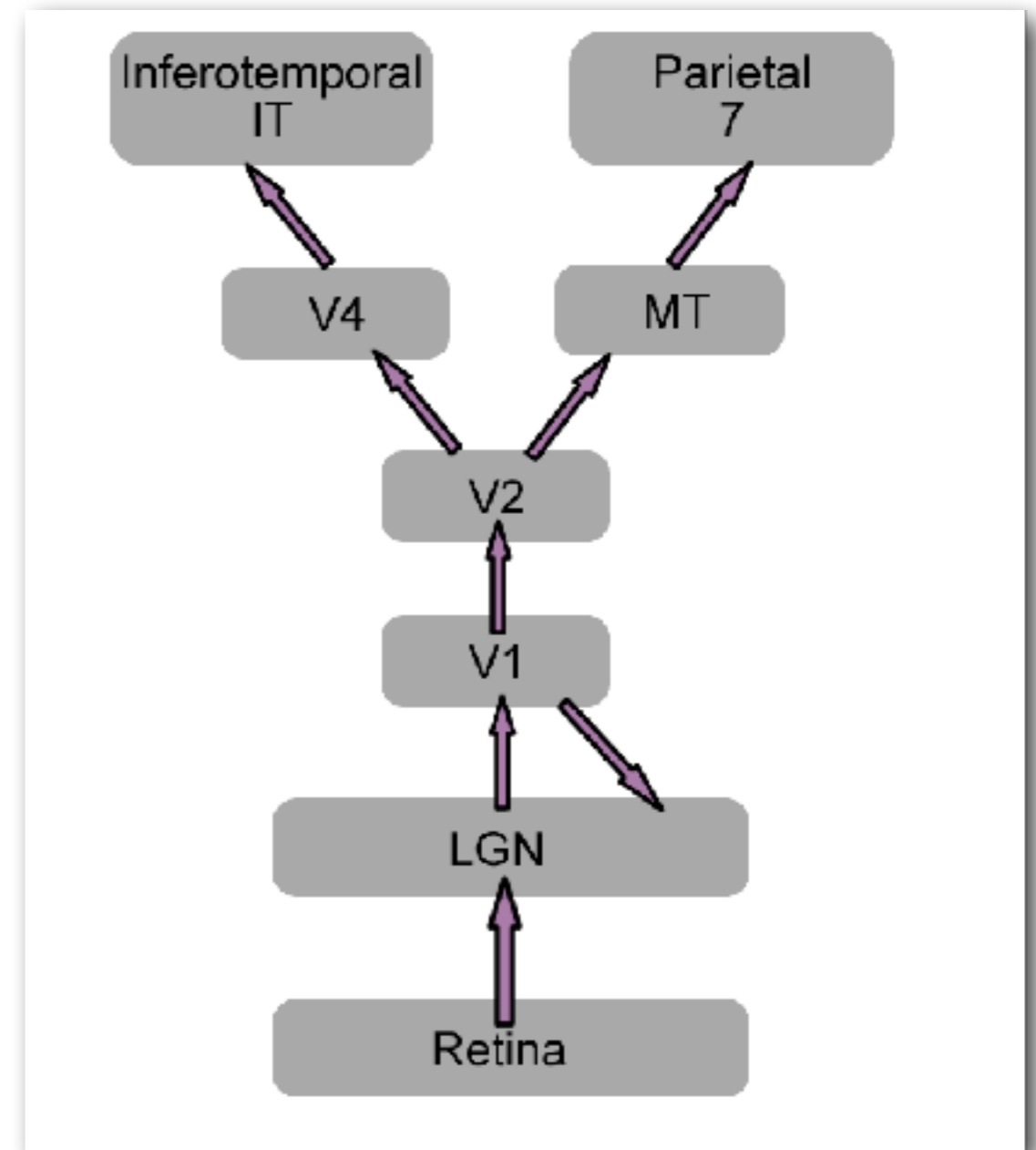
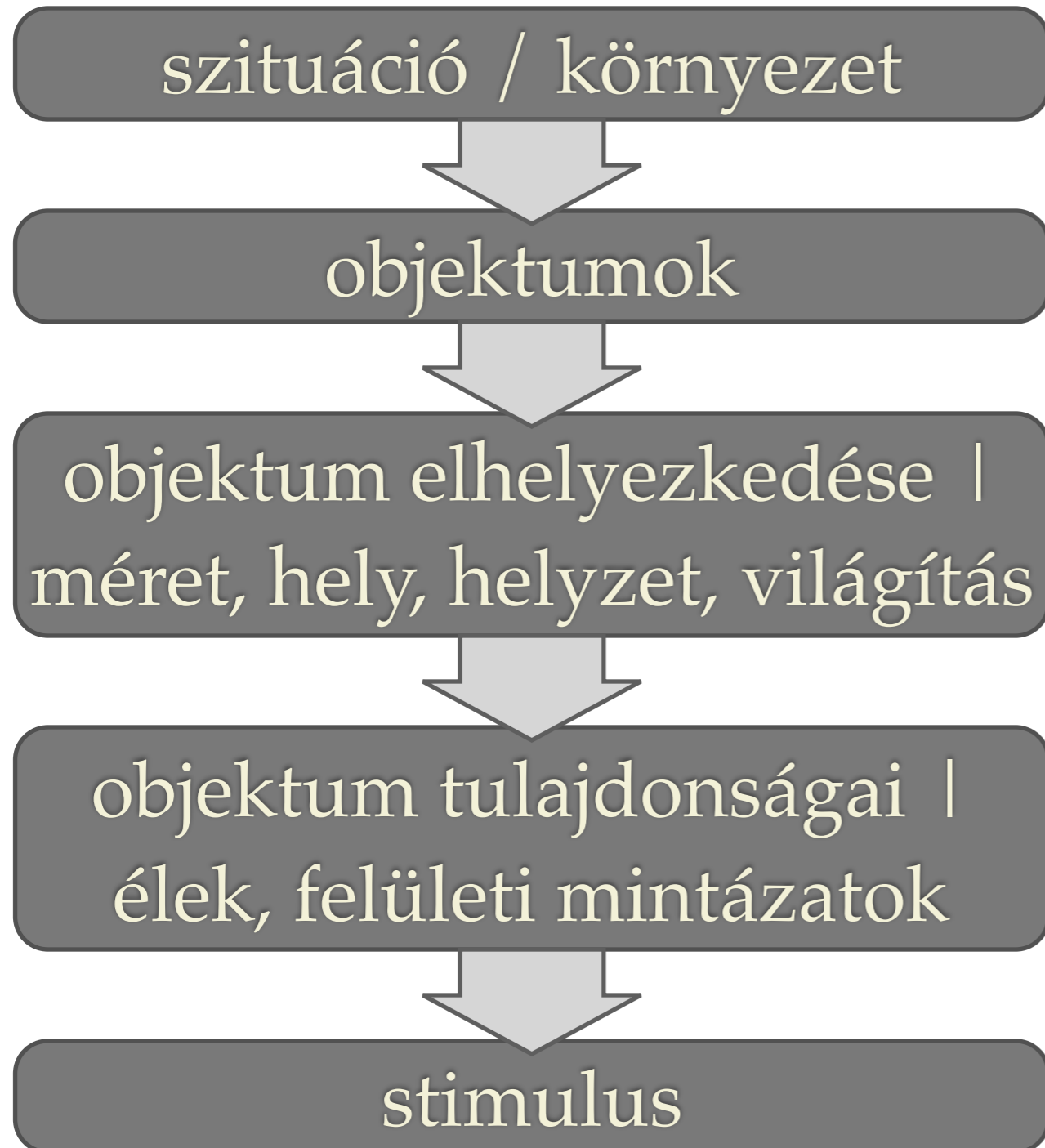
Egyrétegű modellek: filter korrelációk



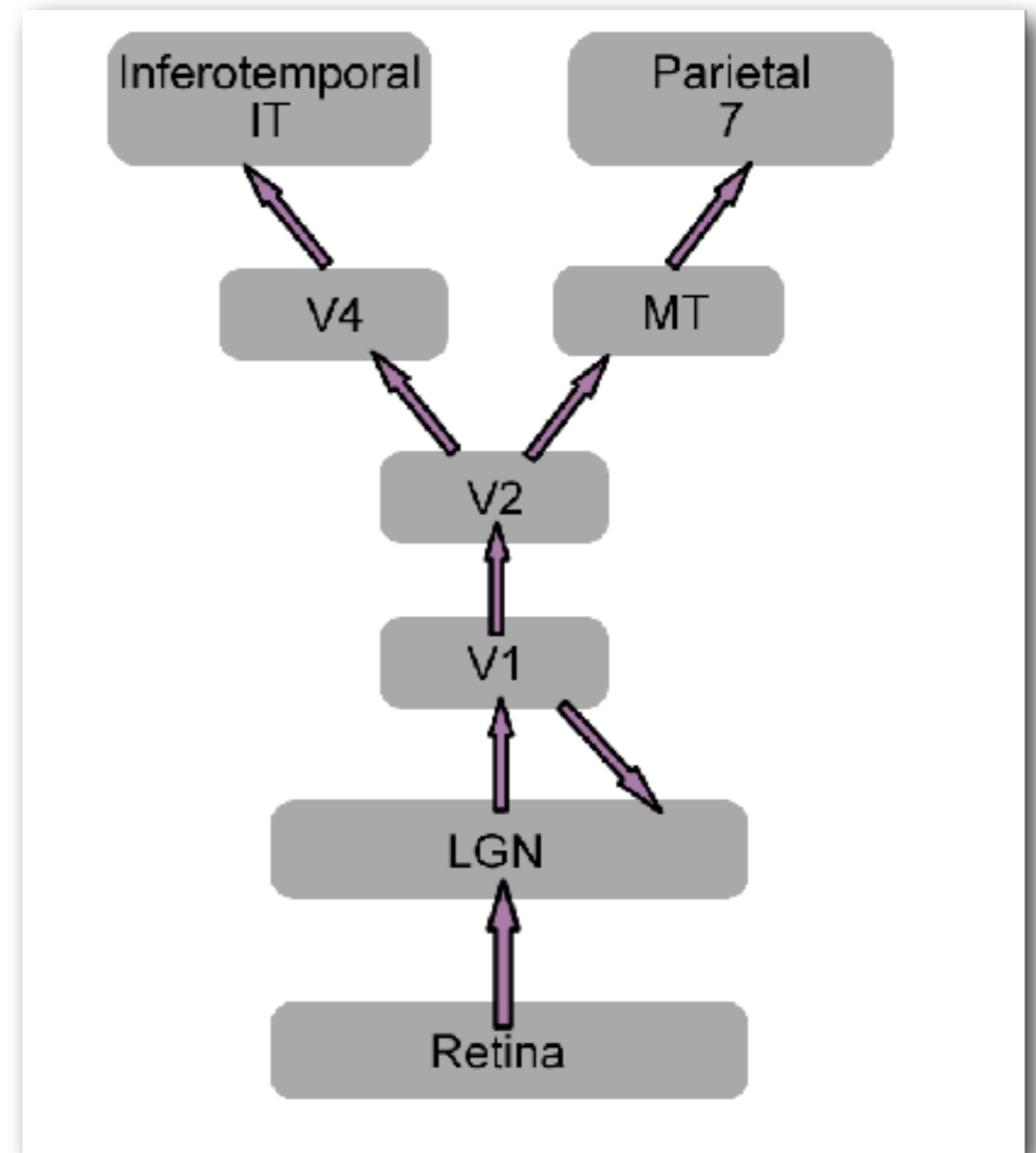
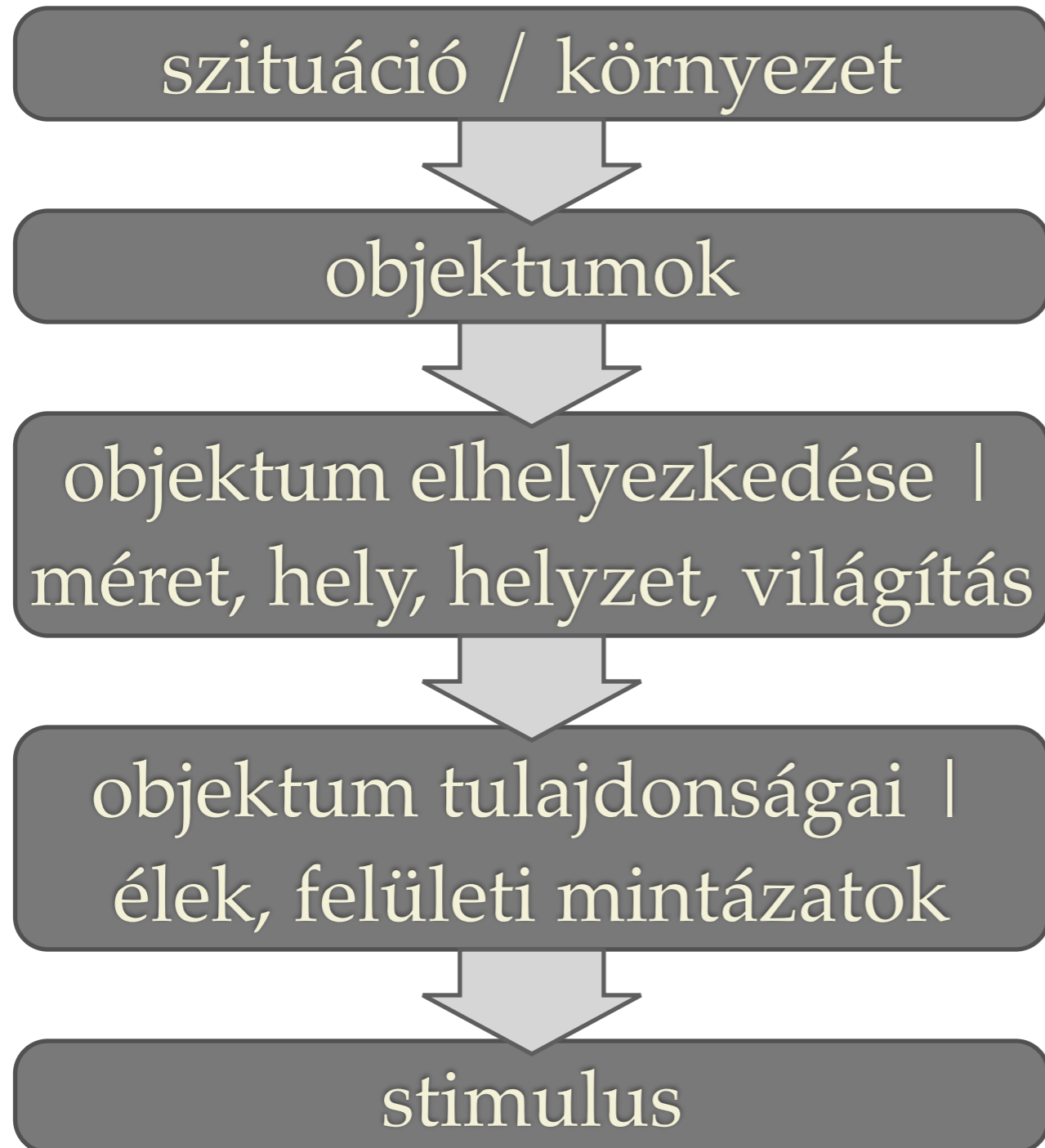
Generatív/rekogníciós modell



Generatív/rekogníciós modell



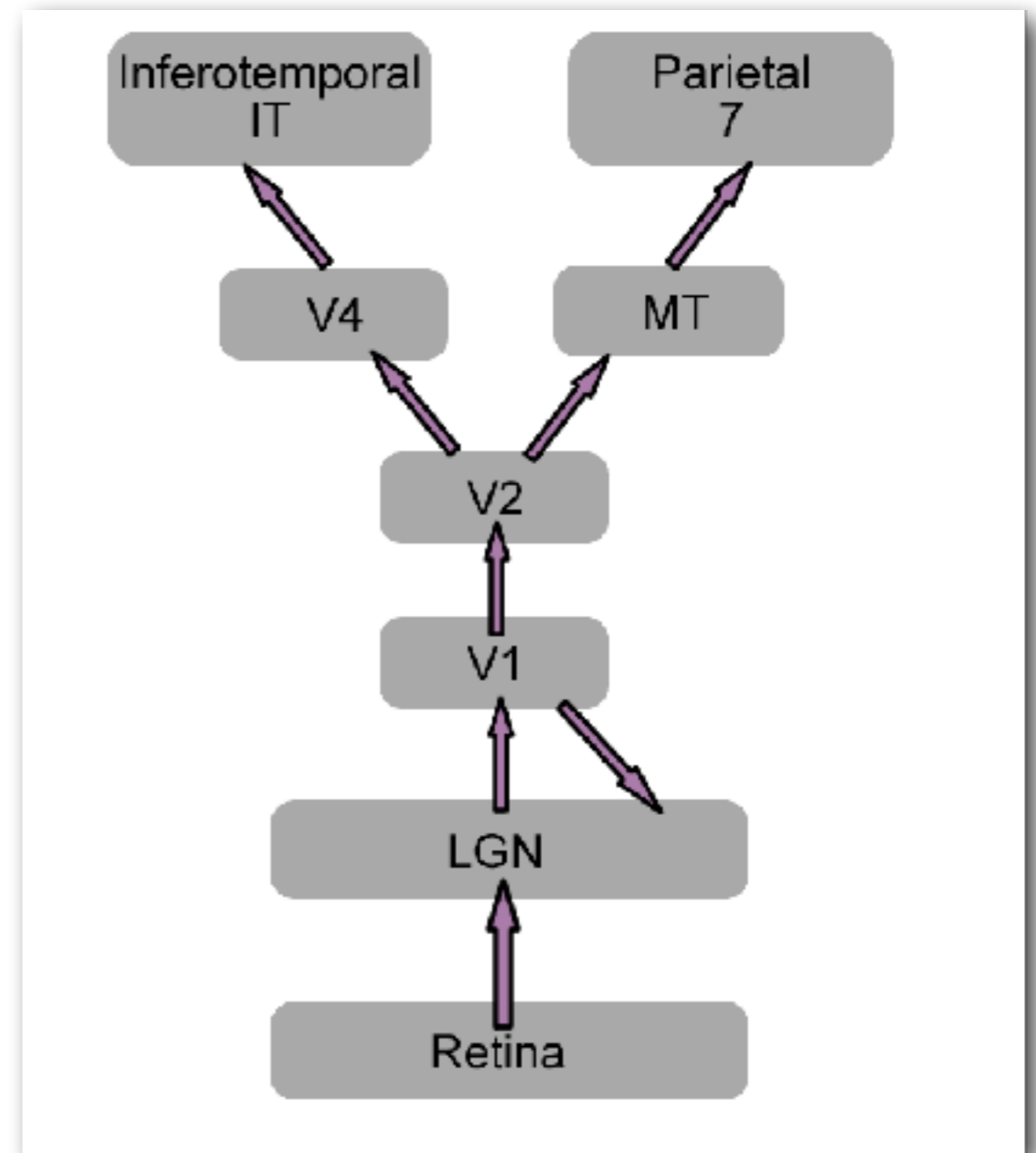
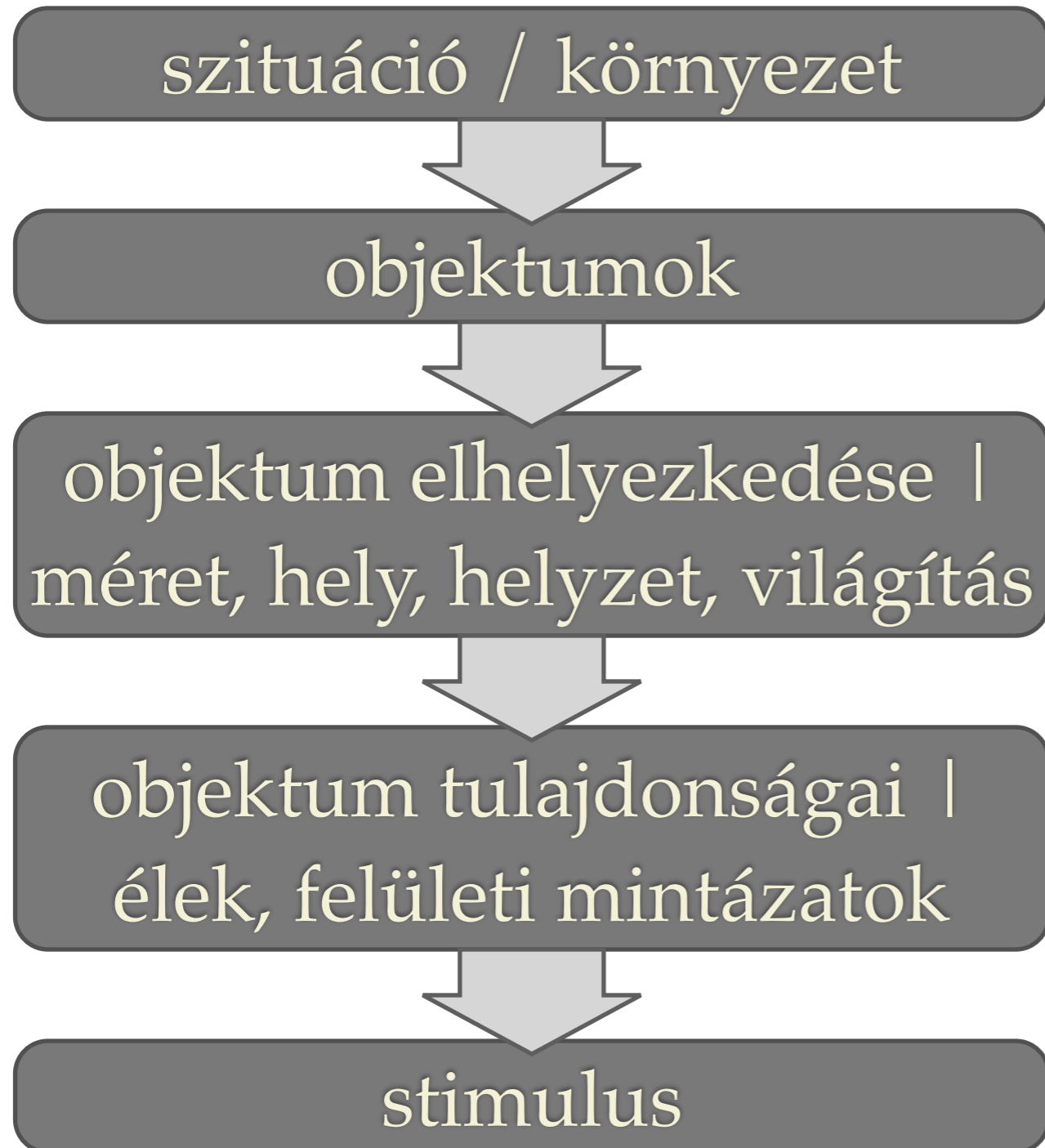
Generatív/rekogníciós modell



Modell definíció -> rekogníció:

$$P(x|z)$$

Generatív/rekogníciós modell

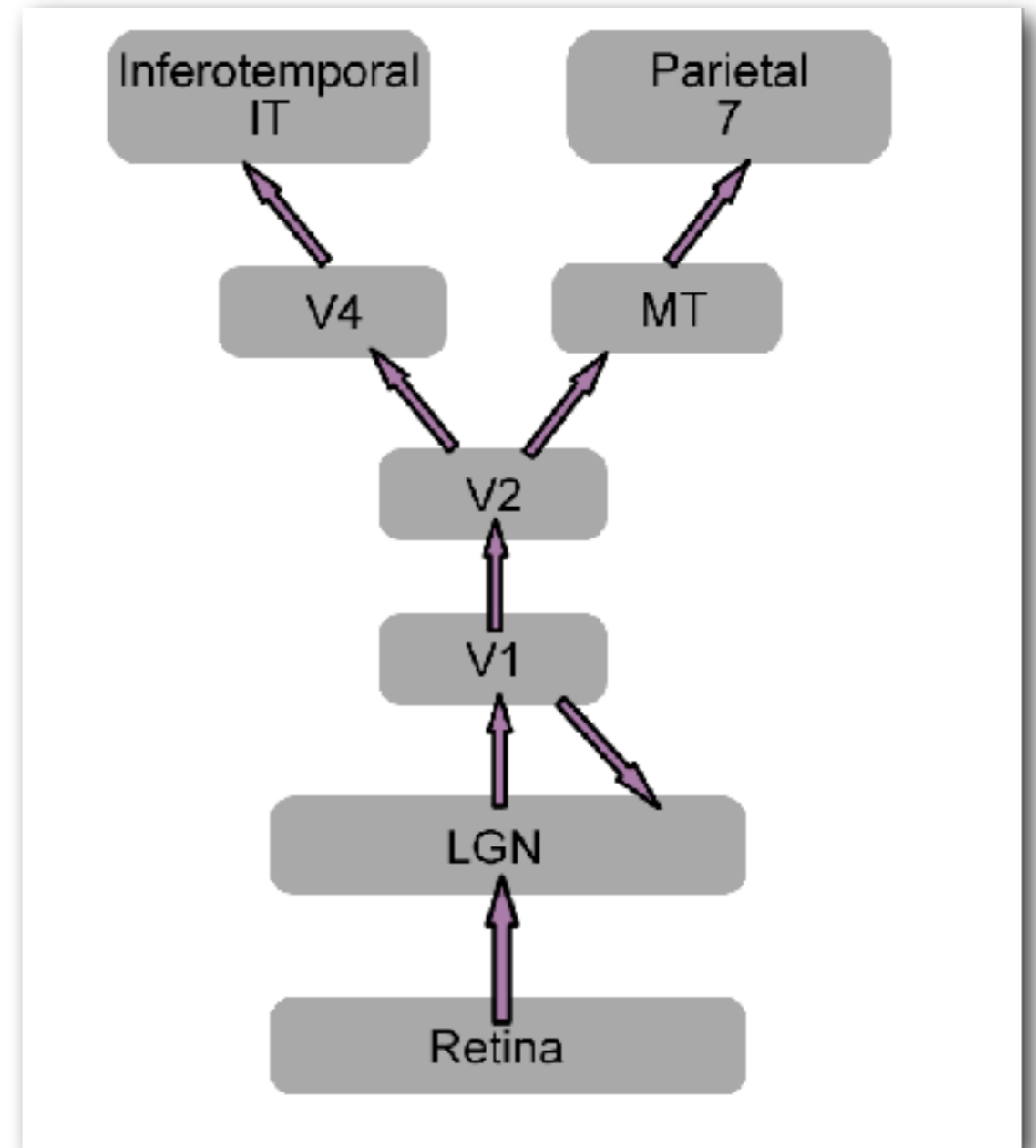
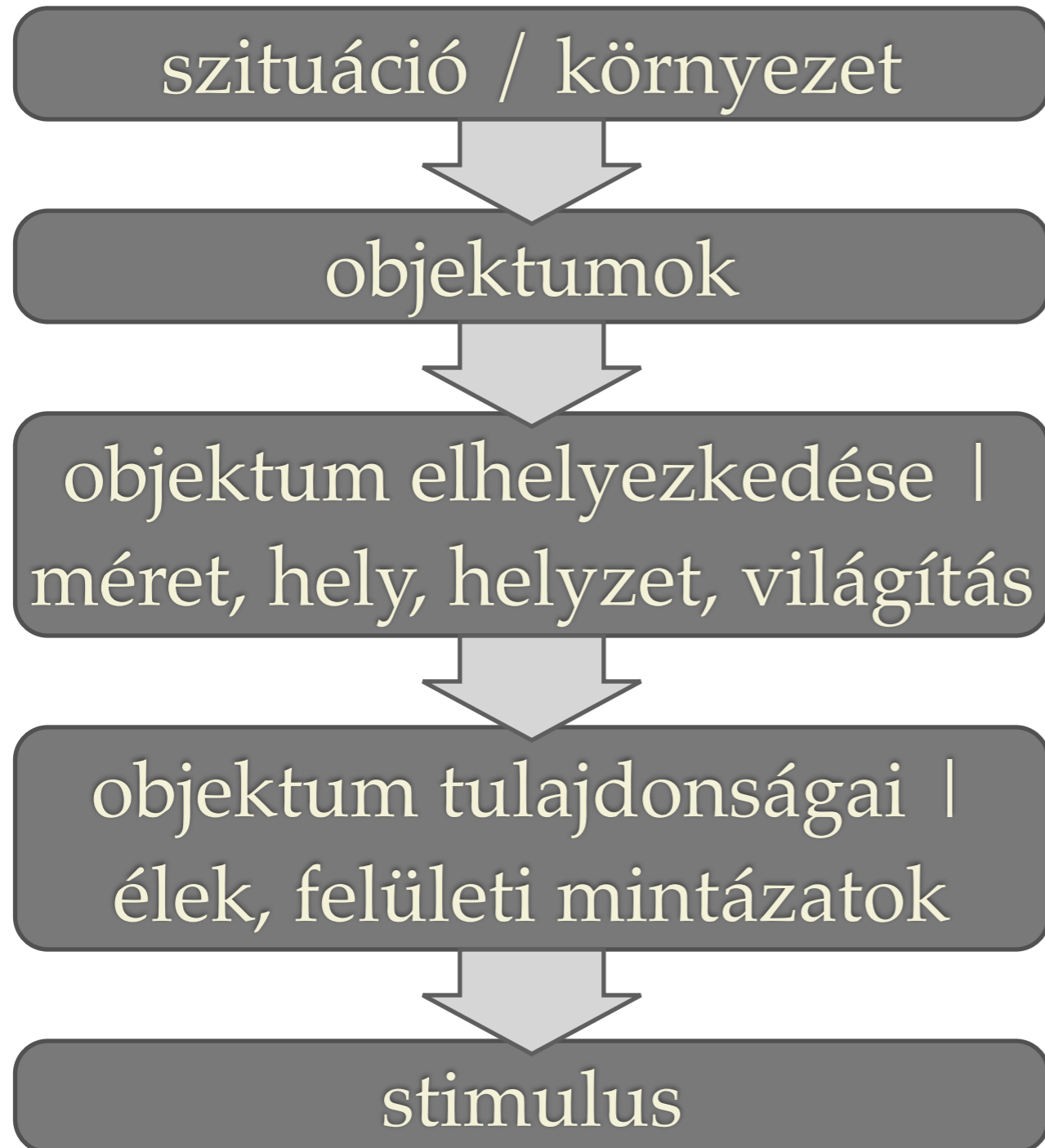


Modell definíció -> rekogníció:

$$P(x|z)$$

Inferencia igénye -> rekogníció:

Generatív/rekogníciós modell



Modell definíció -> rekogníció:

$$P(x|z)$$

Inferencia igénye -> rekogníció:

$$P(z|x)$$

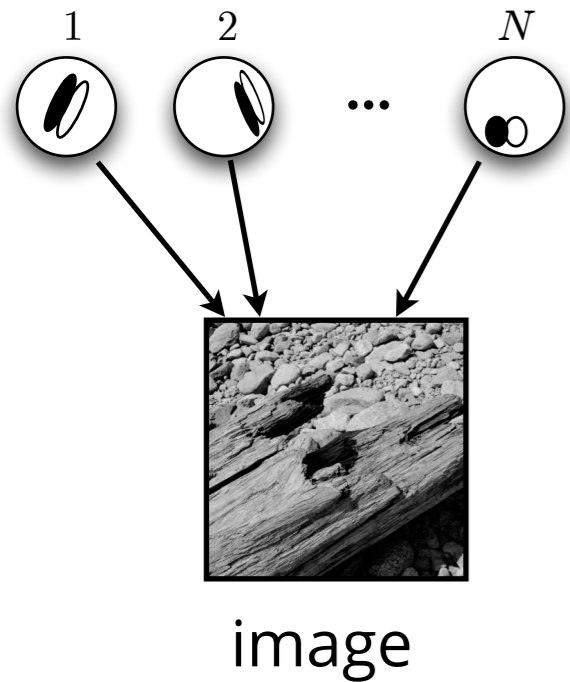
Bayesian inference



image

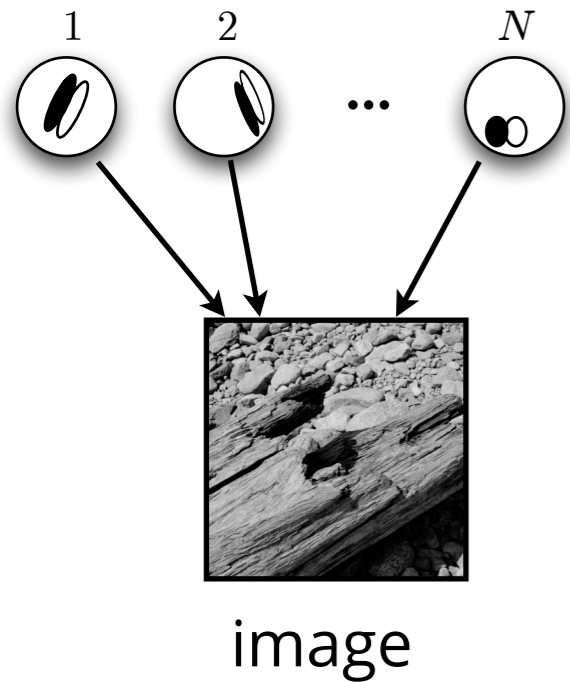
Bayesian inference

linear features



Bayesian inference

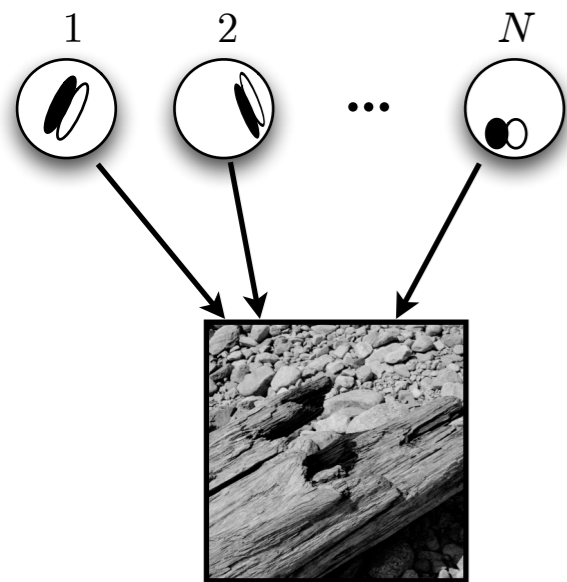
linear features



$$\text{image} = a_1 \text{feature}_1 + a_2 \text{feature}_2 + \dots + a_N \text{feature}_N + \text{noise}$$

Bayesian inference

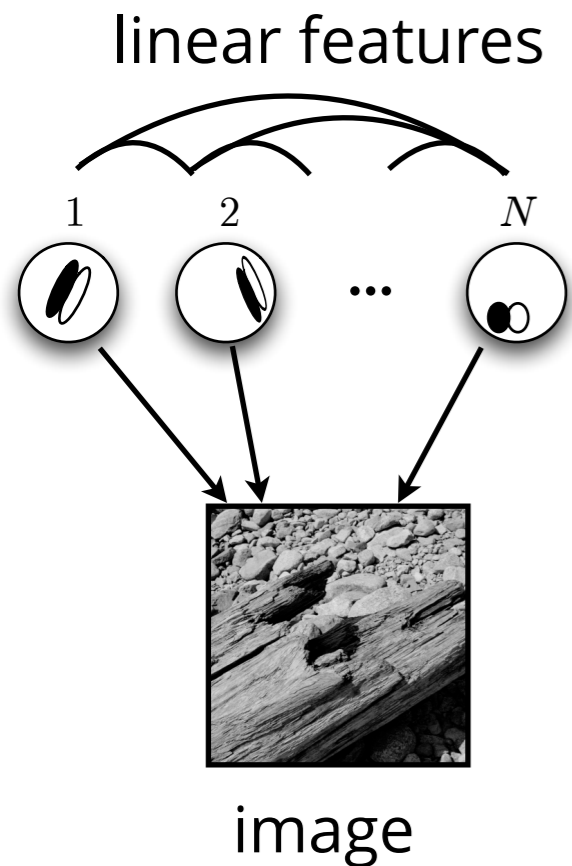
linear features



neural activities
[a_1, a_2, \dots, a_N]

$$\text{image} = a_1 \text{feature}_1 + a_2 \text{feature}_2 + \dots + a_N \text{feature}_N + \text{noise}$$

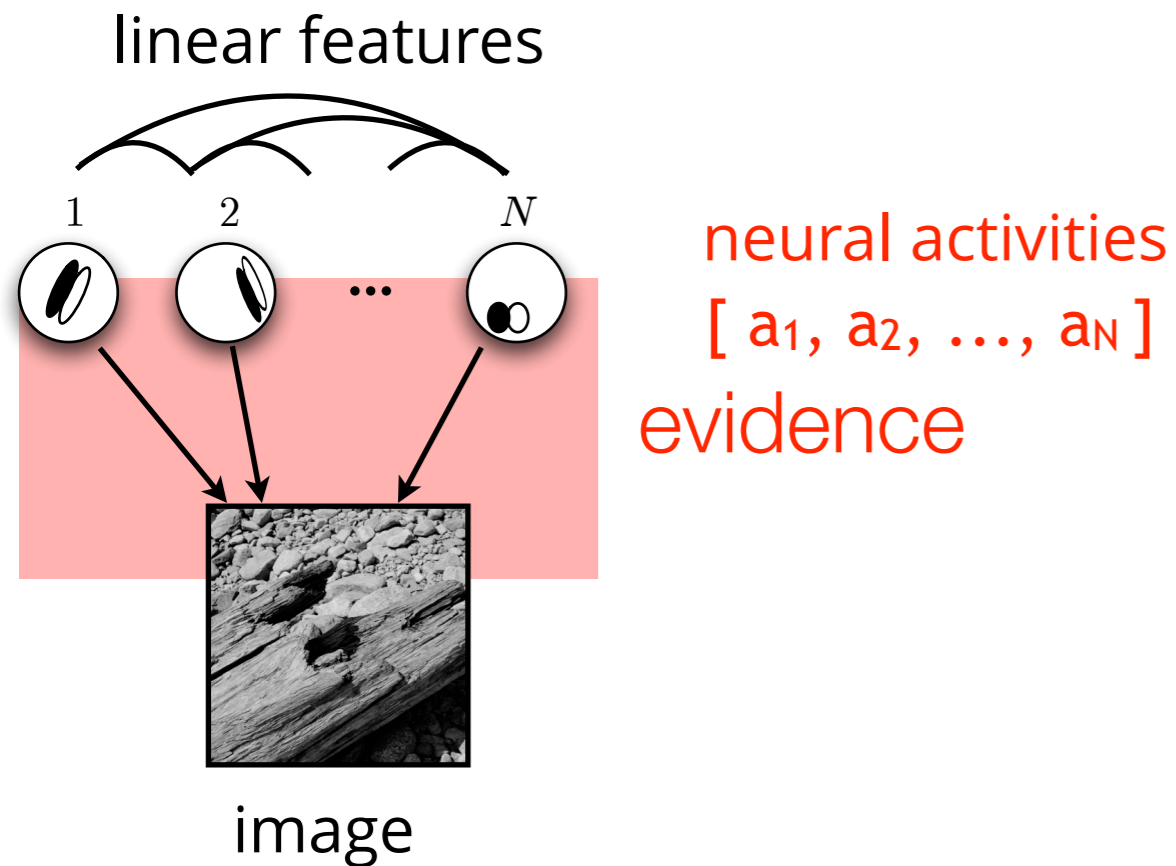
Bayesian inference



neural activities
[a_1, a_2, \dots, a_N]

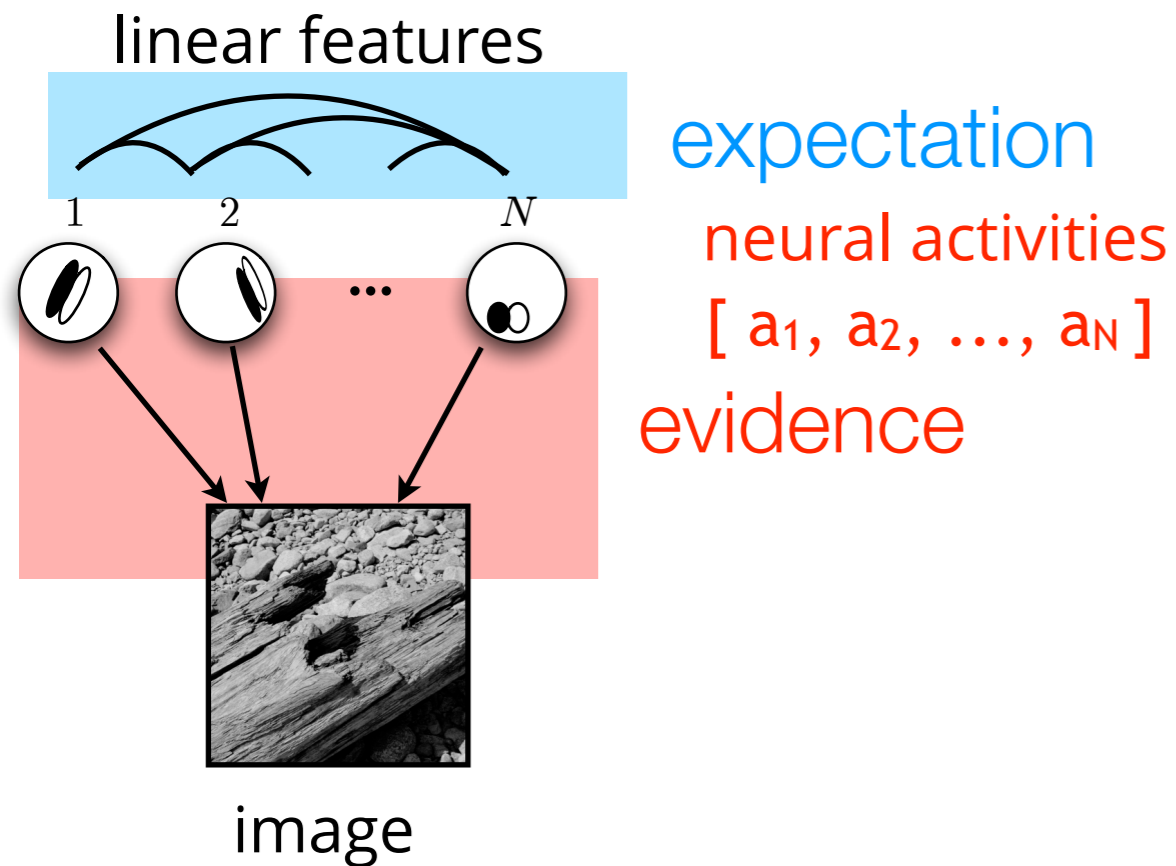
$$\text{image} = a_1 \text{feature}_1 + a_2 \text{feature}_2 + \dots + a_N \text{feature}_N + \text{noise}$$

Bayesian inference



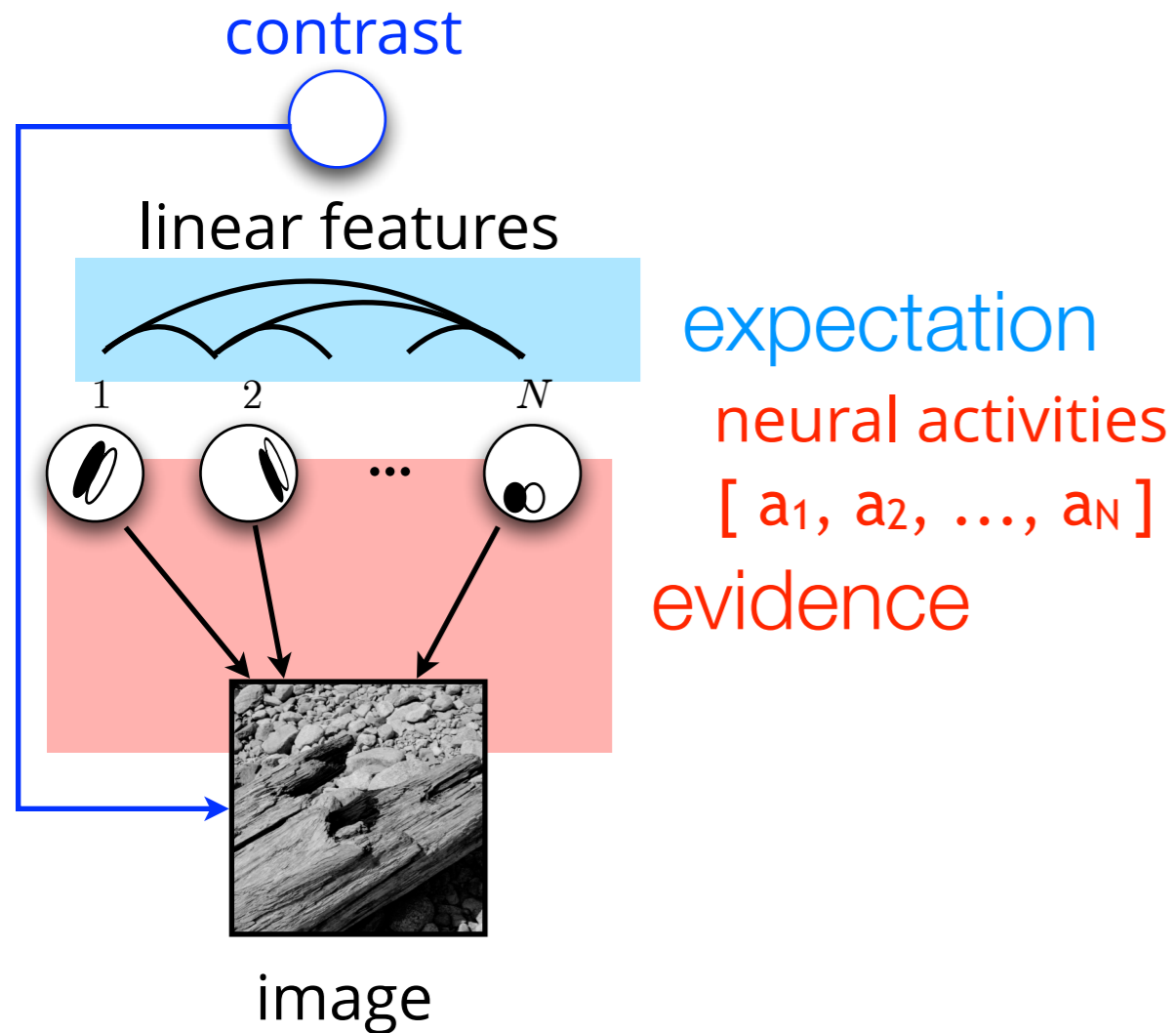
$$\text{image} = a_1 \text{feature}_1 + a_2 \text{feature}_2 + \dots + a_N \text{feature}_N + \text{noise}$$

Bayesian inference



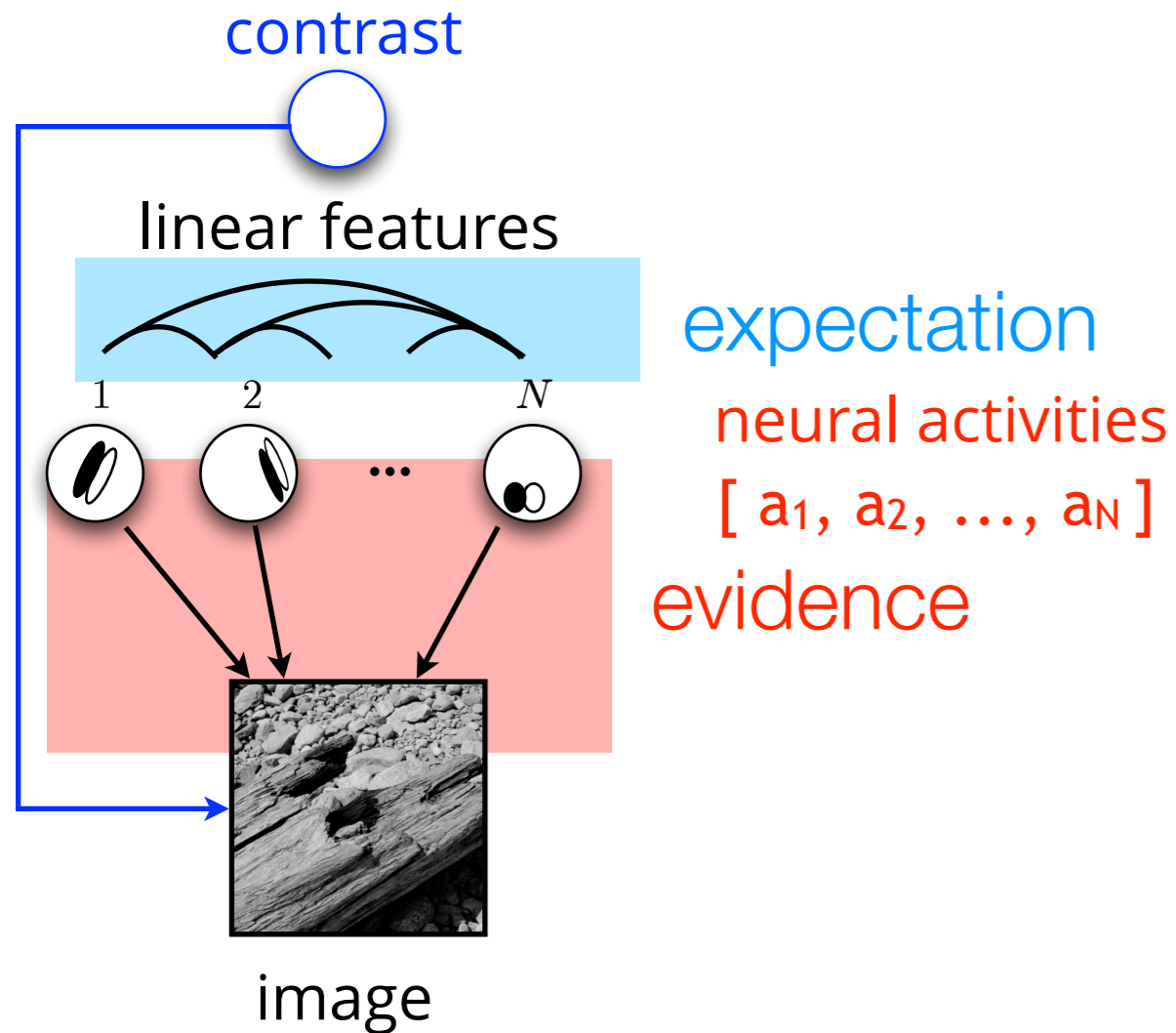
$$\text{image} = a_1 \text{feature}_1 + a_2 \text{feature}_2 + \dots + a_N \text{feature}_N + \text{noise}$$

Bayesian inference



$$\text{image} = \text{contrast} \times \left(a_1 \text{ feature}_1 + a_2 \text{ feature}_2 + \dots + a_N \text{ feature}_N \right) + \text{noise}$$

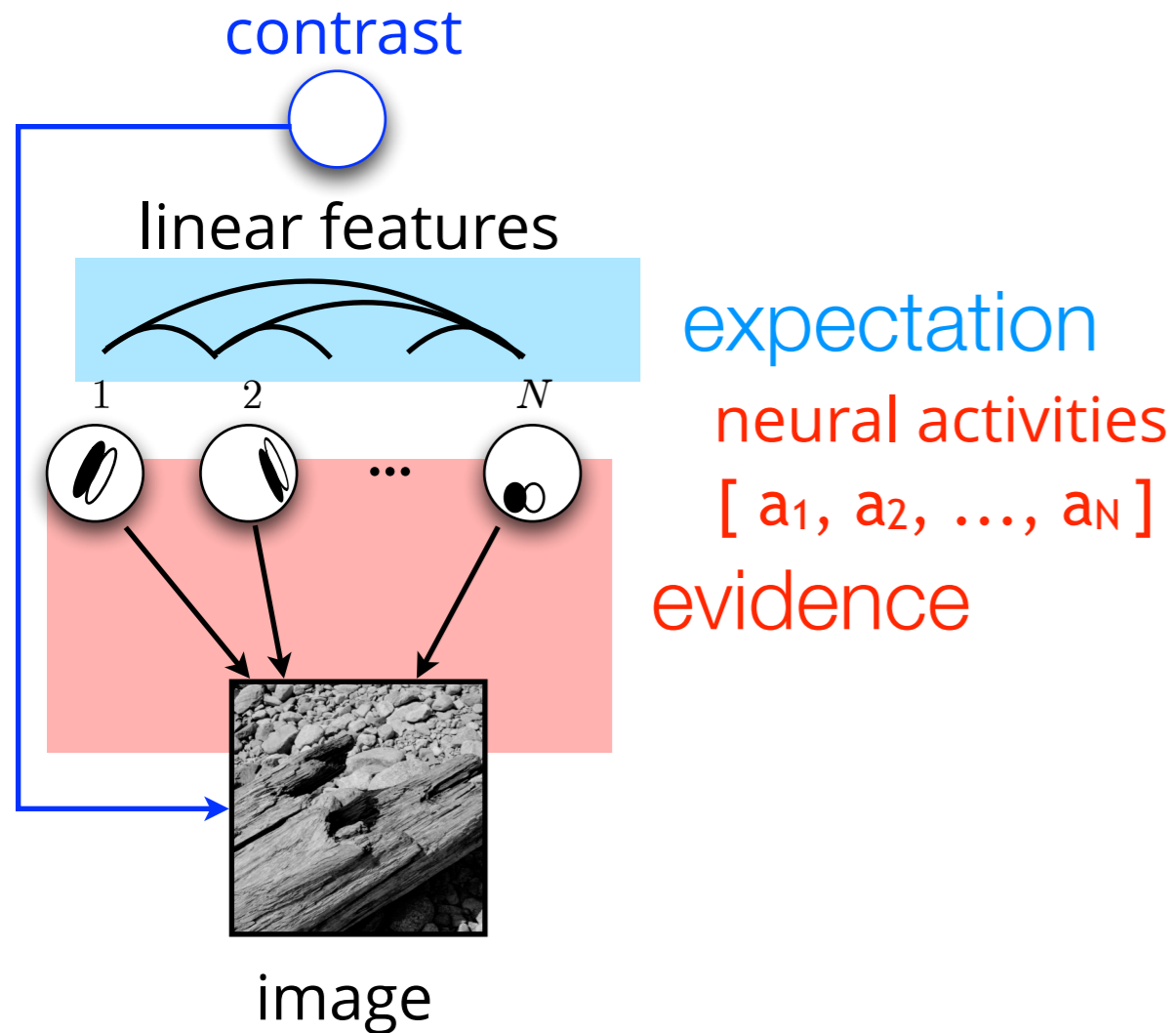
Bayesian inference



$$\text{image} = \text{contrast} \times \left(a_1 \text{feature}_1 + a_2 \text{feature}_2 + \dots + a_N \text{feature}_N \right) + \text{noise}$$

$\underbrace{P(\text{image} | a_1, a_2, \dots, a_N, c)}_{\text{sensory evidence}}$

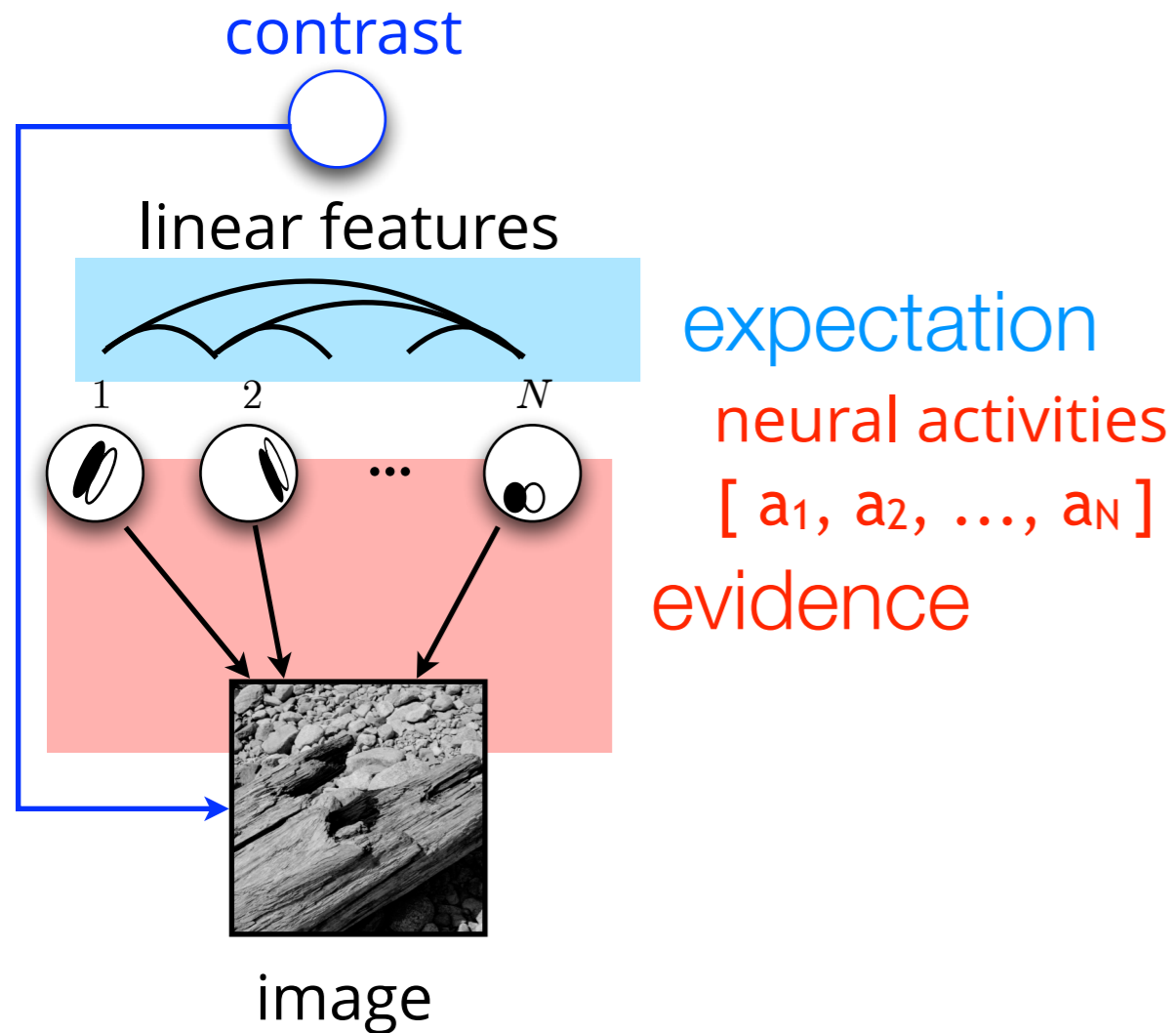
Bayesian inference



$$\text{image} = \text{contrast} \times \left(a_1 \text{feature}_1 + a_2 \text{feature}_2 + \dots + a_N \text{feature}_N \right) + \text{noise}$$

$$\underbrace{P(a_1, a_2, \dots, a_N | \text{image}, c)}_{\text{posterior}} \propto \underbrace{P(a_1, a_2, \dots, a_N | c)}_{\text{prior}} \times \underbrace{P(\text{image} | a_1, a_2, \dots, a_N, c)}_{\text{sensory evidence}}$$

Bayesian inference



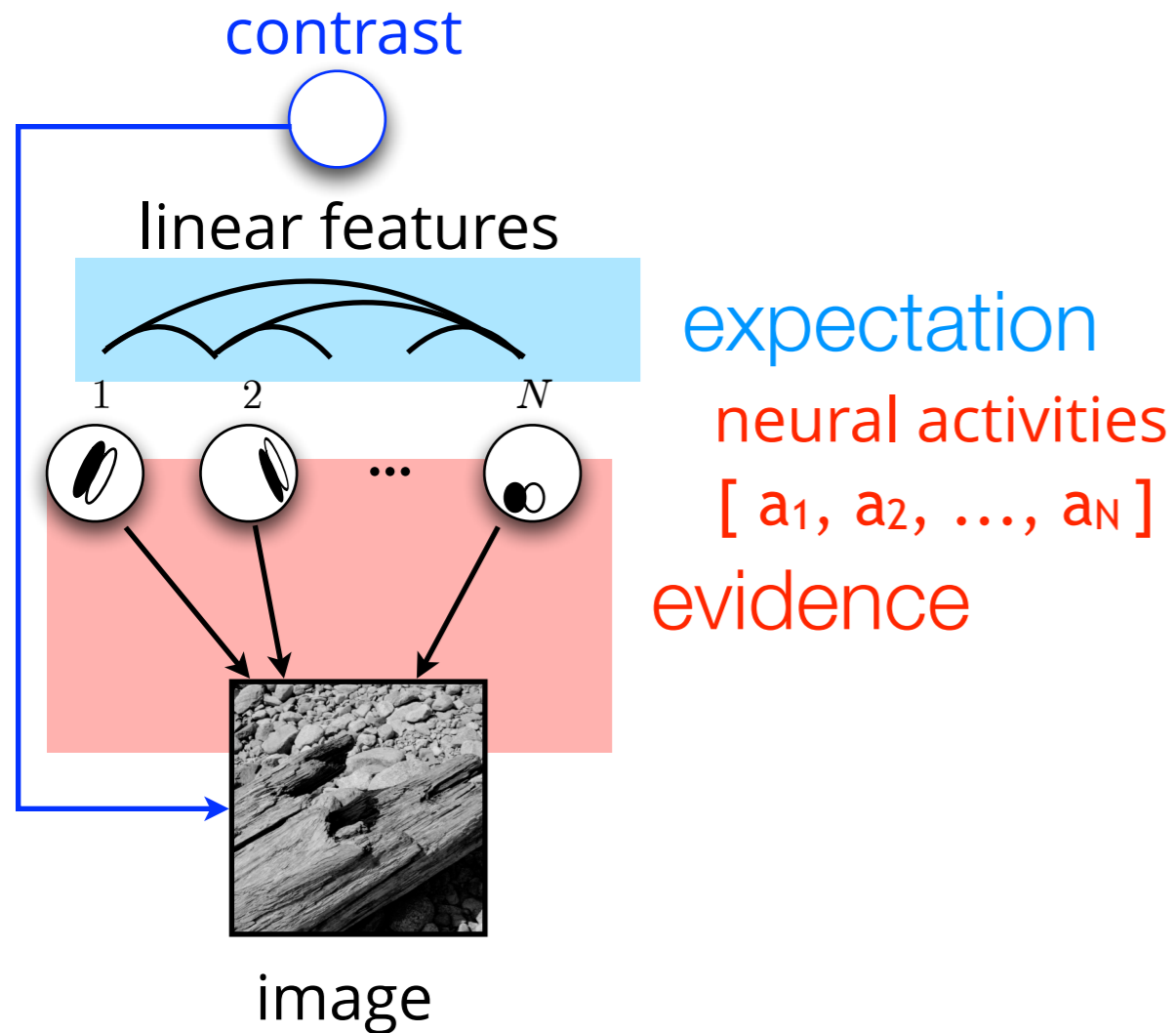
Demonstrated efficiency in:

- ★ pattern-completion
- ★ compression
- ★ denoising

$$\text{image} = \text{contrast} \times \left(a_1 \text{feature}_1 + a_2 \text{feature}_2 + \dots + a_N \text{feature}_N \right) + \text{noise}$$

$$\underbrace{P(a_1, a_2, \dots, a_N | \text{image}, c)}_{\text{posterior}} \propto \underbrace{P(a_1, a_2, \dots, a_N | c)}_{\text{prior}} \times \underbrace{P(\text{image} | a_1, a_2, \dots, a_N, c)}_{\text{sensory evidence}}$$

Bayesian inference



Demonstrated efficiency in:

- ★ pattern-completion
- ★ compression
- ★ denoising

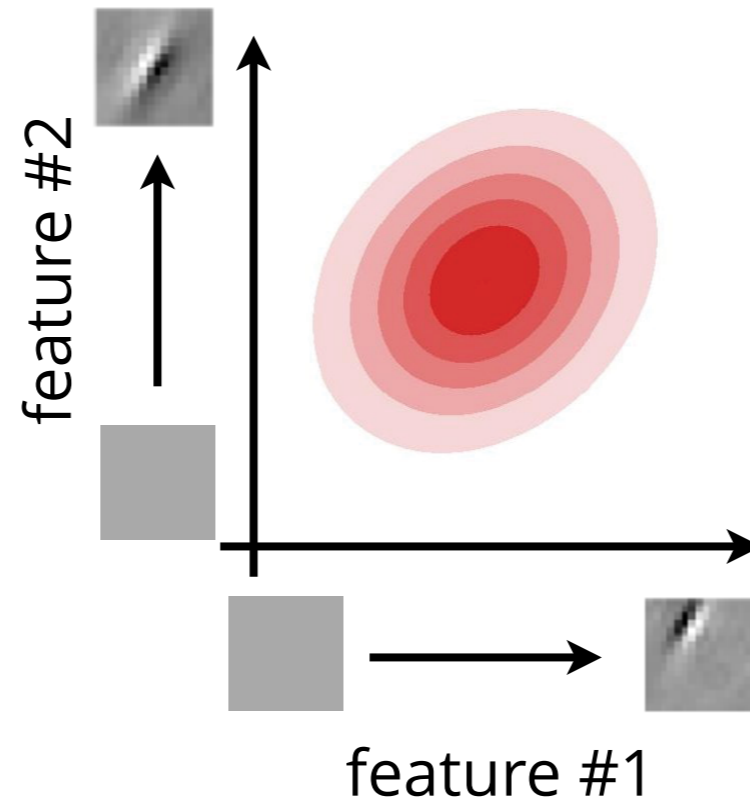
the parametric form of both
evidence and expectation is determined by
natural image statistics

mean responses

$$P(a_1, a_2 \mid \text{image}, c)$$

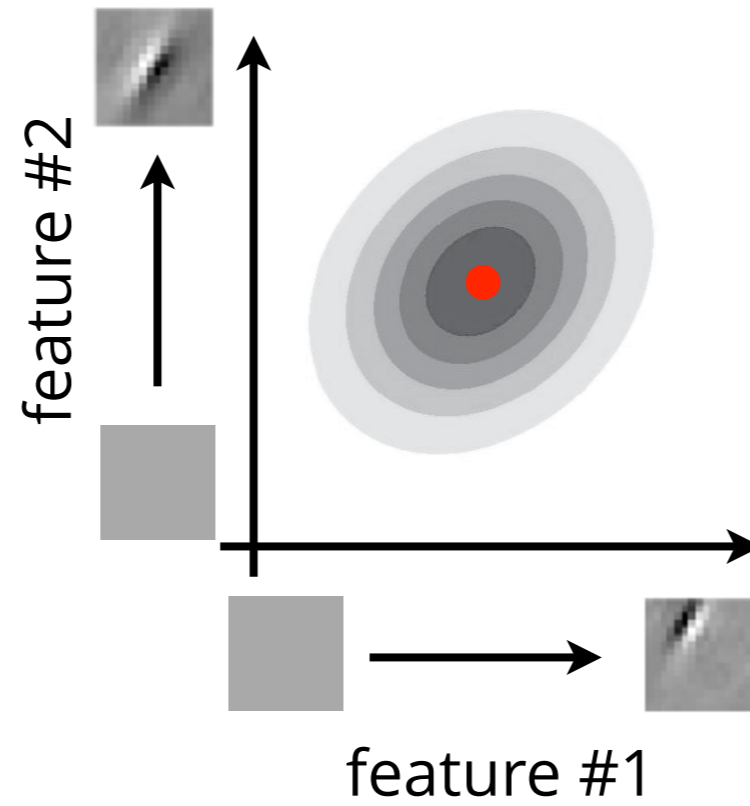
mean responses

$$P(a_1, a_2 | \text{image}, c)$$

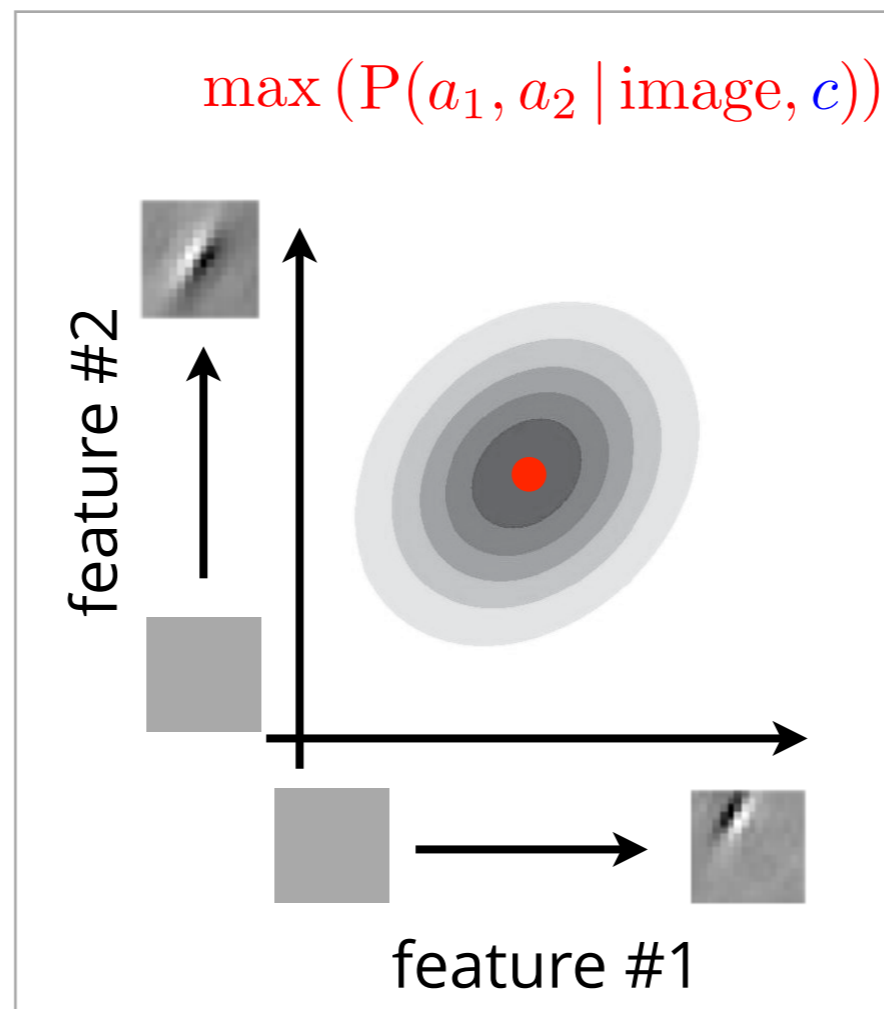


mean responses

$$\max (P(a_1, a_2 | \text{image}, c))$$



mean responses



traditional theories

e.g. Olshausen & Field, Nature 1996,
Schwartz & Simoncelli, Nat Neurosci
2001

mean response \leadsto maximum a posteriori inference

roadmap

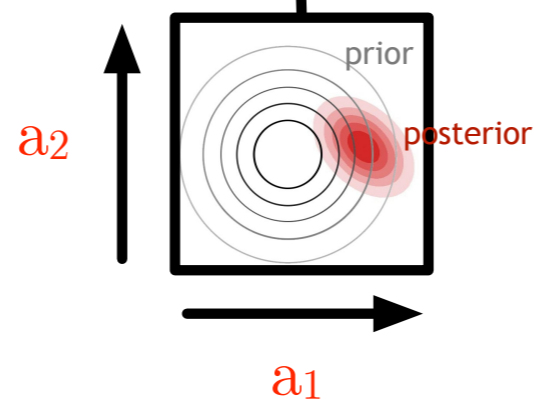
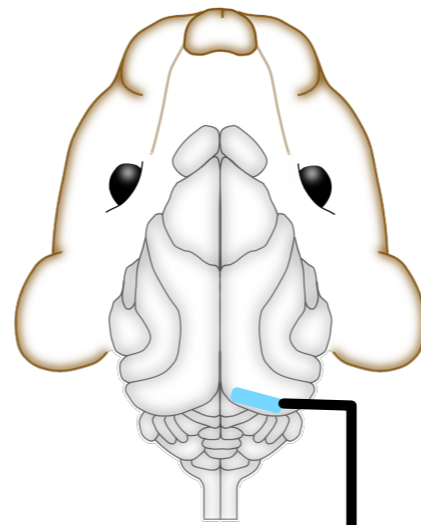
- image model
- consequence of the representation of prior
- stimulus-dependence of variability
- stimulus dependence of covariability of multiple neurons

inference and uncertainty

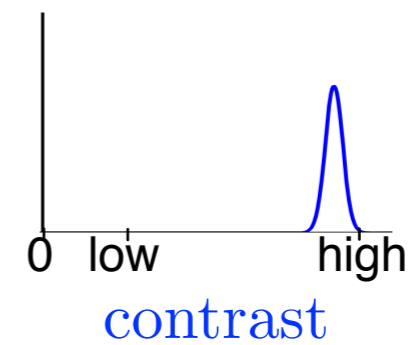
$$\underbrace{P(a_1, a_2, \dots, a_N \mid \text{image}, \mathbf{c})}_{\text{posterior}} \propto \underbrace{P(a_1, a_2, \dots, a_N \mid \mathbf{c})}_{\text{prior}} \times \underbrace{P(\text{image} \mid a_1, a_2, \dots, a_N, \mathbf{c})}_{\text{sensory evidence}}$$

inference and uncertainty

$$\underbrace{P(a_1, a_2, \dots, a_N \mid \text{image}, c)}_{\text{posterior}} \propto \underbrace{P(a_1, a_2, \dots, a_N \mid c)}_{\text{prior}} \times \underbrace{P(\text{image} \mid a_1, a_2, \dots, a_N, c)}_{\text{sensory evidence}}$$

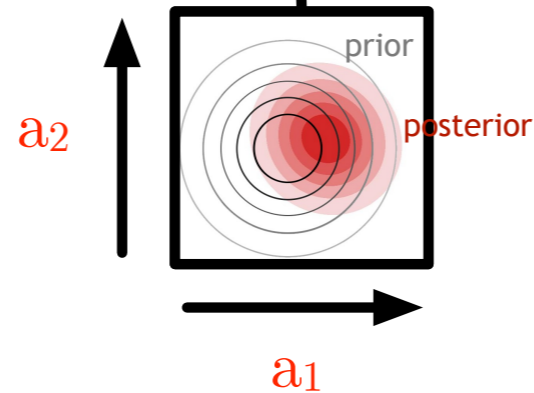
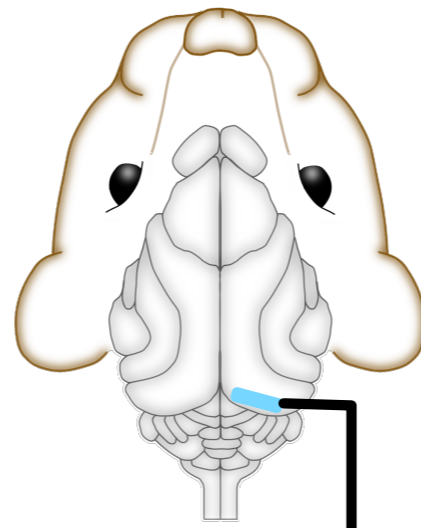
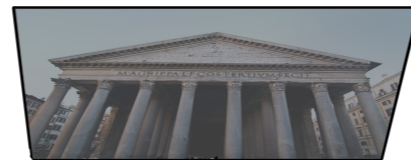


$P(\text{contrast} \mid \text{image})$

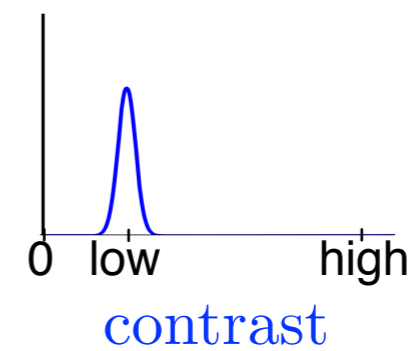


inference and uncertainty

$$\underbrace{P(a_1, a_2, \dots, a_N \mid \text{image}, c)}_{\text{posterior}} \propto \underbrace{P(a_1, a_2, \dots, a_N \mid c)}_{\text{prior}} \times \underbrace{P(\text{image} \mid a_1, a_2, \dots, a_N, c)}_{\text{sensory evidence}}$$

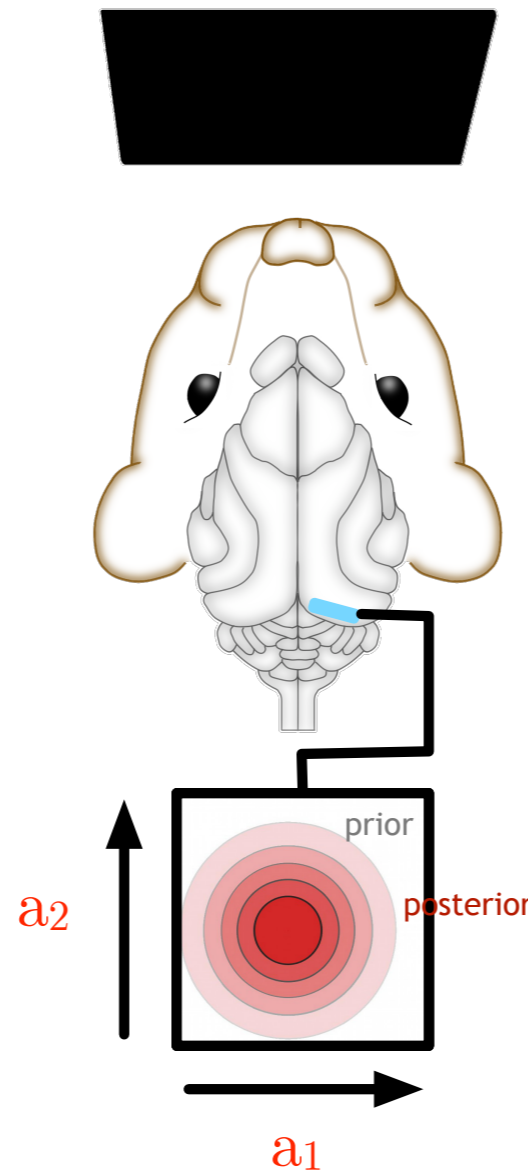


$P(\text{contrast} \mid \text{image})$

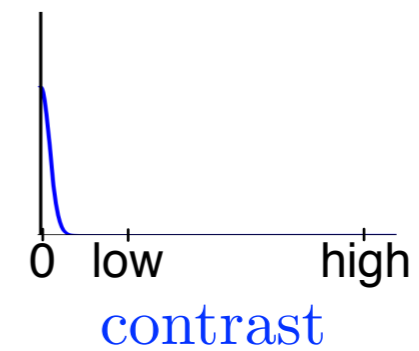


inference and uncertainty

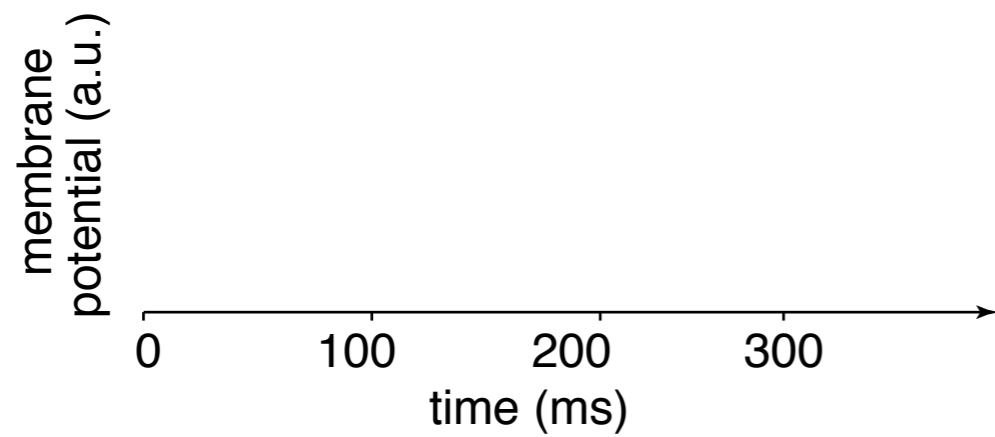
$$\underbrace{P(a_1, a_2, \dots, a_N \mid \text{image}, c)}_{\text{posterior}} \propto \underbrace{P(a_1, a_2, \dots, a_N \mid c)}_{\text{prior}} \times \underbrace{P(\text{image} \mid a_1, a_2, \dots, a_N, c)}_{\text{sensory evidence}}$$



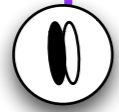
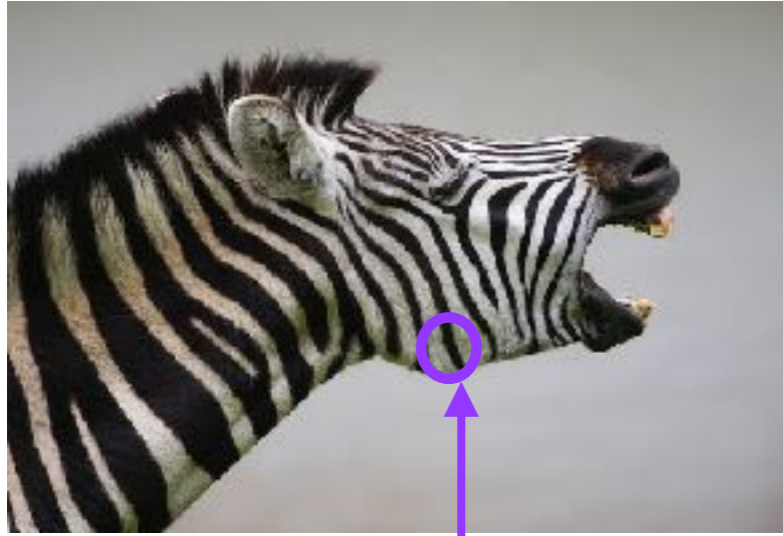
$P(\text{contrast} \mid \text{image})$



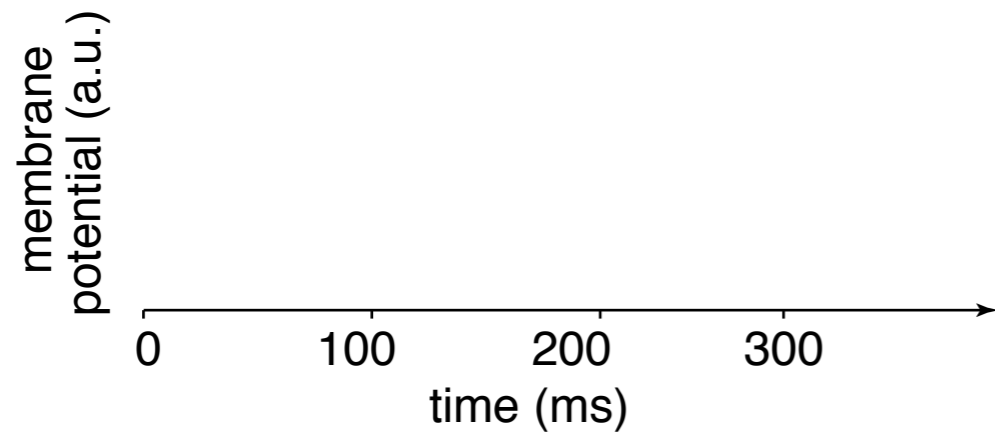
stochastic sampling



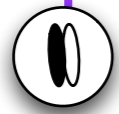
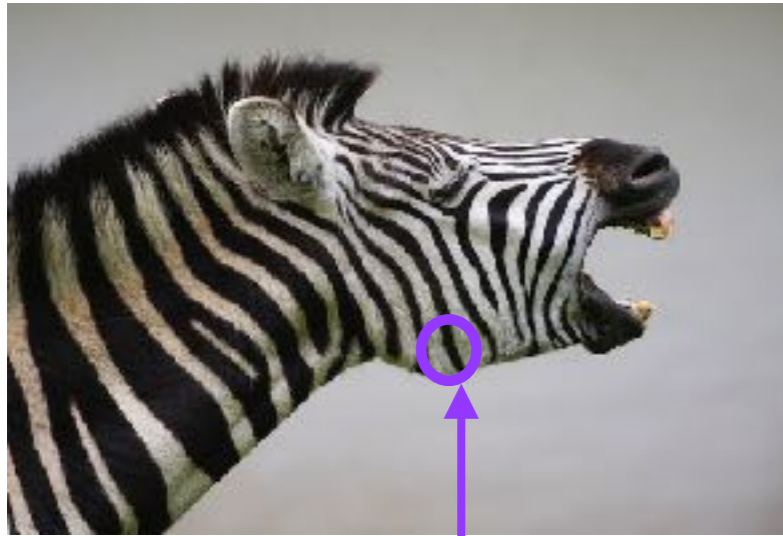
stochastic sampling



feature #1

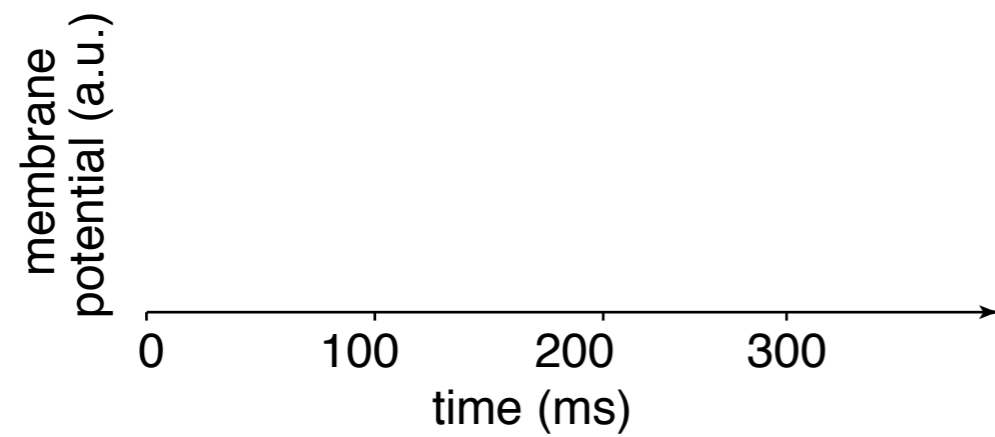
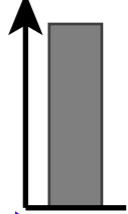


stochastic sampling

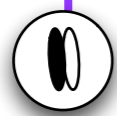
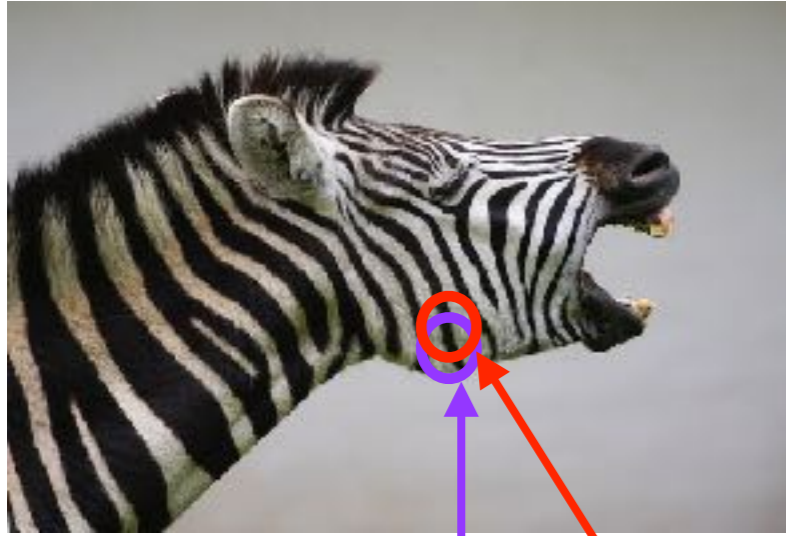


feature #1

y_1



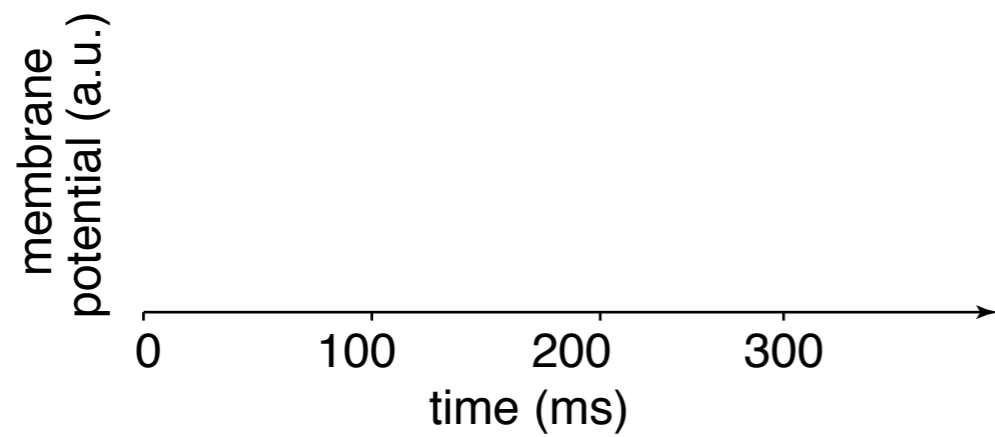
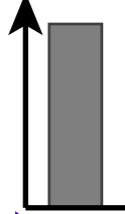
stochastic sampling



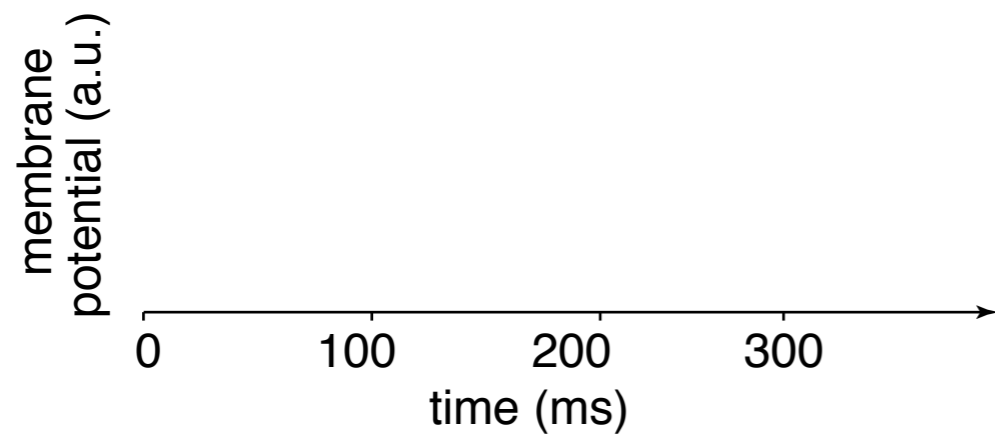
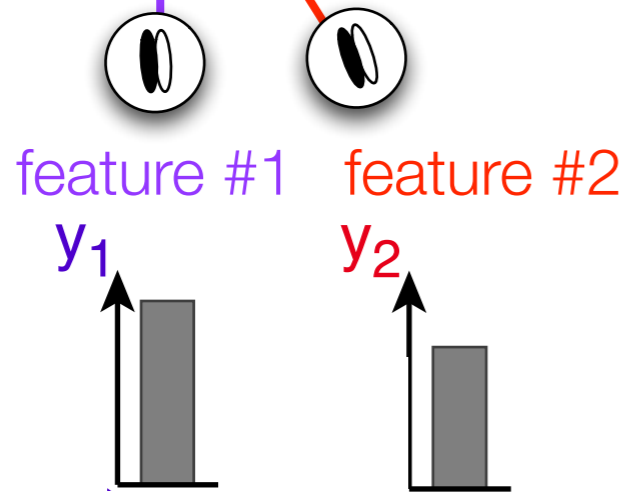
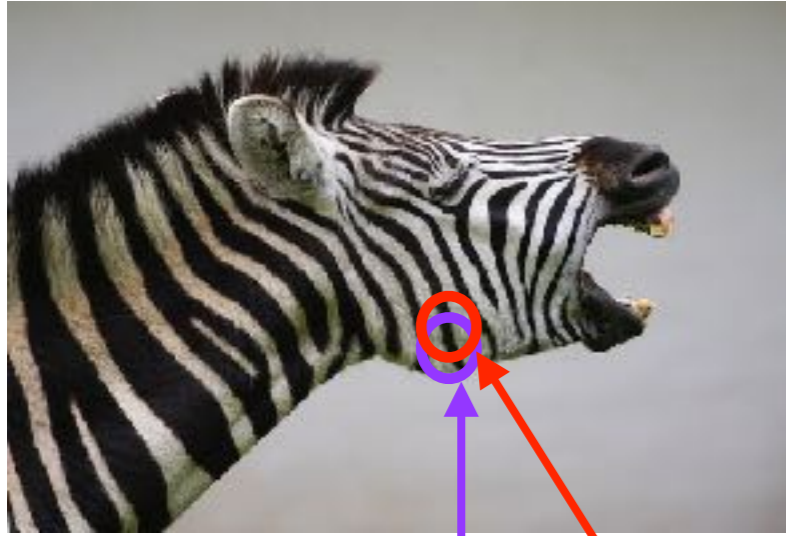
feature #1

feature #2

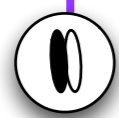
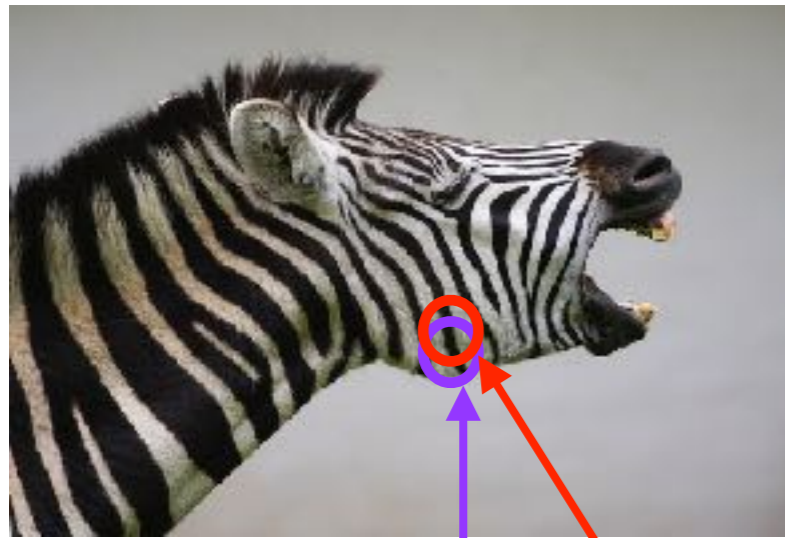
y_1



stochastic sampling

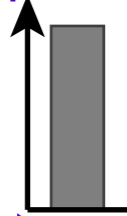


stochastic sampling



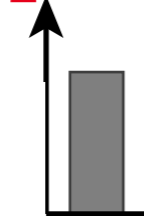
feature #1

y_1

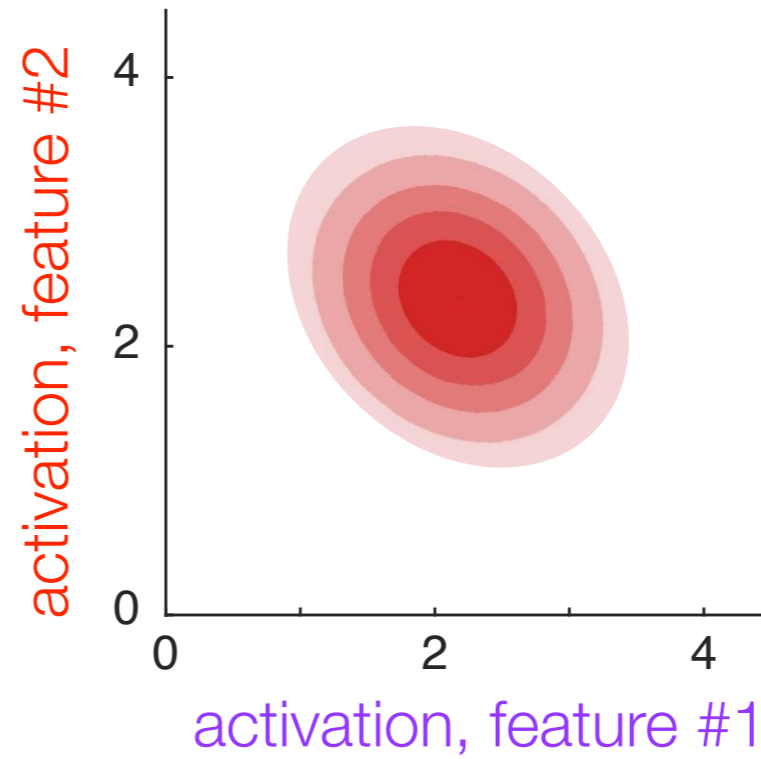
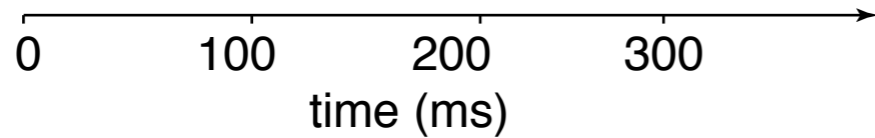


feature #2

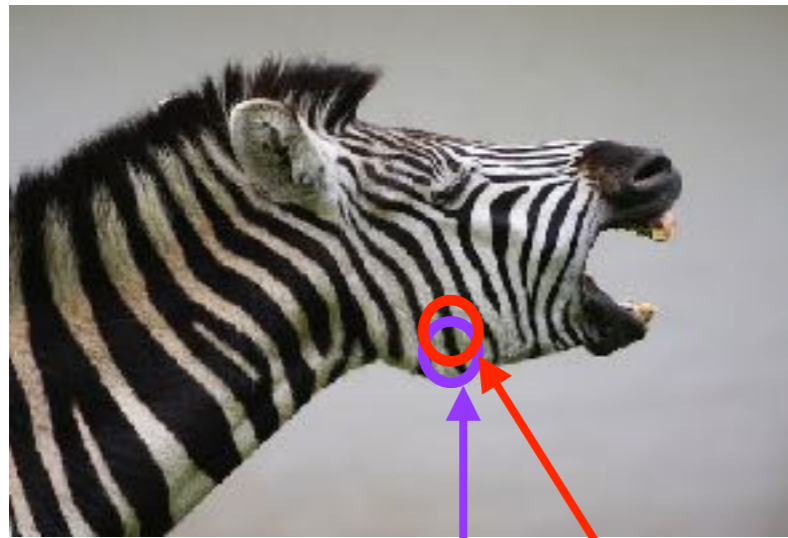
y_2



membrane potential (a.u.)

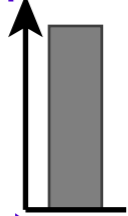


stochastic sampling



feature #1

y_1

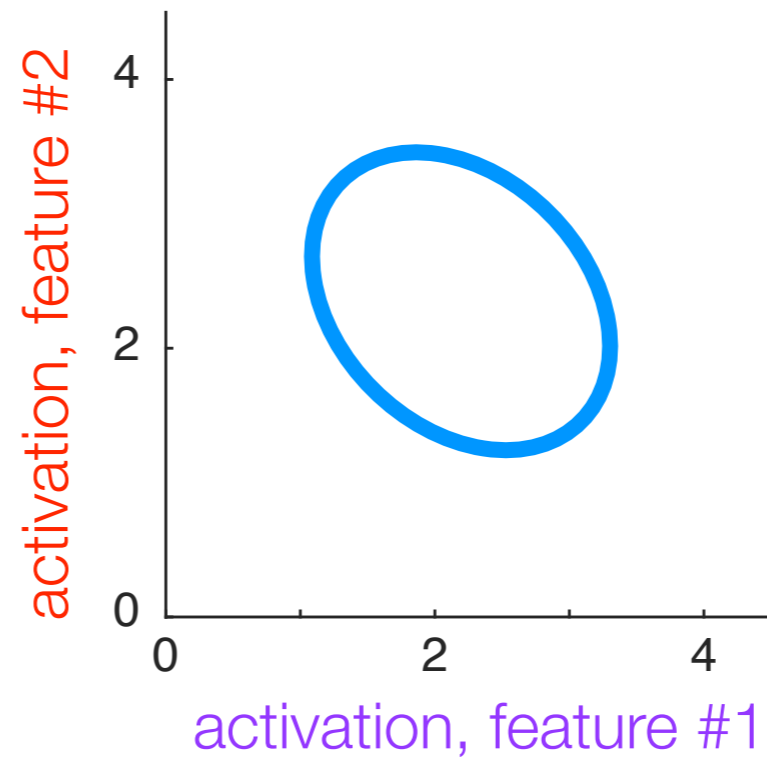
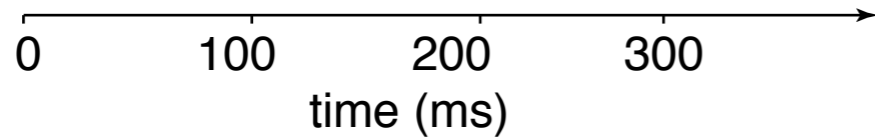


feature #2

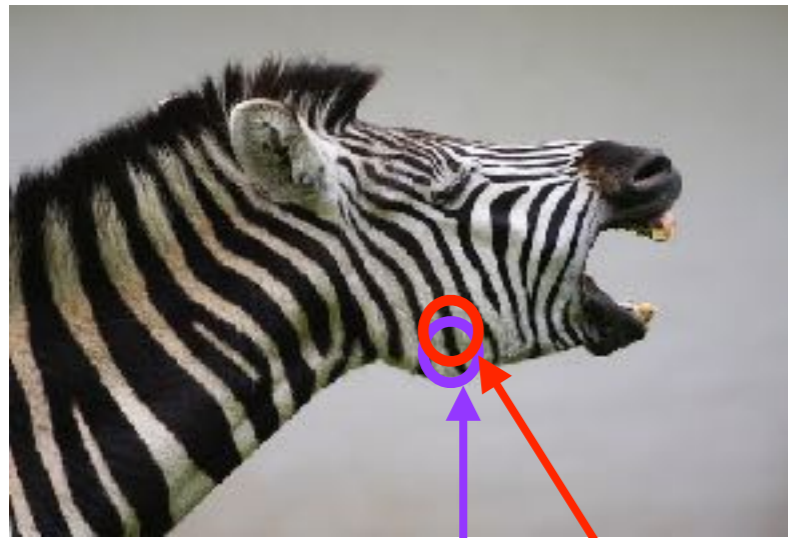
y_2



membrane potential (a.u.)

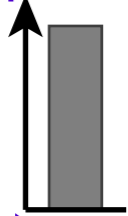


stochastic sampling



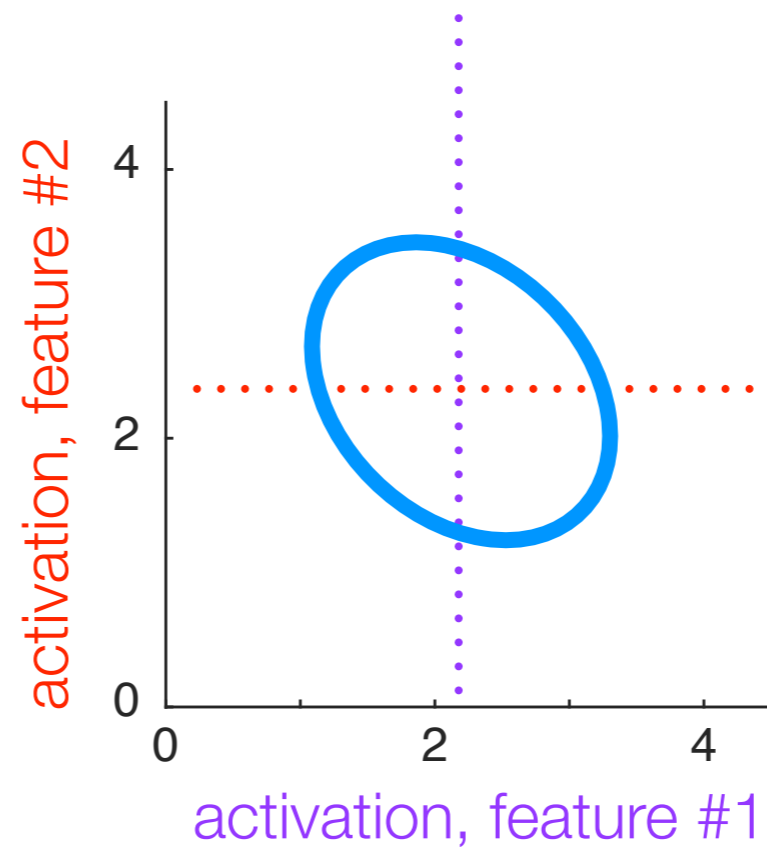
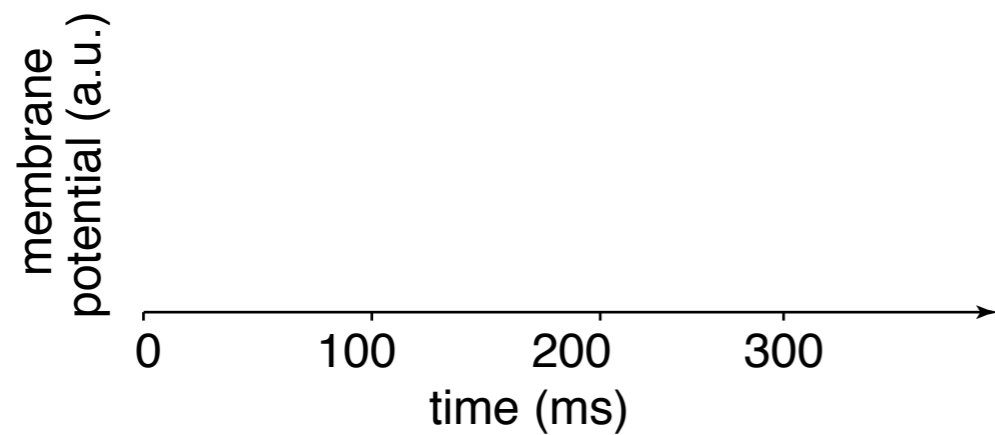
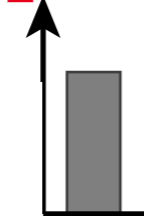
feature #1

y_1



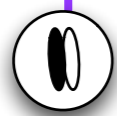
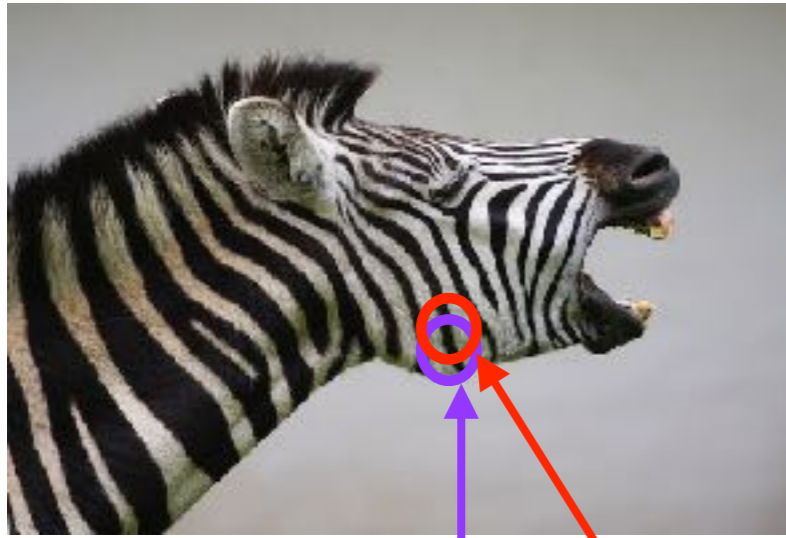
feature #2

y_2



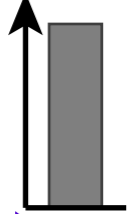
mean activations

stochastic sampling



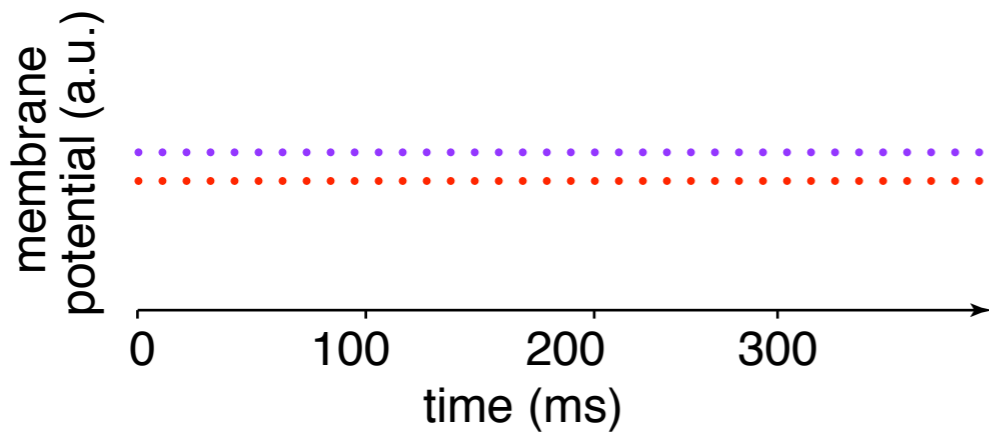
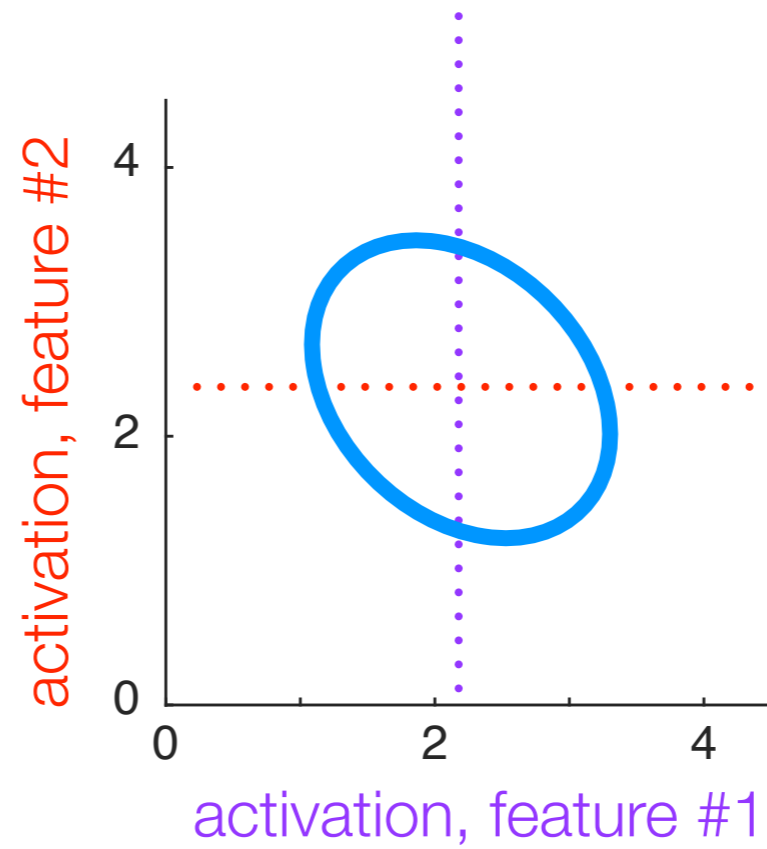
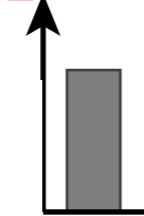
feature #1

y_1

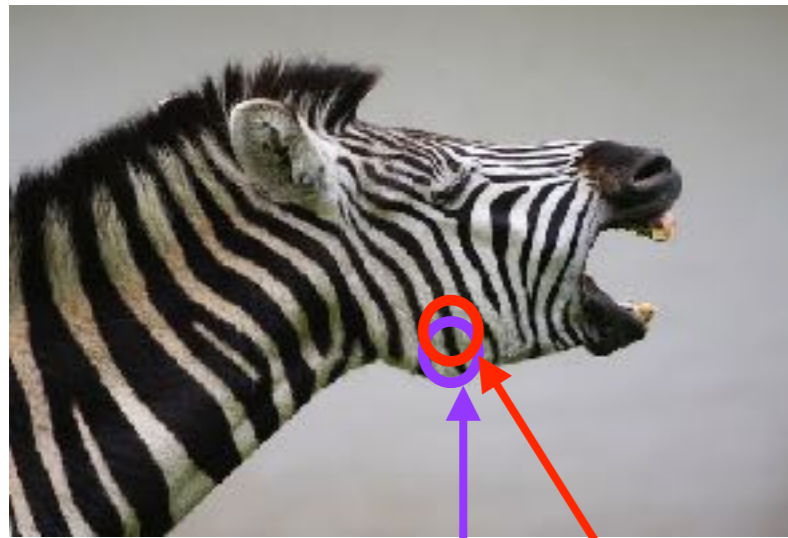


feature #2

y_2



stochastic sampling

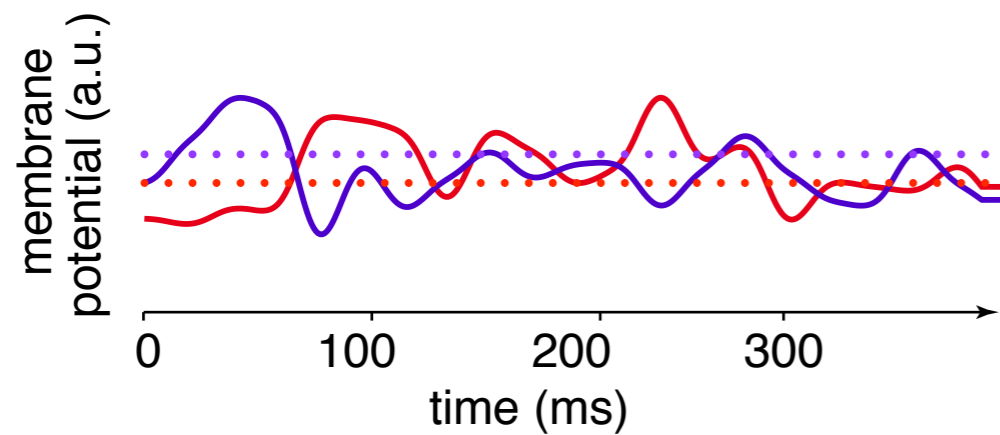
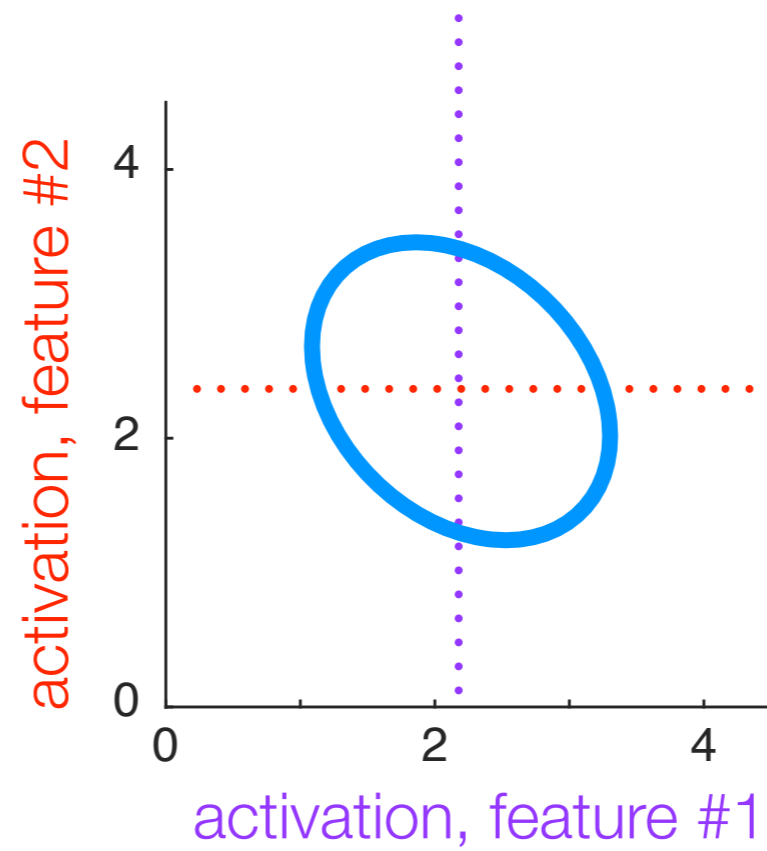
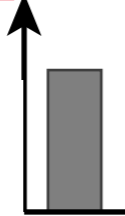
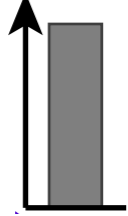


feature #1

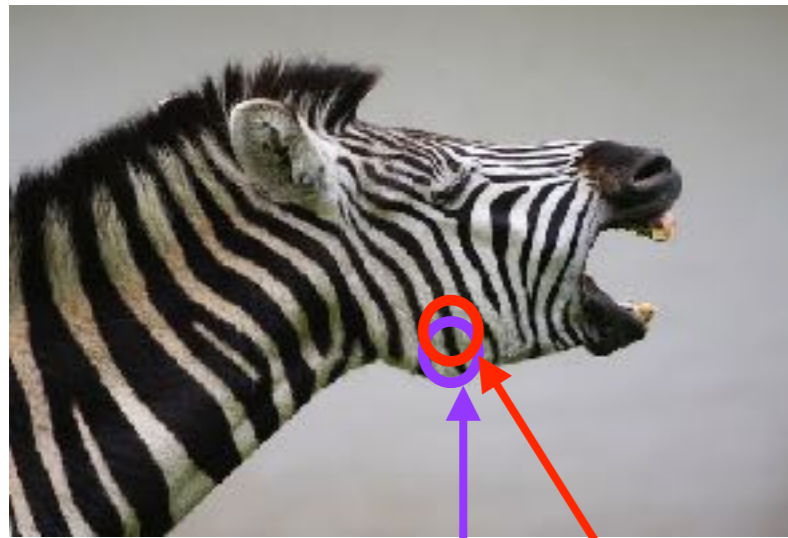
feature #2

y_1

y_2



stochastic sampling

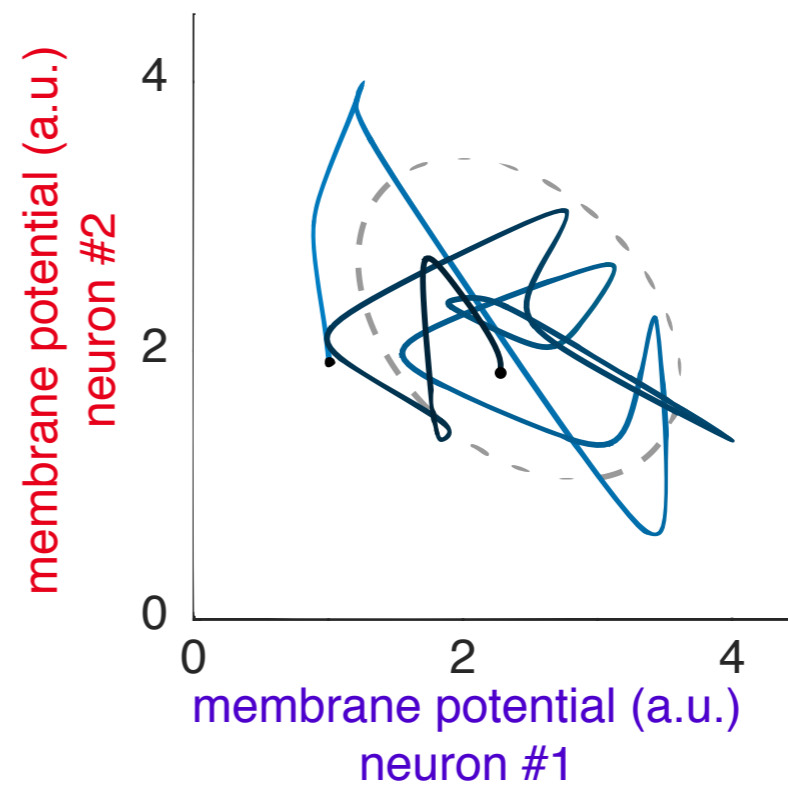
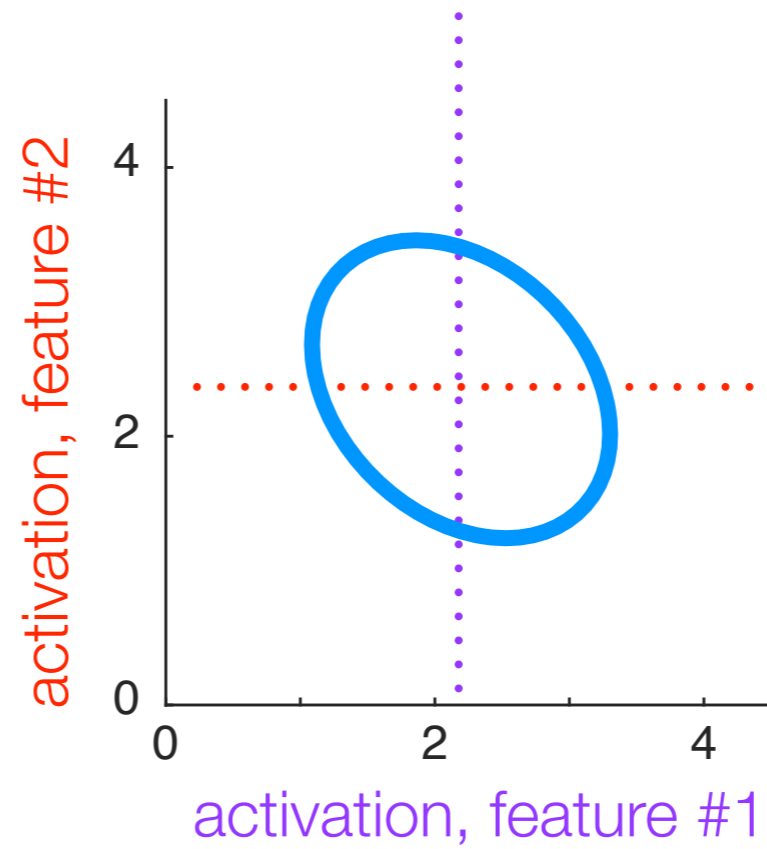
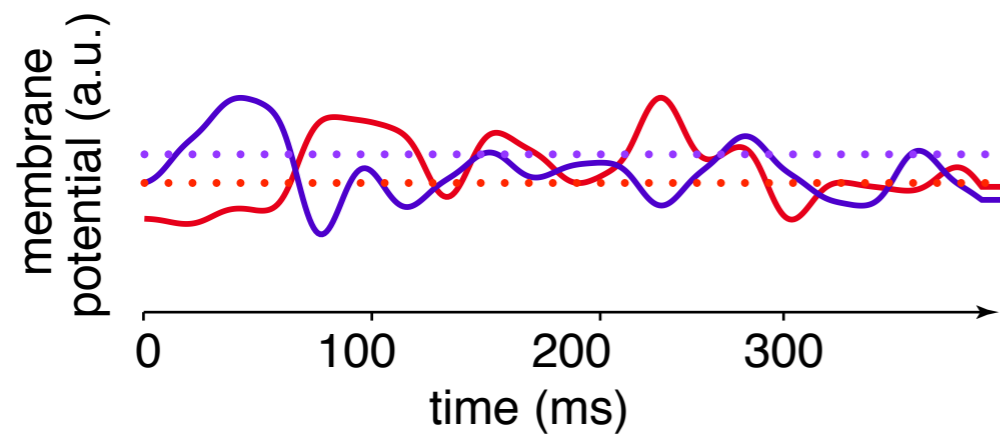
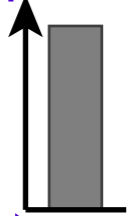


feature #1

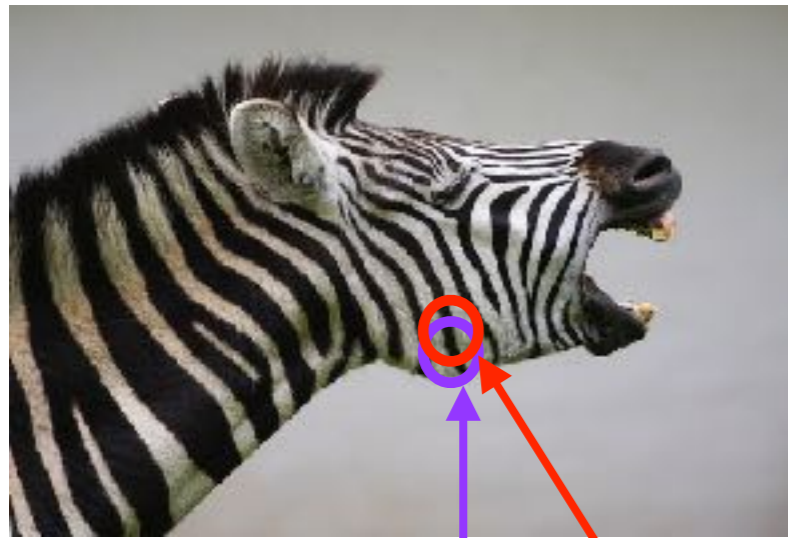
feature #2

y_1

y_2

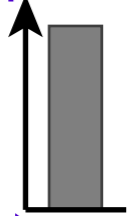


stochastic sampling



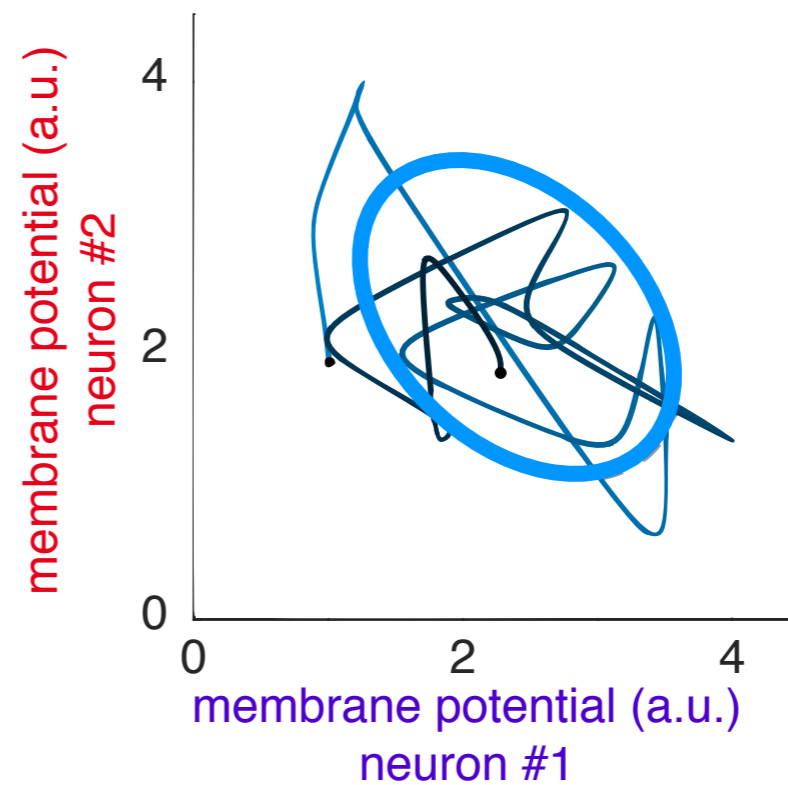
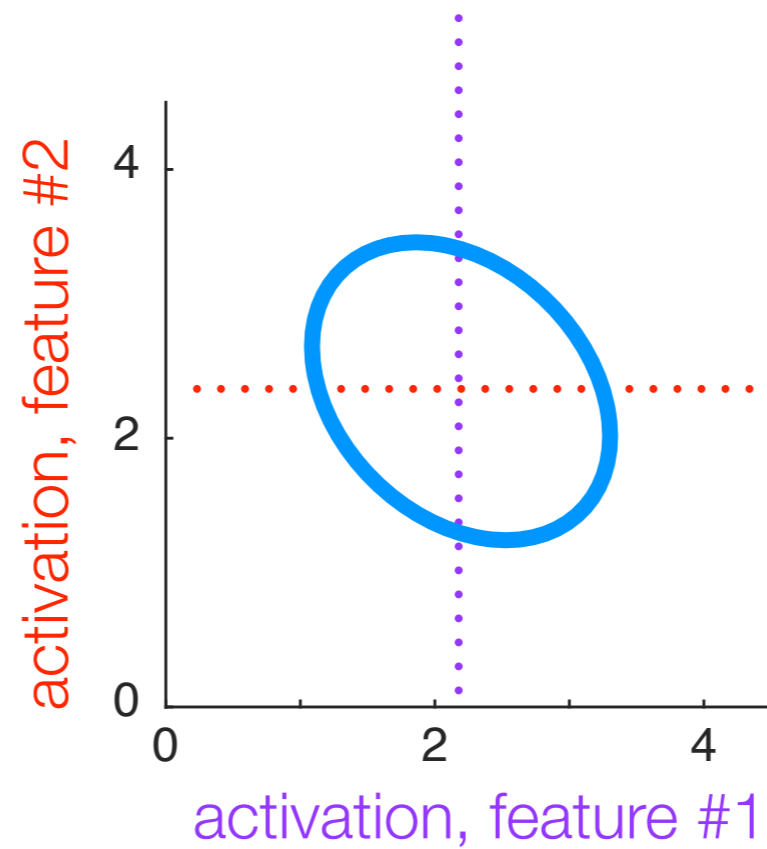
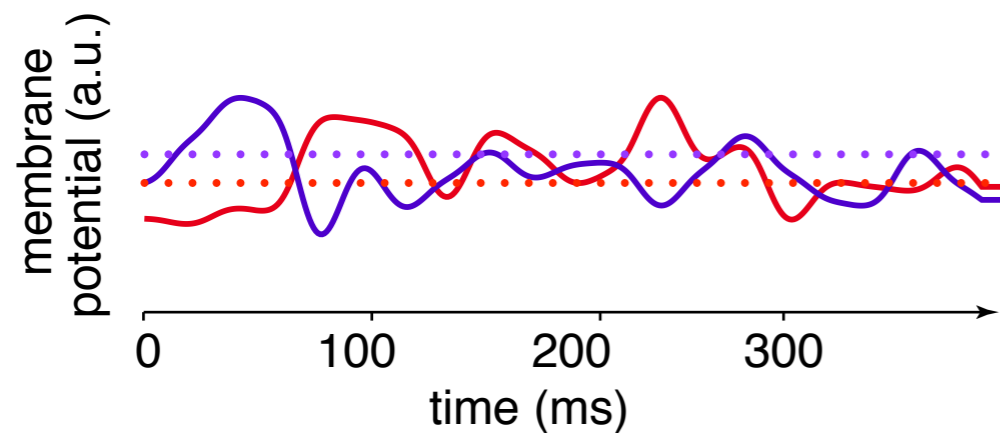
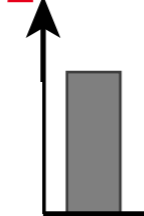
feature #1

y_1

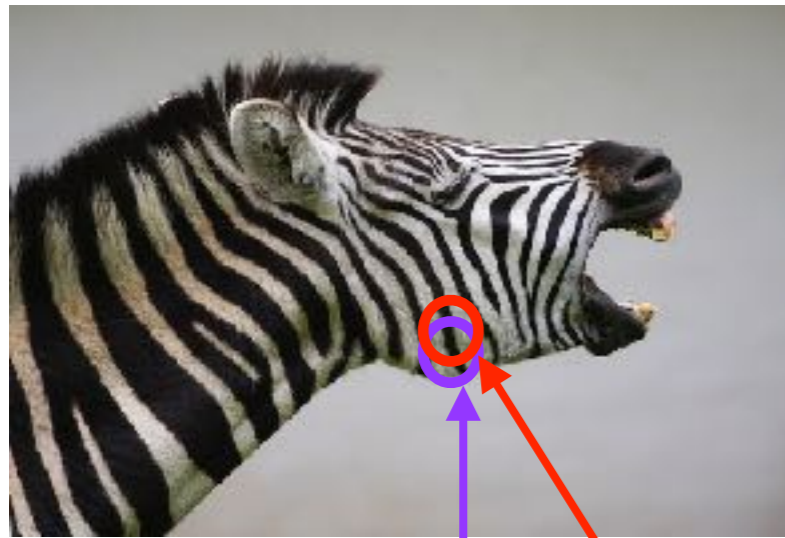


feature #2

y_2

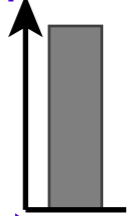


stochastic sampling



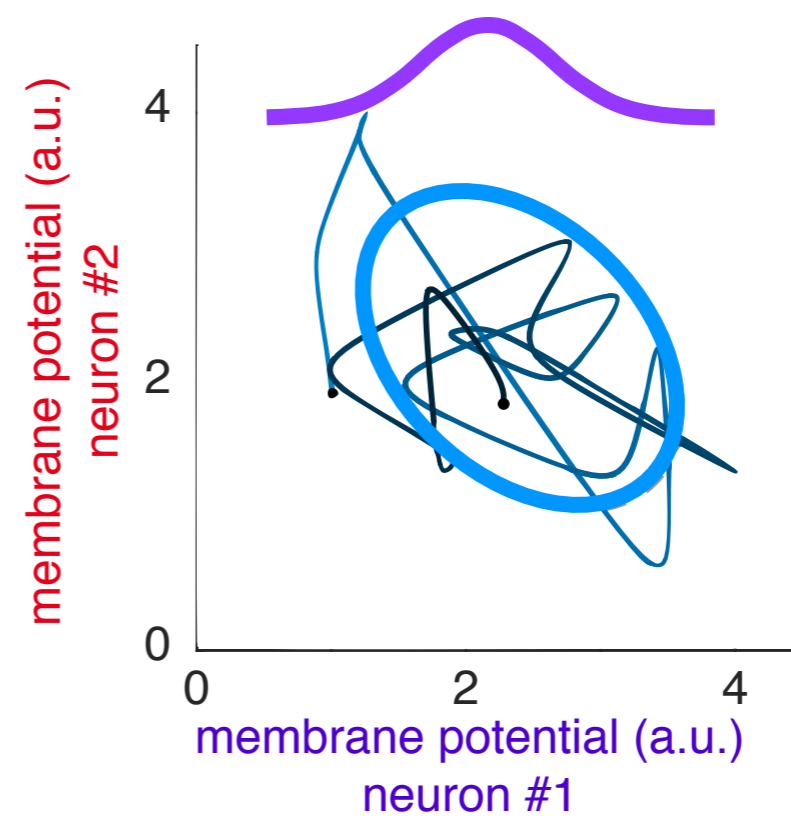
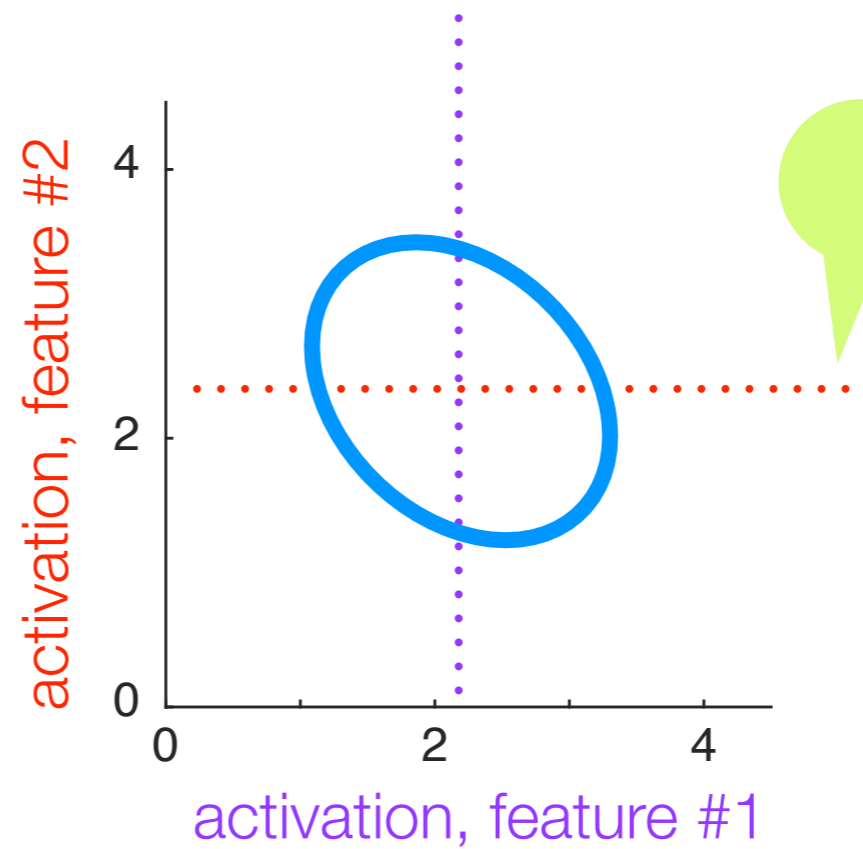
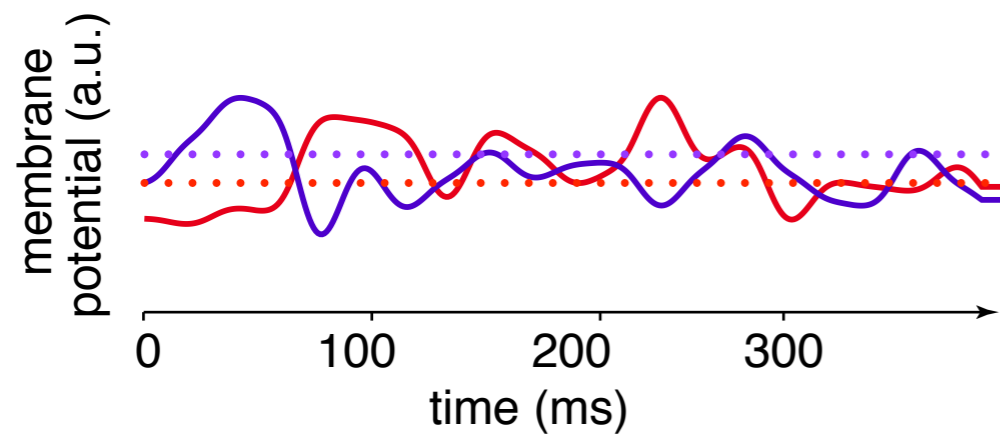
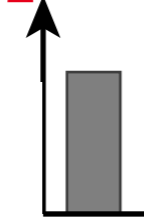
feature #1

y_1

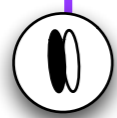
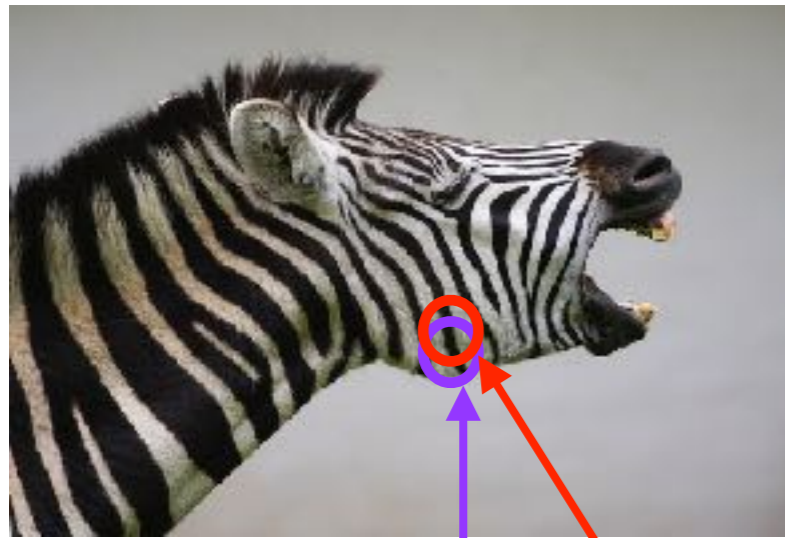


feature #2

y_2



stochastic sampling

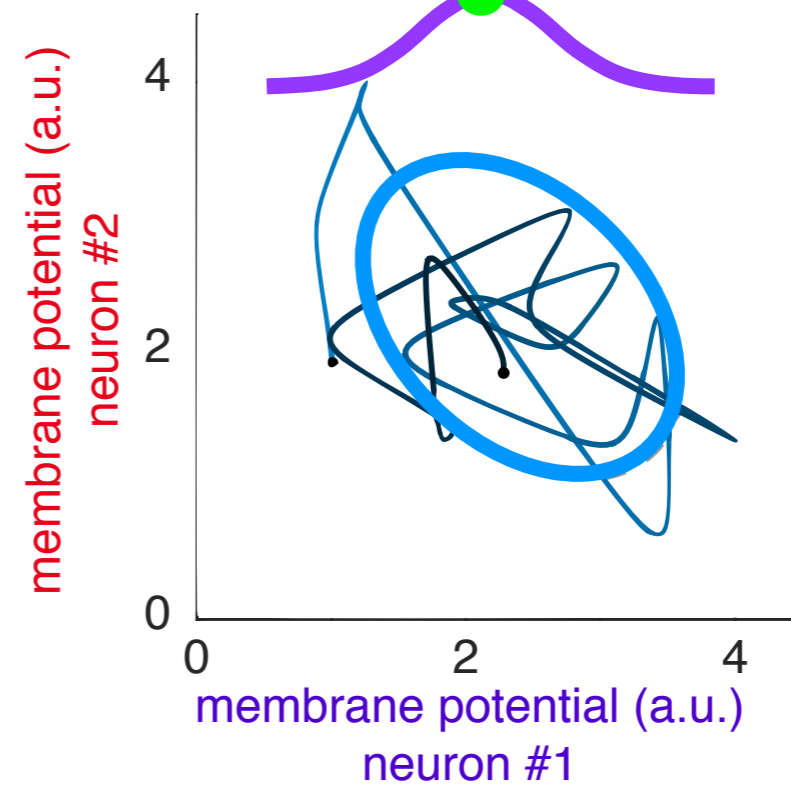
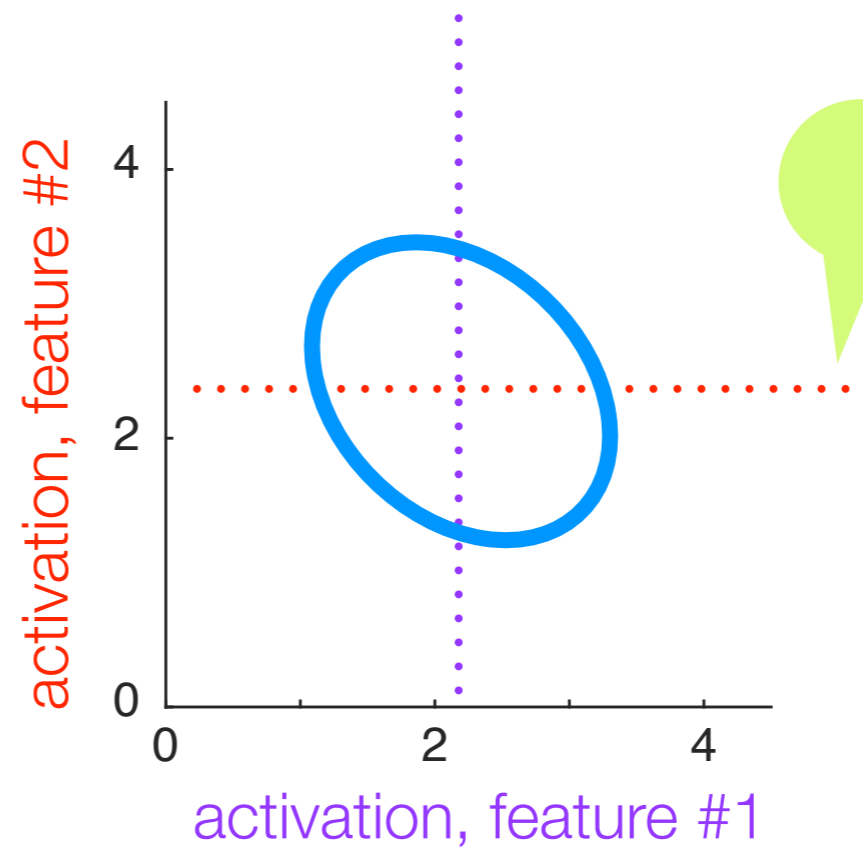
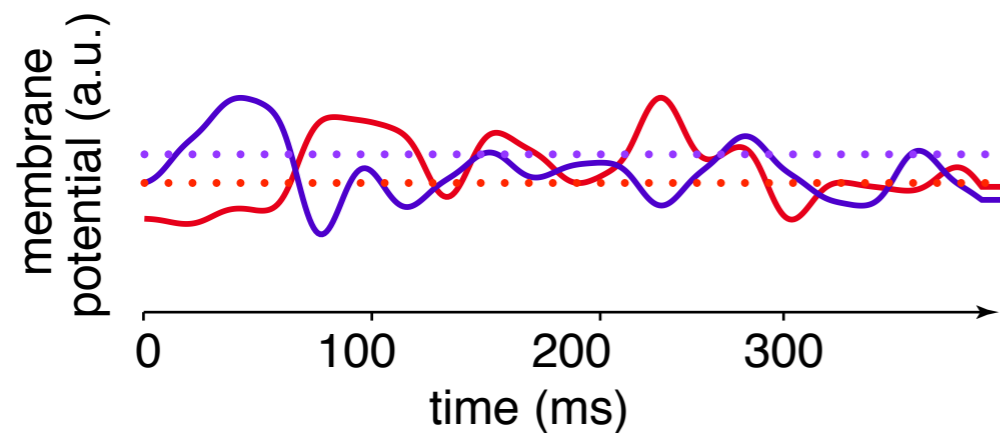
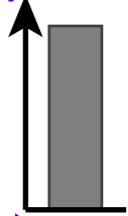


feature #1

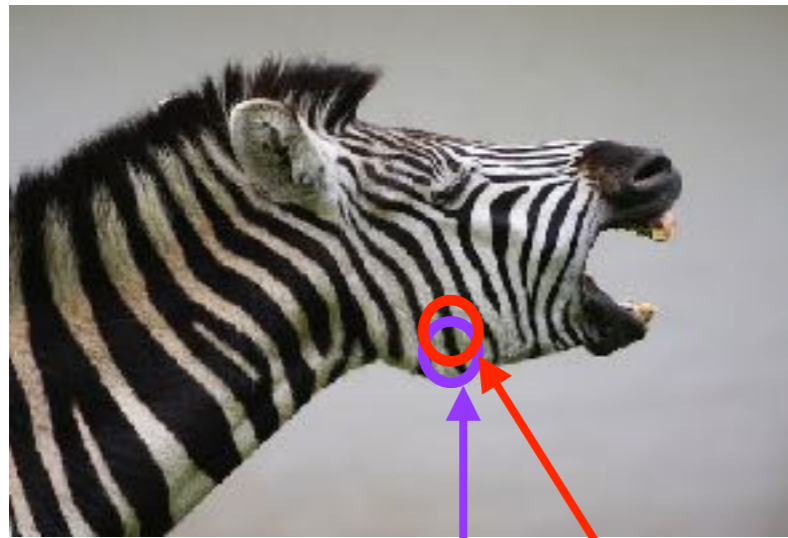
feature #2

y_1

y_2



stochastic sampling

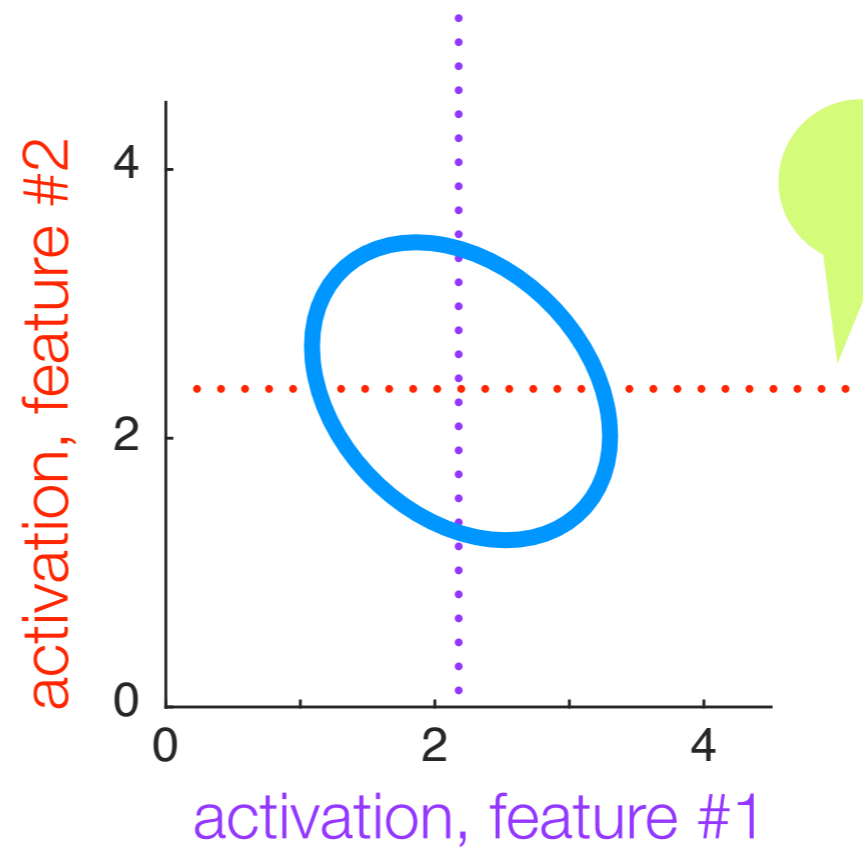
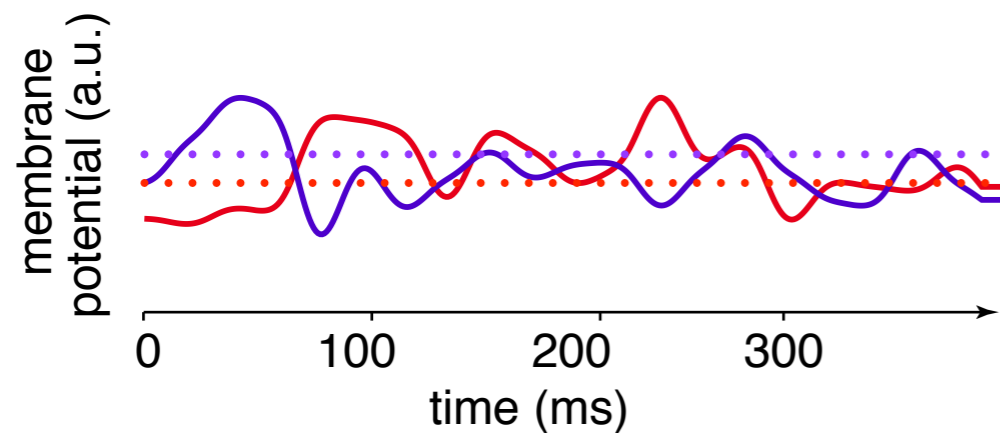
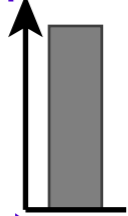


feature #1

feature #2

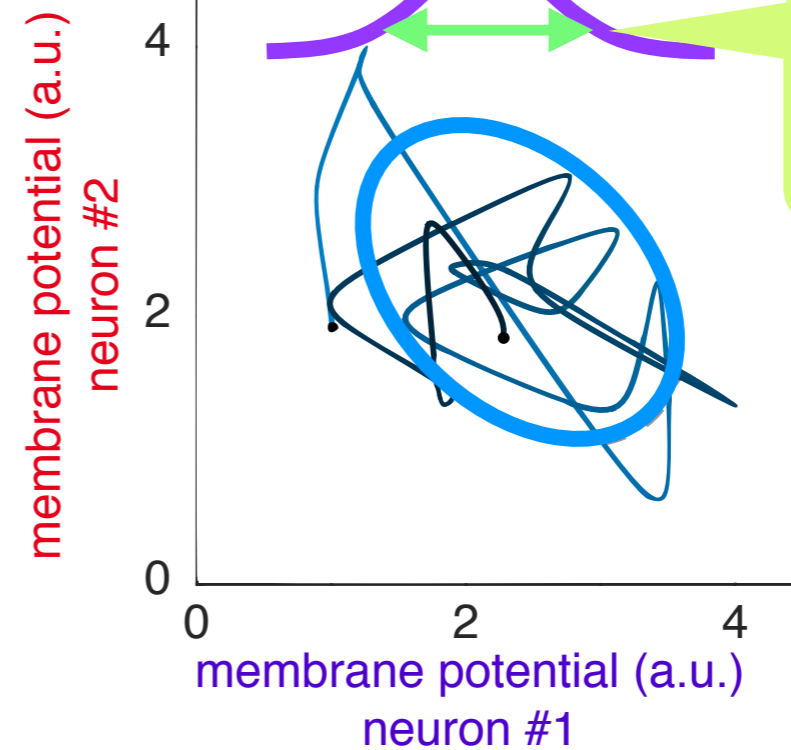
y_1

y_2



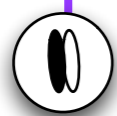
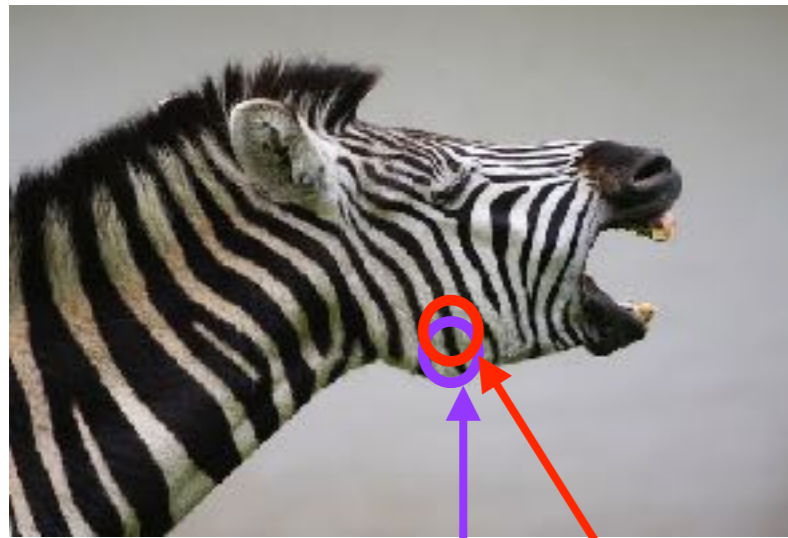
mean activations

mean response



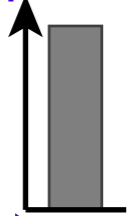
response variance

stochastic sampling



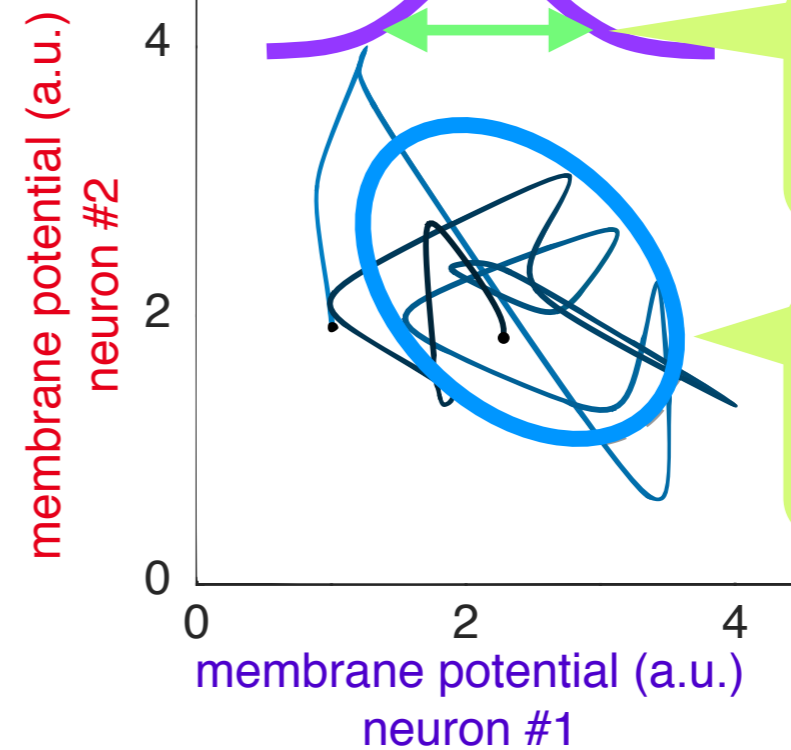
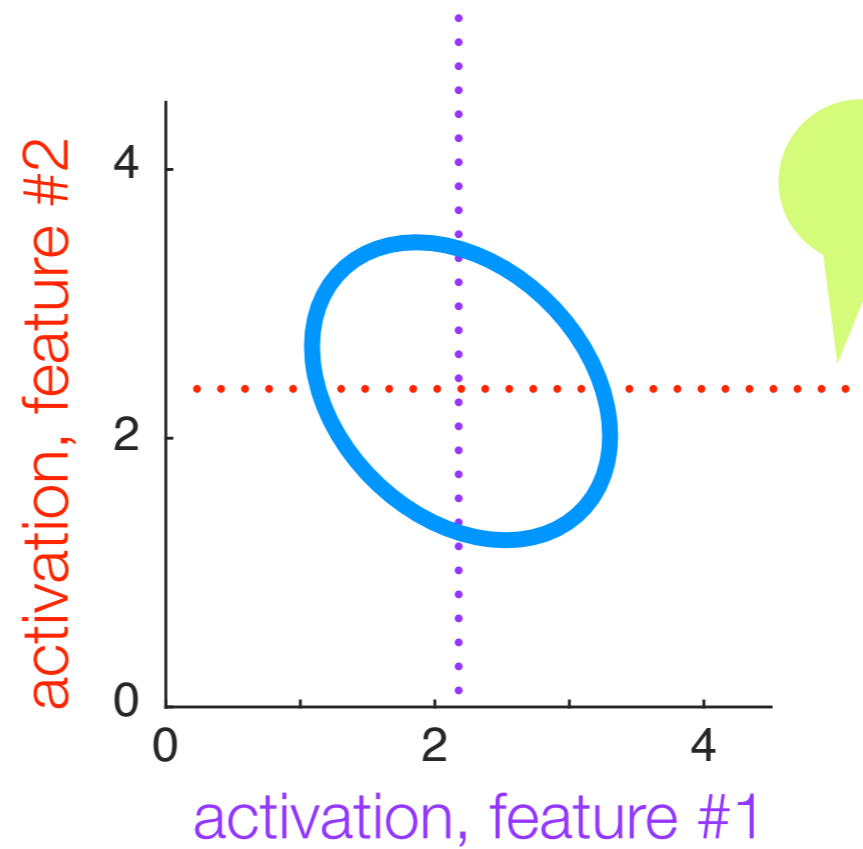
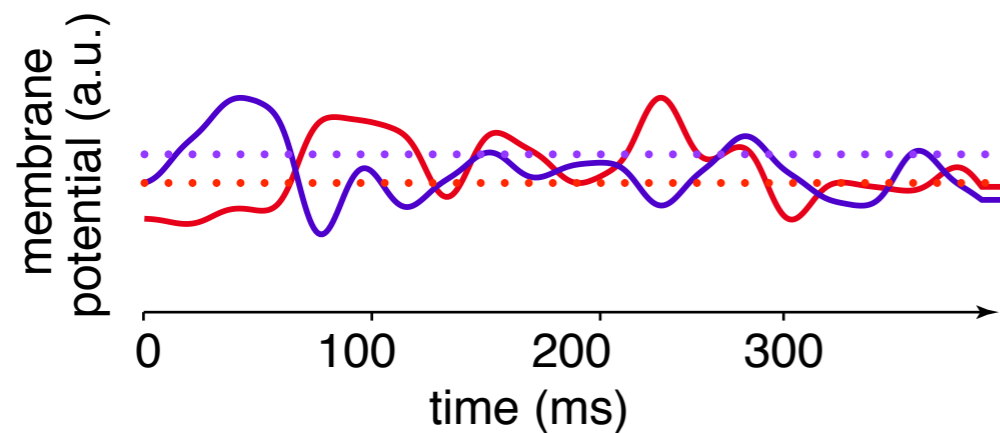
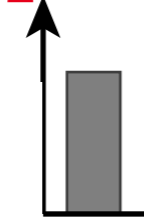
feature #1

y_1

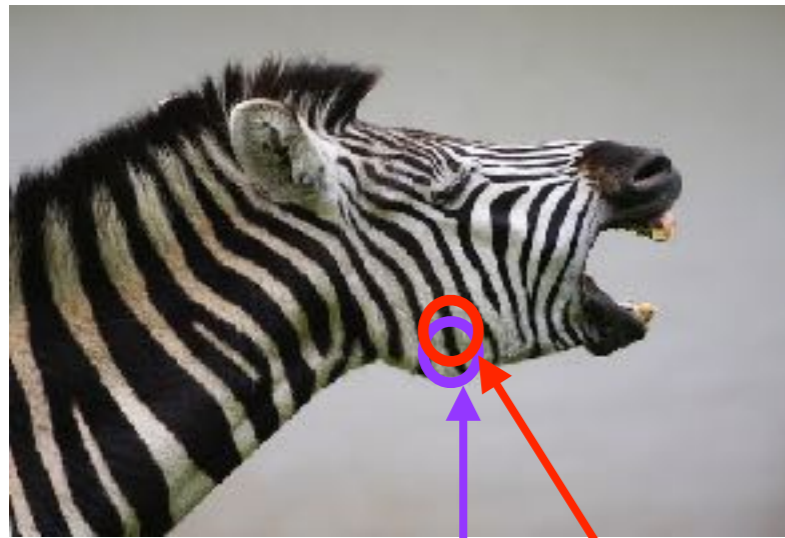


feature #2

y_2



stochastic sampling

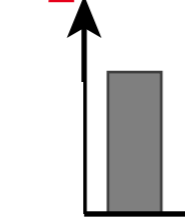
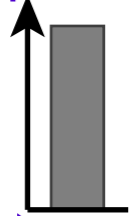


feature #1

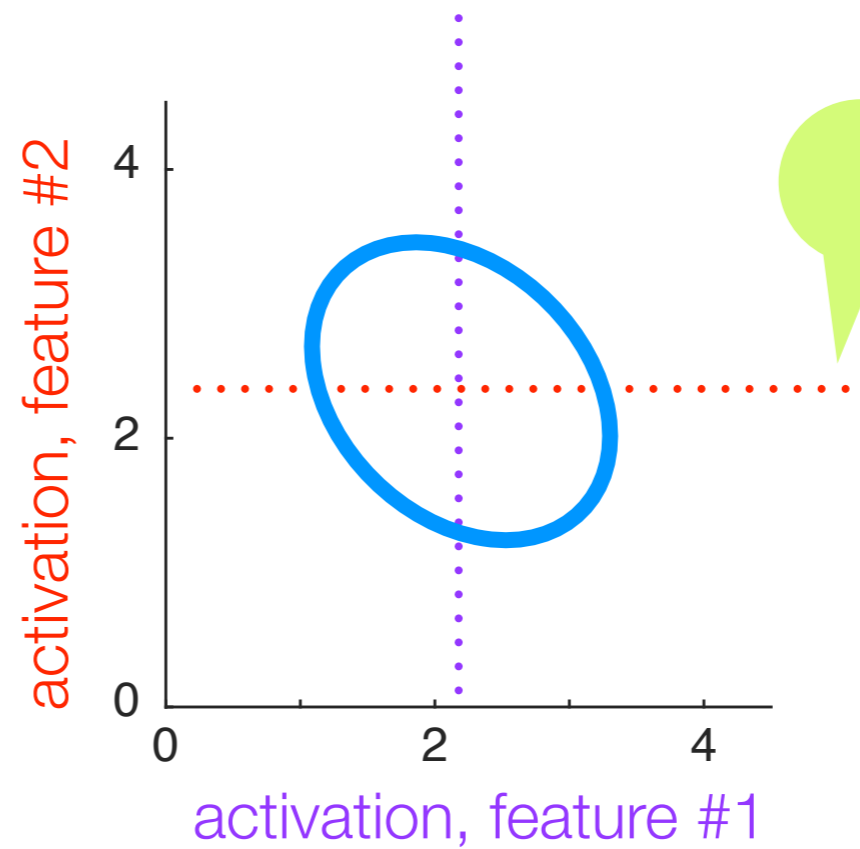
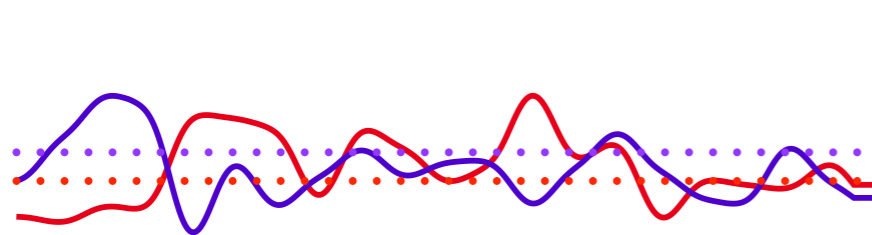
feature #2

y_1

y_2



membrane potential (a.u.)

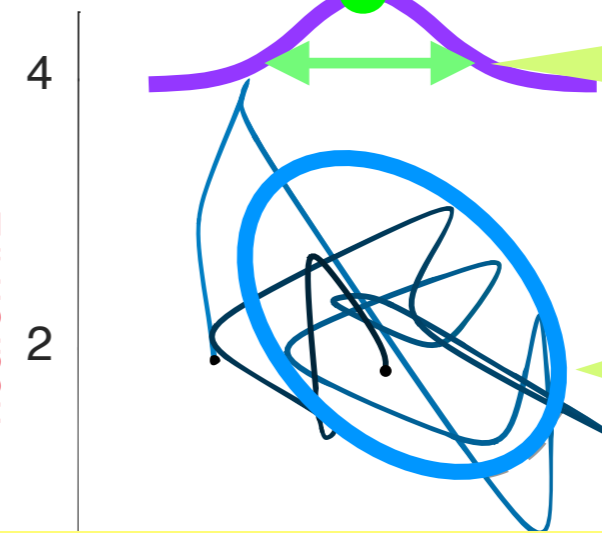


mean activations

activation, feature #1

mean response

membrane potential (a.u.)
neuron #2



response variance

response correlation

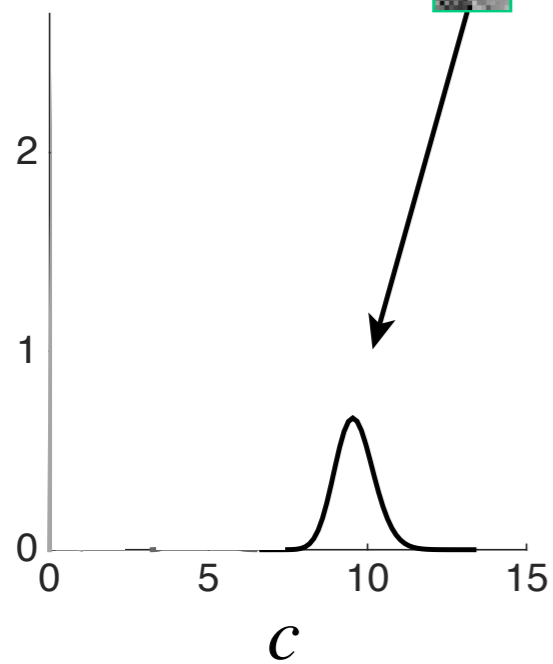
changes in inferences need to be reflected in the response statistics

Stimulus-dependence of variance

$$\underbrace{P(a_1, a_2, \dots, a_N \mid \text{image}, \mathbf{c})}_{\text{posterior}} \propto \underbrace{P(a_1, a_2, \dots, a_N \mid \mathbf{c})}_{\text{prior}} \times \underbrace{P(\text{image} \mid a_1, a_2, \dots, a_N, \mathbf{c})}_{\text{sensory evidence}}$$

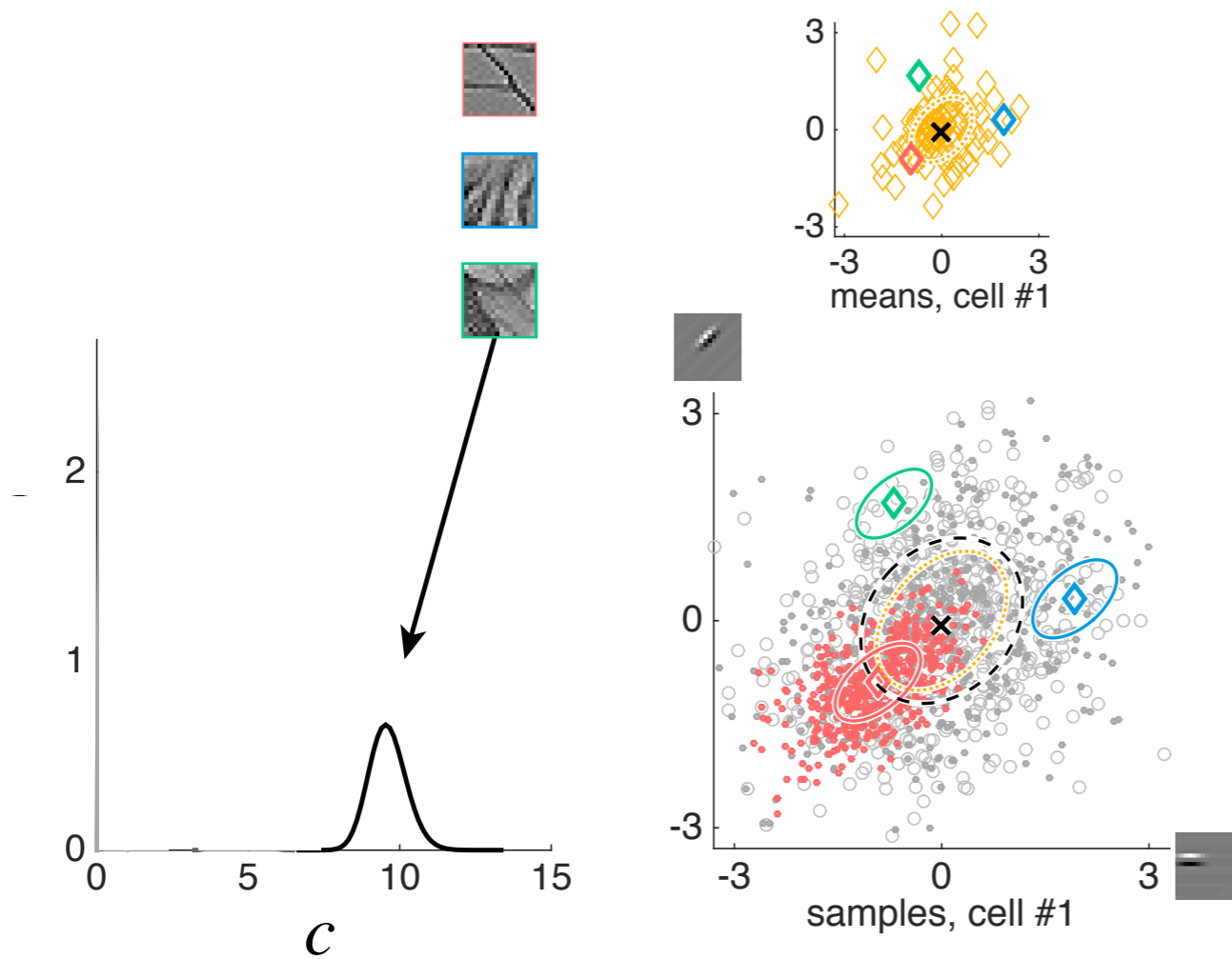
Stimulus-dependence of variance

$$\underbrace{P(a_1, a_2, \dots, a_N \mid \text{image}, c)}_{\text{posterior}} \propto \underbrace{P(a_1, a_2, \dots, a_N \mid c)}_{\text{prior}} \times \underbrace{P(\text{image} \mid a_1, a_2, \dots, a_N, c)}_{\text{sensory evidence}}$$



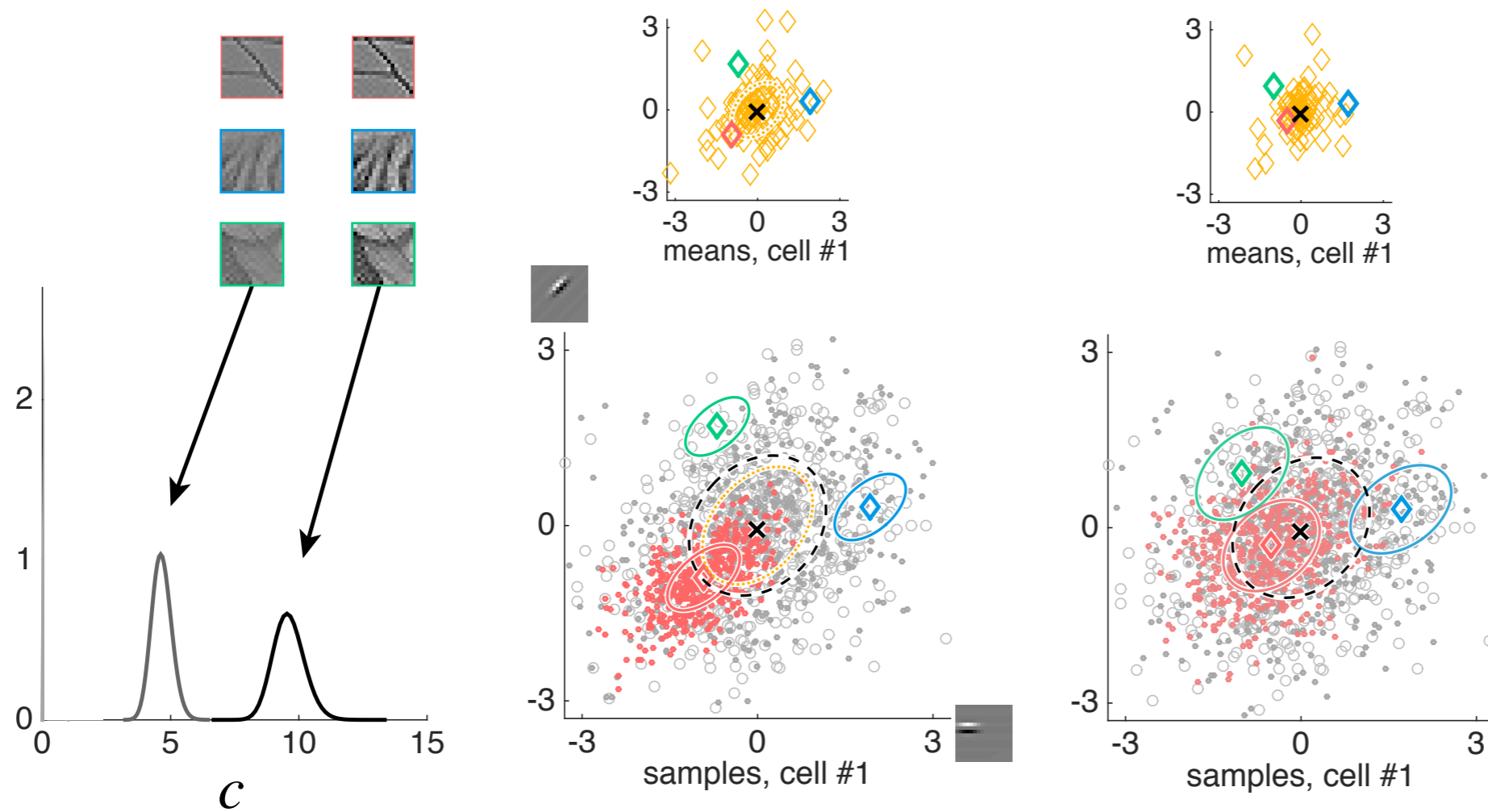
Stimulus-dependence of variance

$$\underbrace{P(a_1, a_2, \dots, a_N \mid \text{image}, c)}_{\text{posterior}} \propto \underbrace{P(a_1, a_2, \dots, a_N \mid c)}_{\text{prior}} \times \underbrace{P(\text{image} \mid a_1, a_2, \dots, a_N, c)}_{\text{sensory evidence}}$$



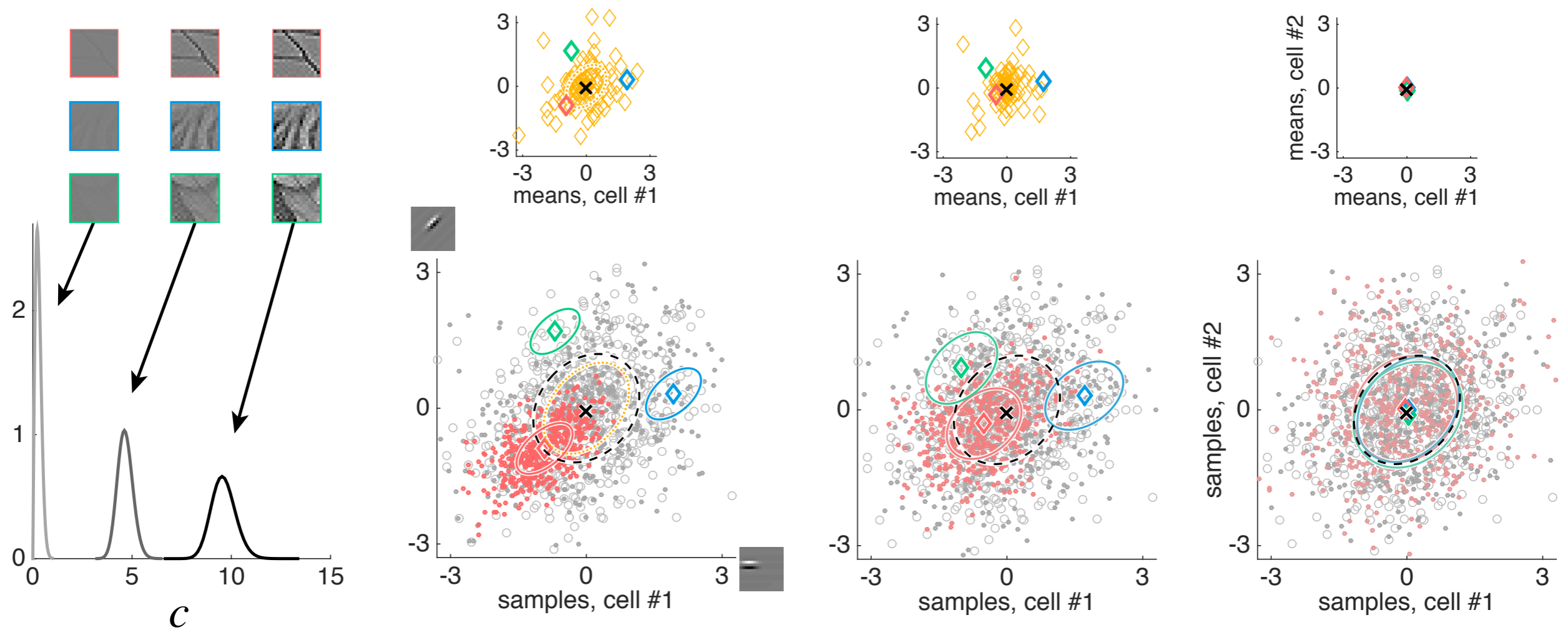
Stimulus-dependence of variance

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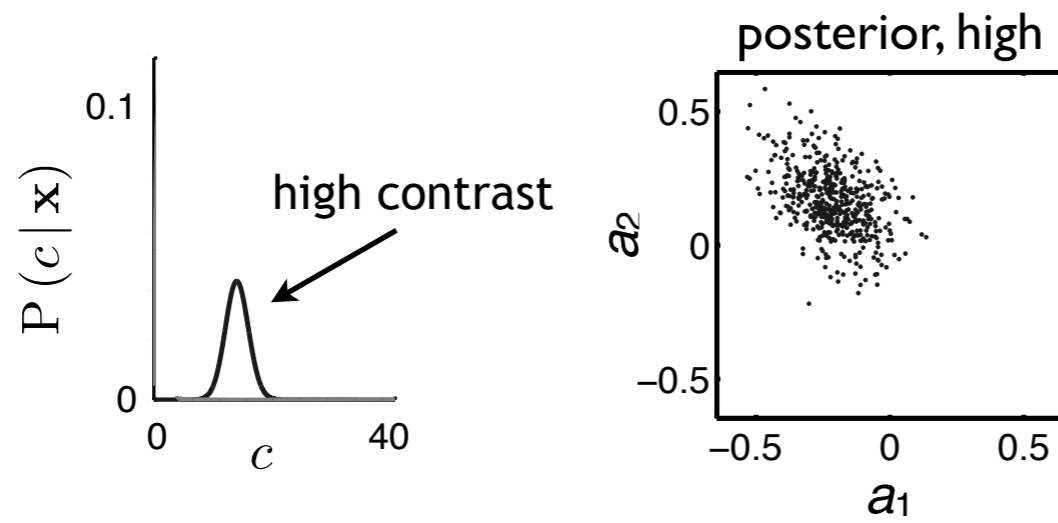


roadmap

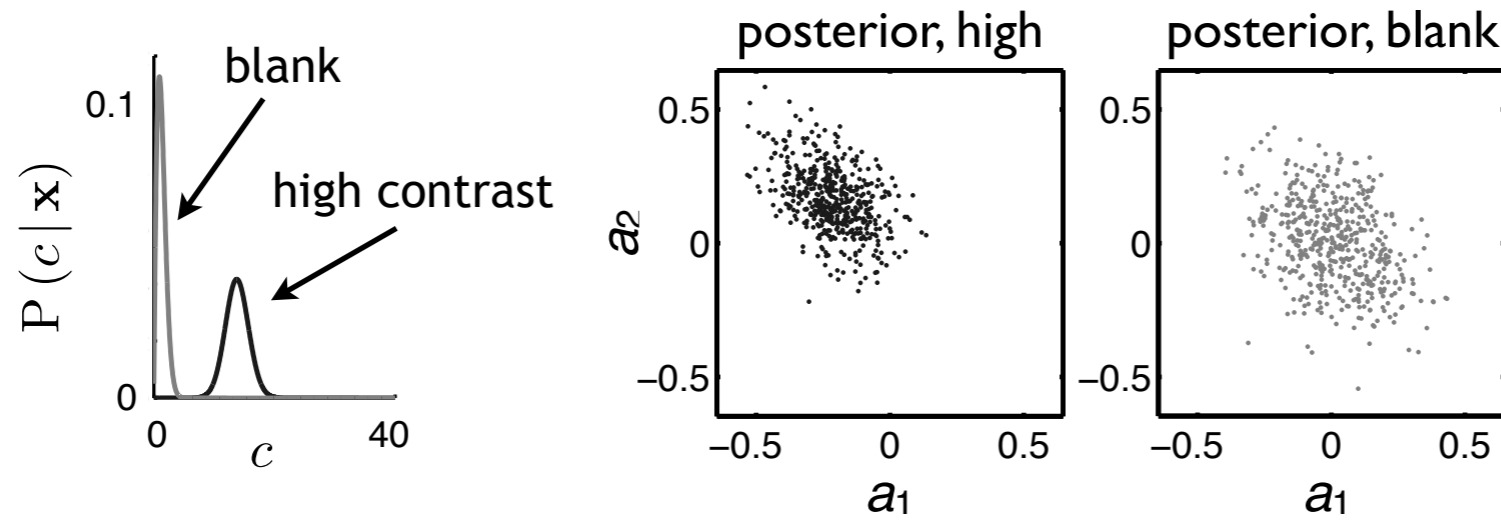
- image model
- consequence of the representation of prior
- **stimulus-dependence of variability**
- stimulus dependence of covariability of multiple neurons

Stimulus onset quenches neural variability

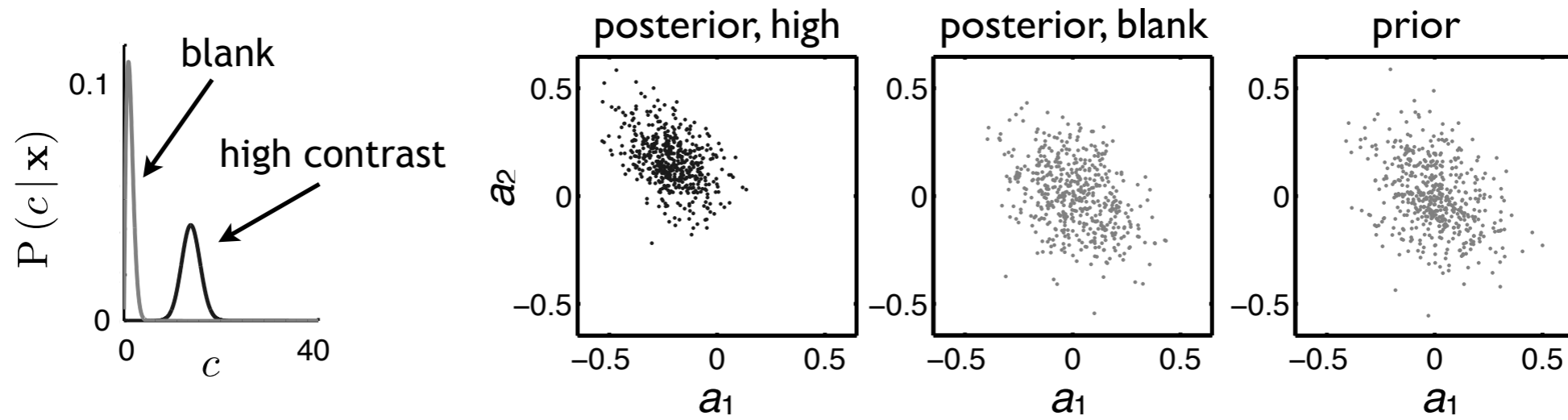
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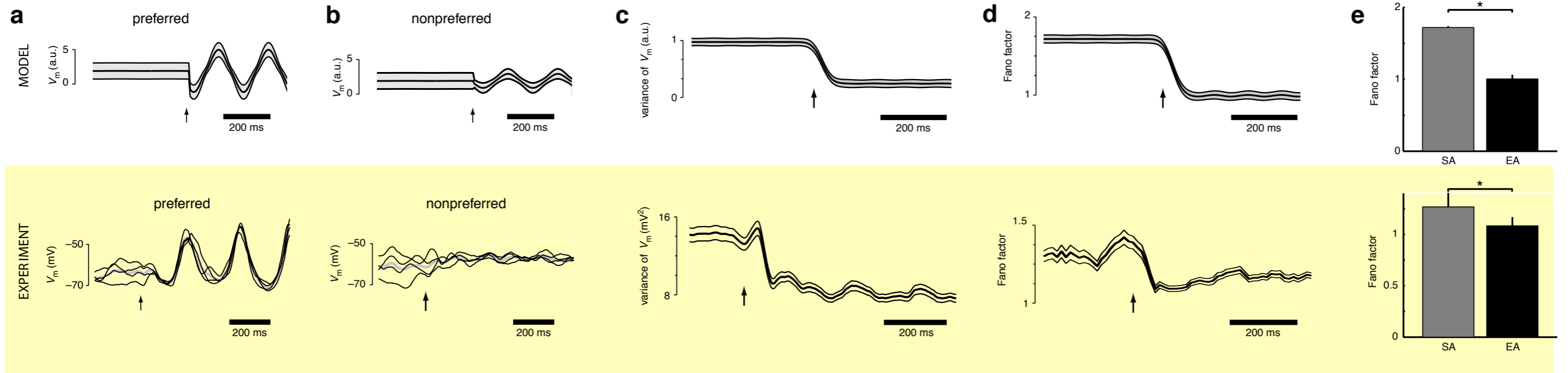
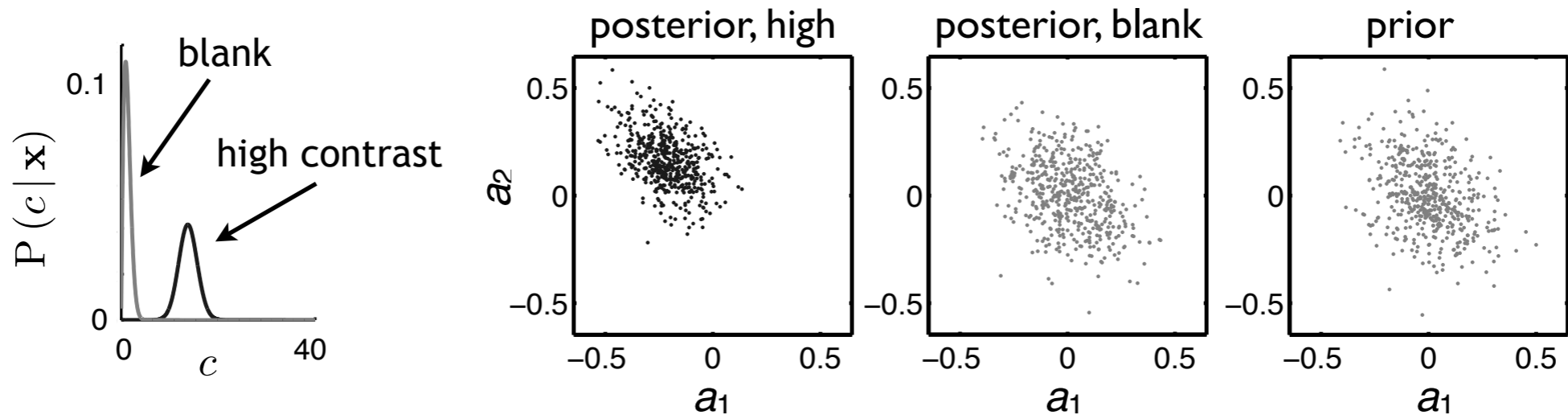
Stimulus onset quenches neural variability



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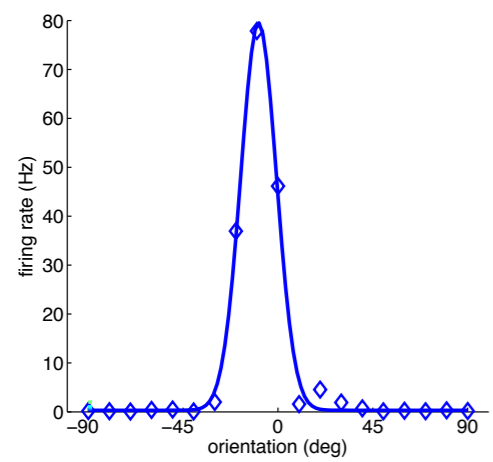
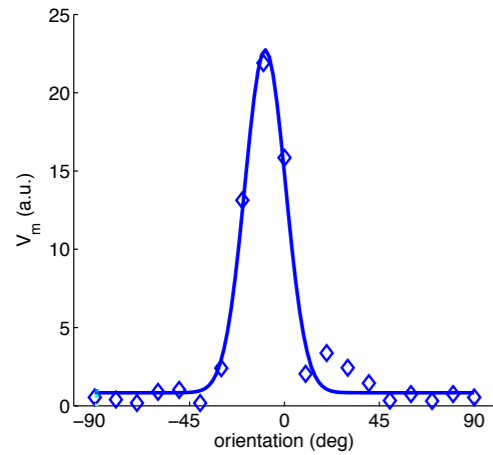


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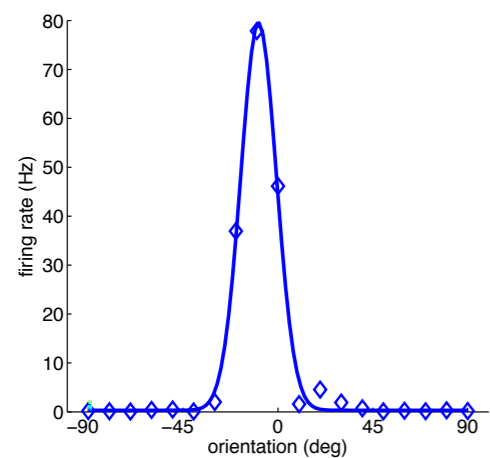
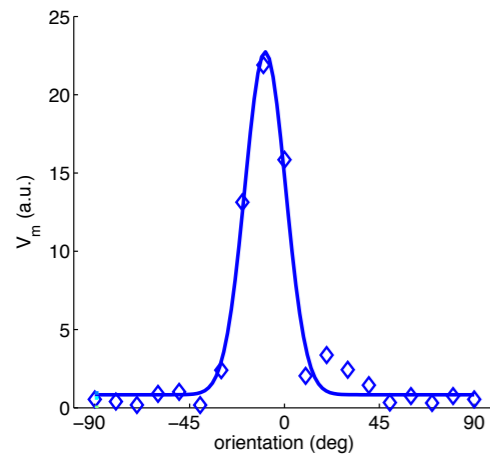
Orientation-dependence of response statistics

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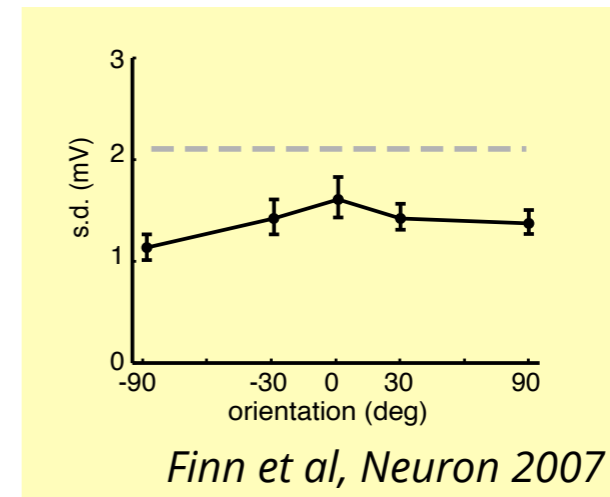
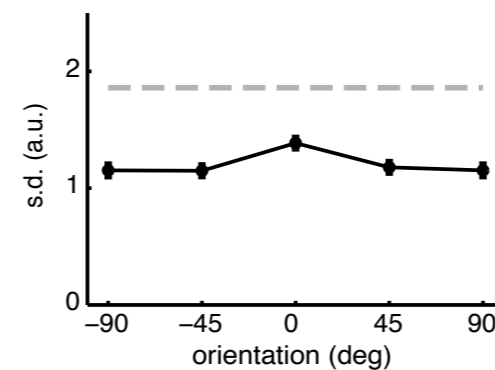
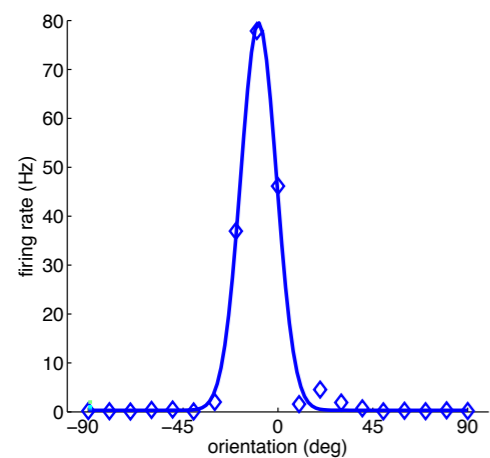
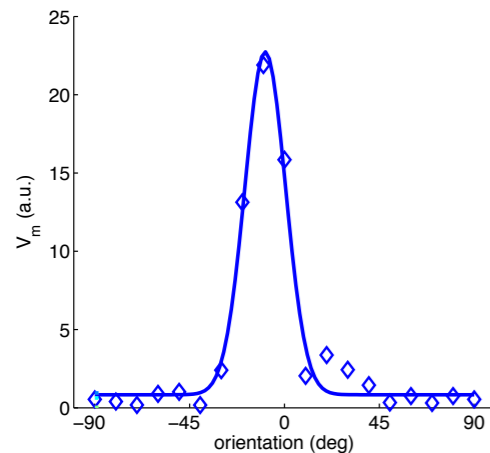
Orientation-dependence of response statistics

- orientation has a big impact on response mean
- however, no change in uncertainty is expected
- no significant change in variance is expected in membrane potential



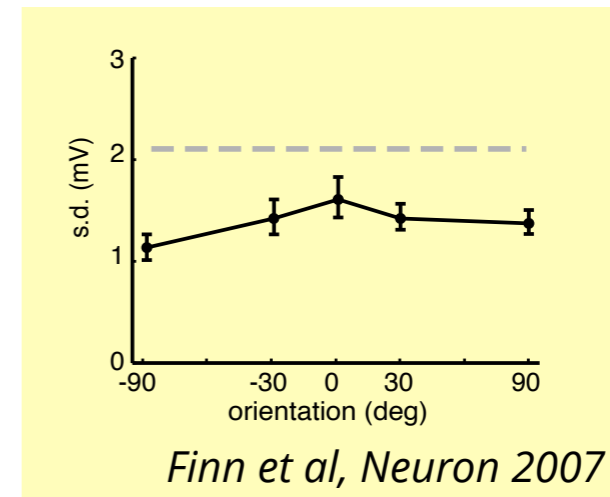
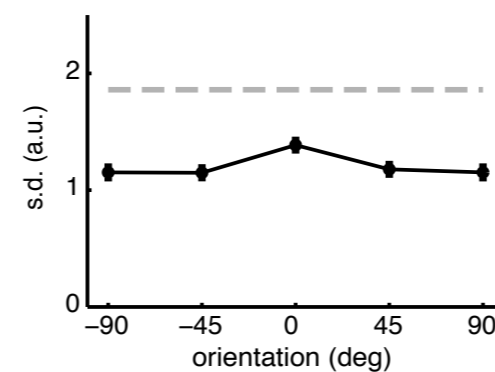
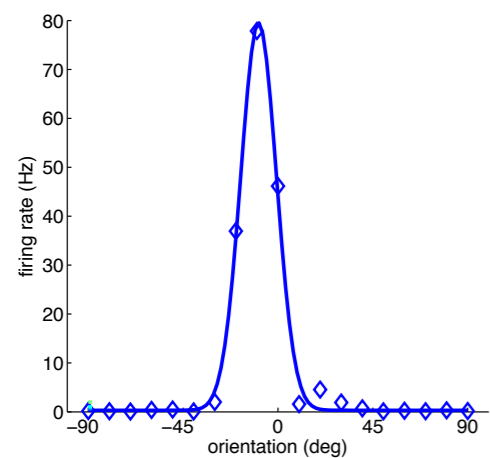
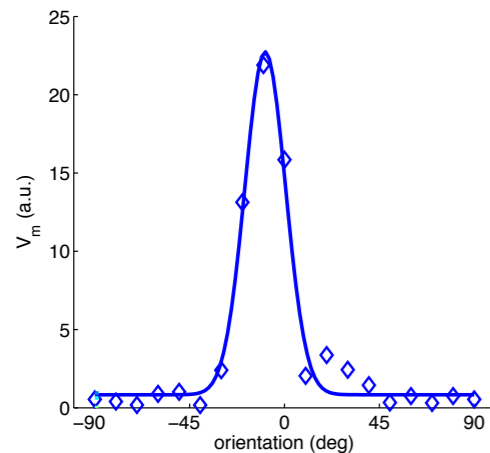
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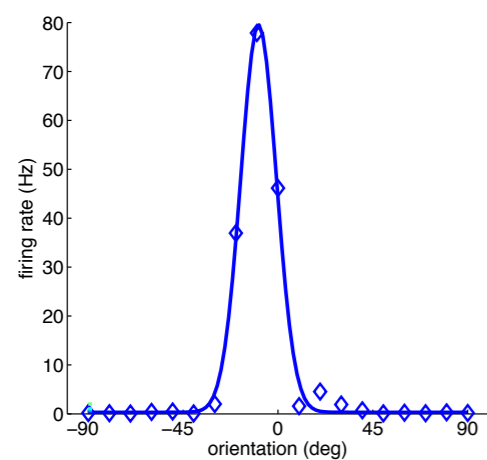
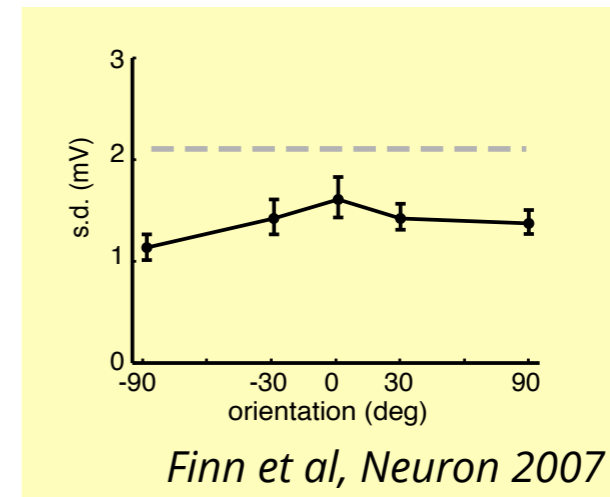
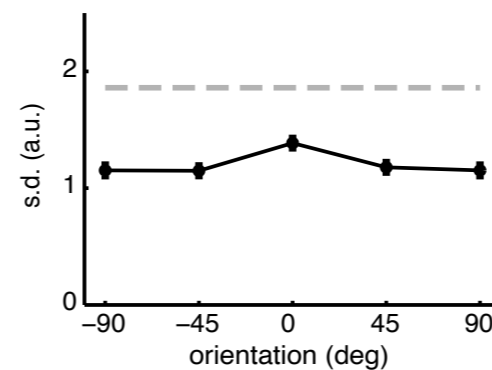
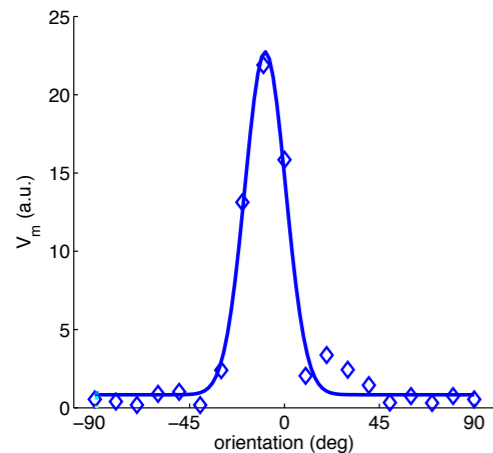
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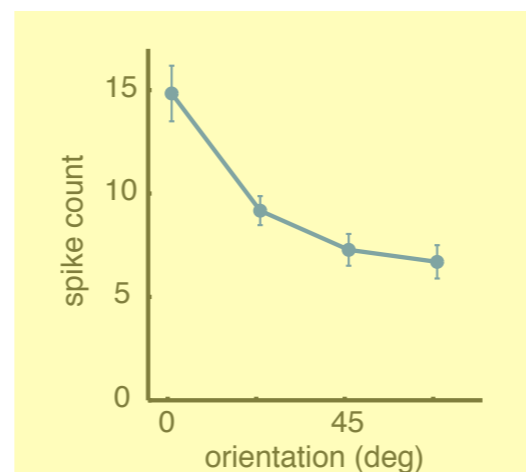
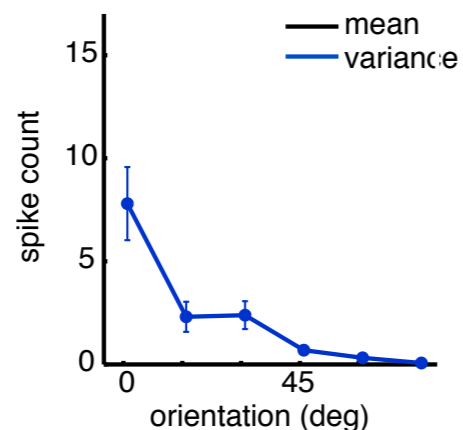
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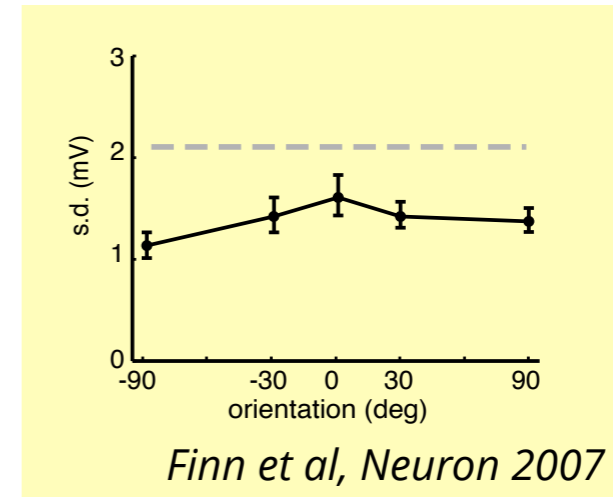
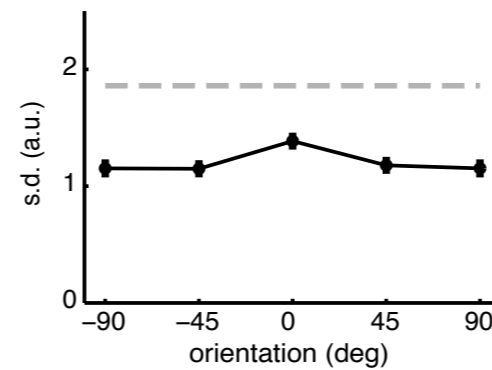
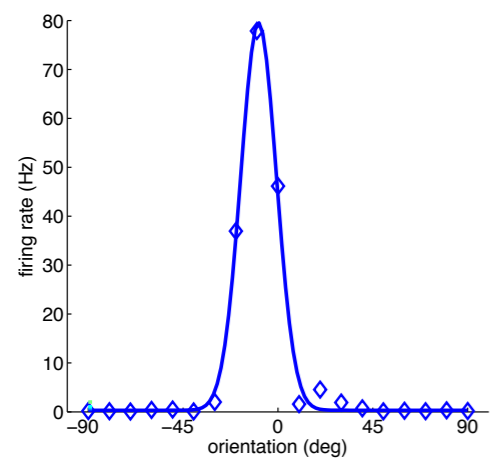
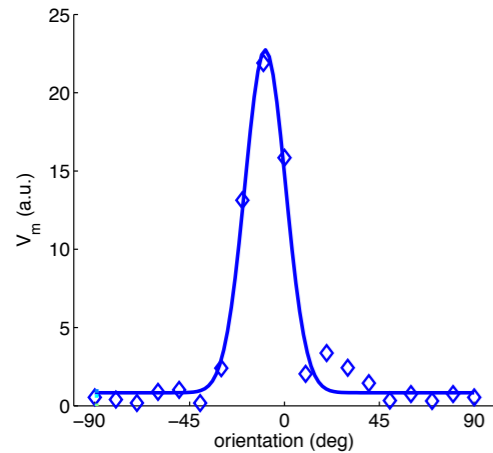


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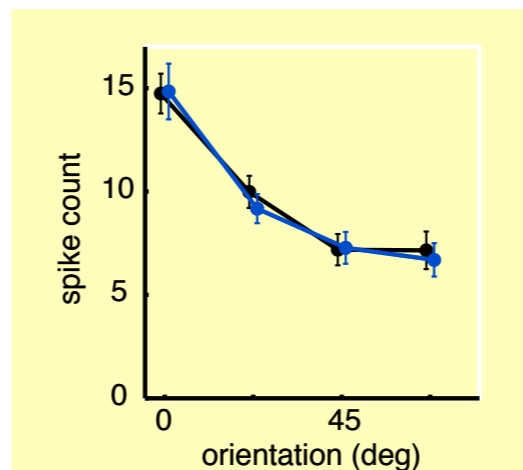
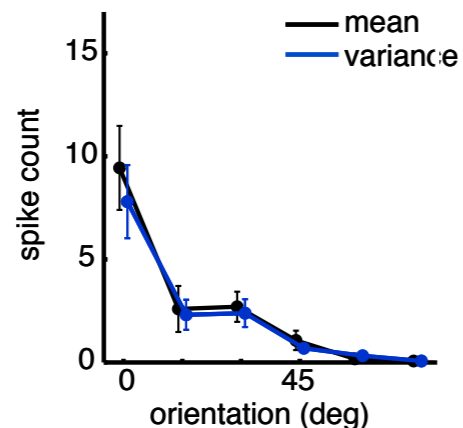


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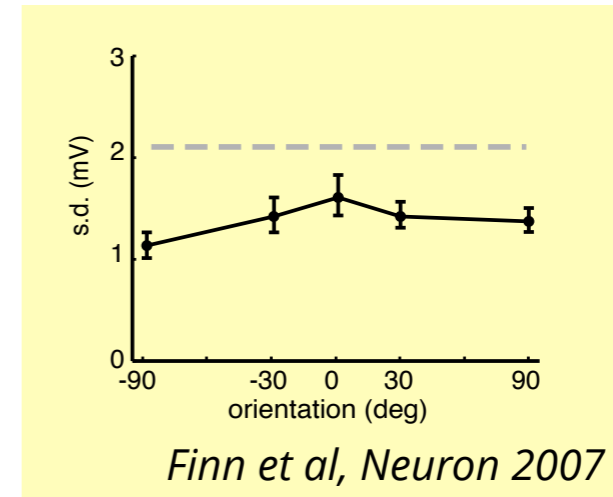
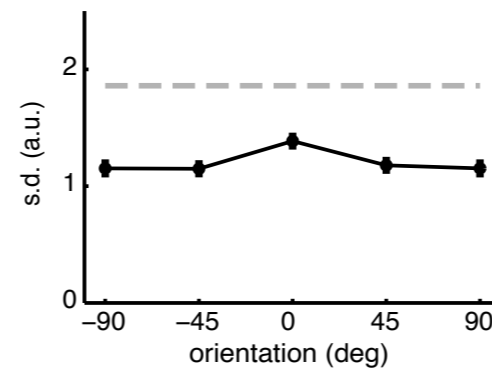
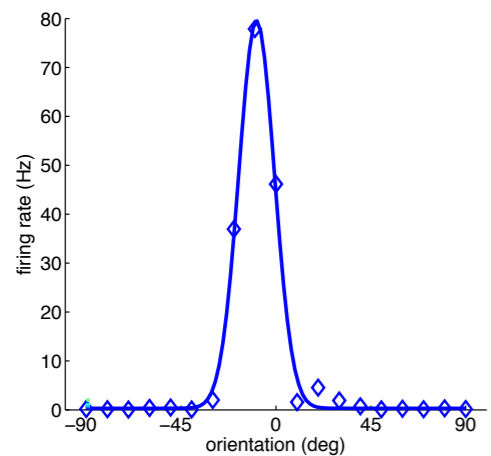
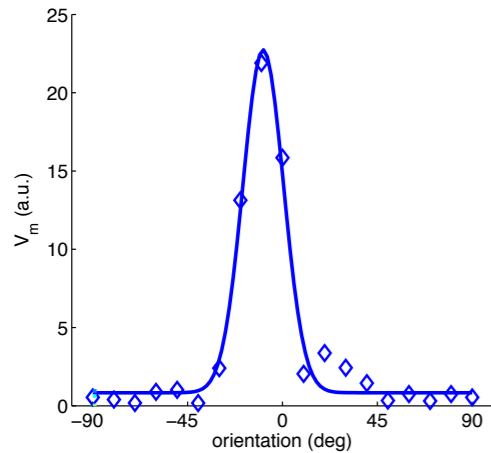


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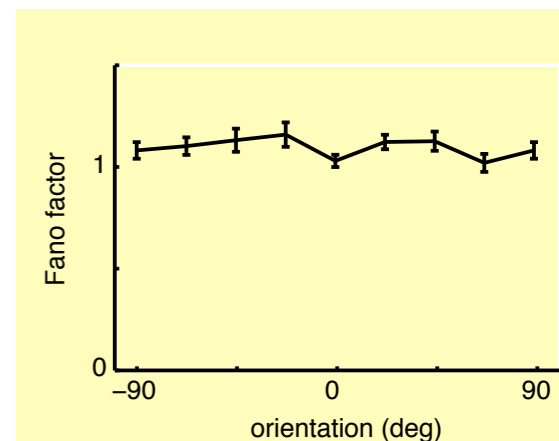
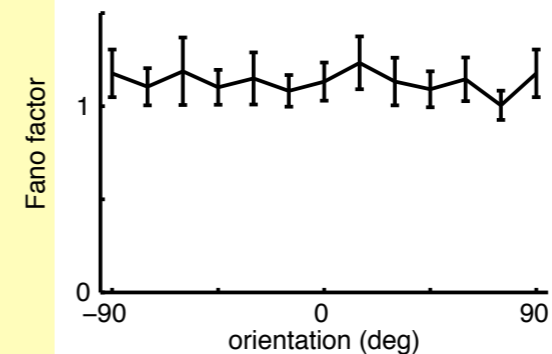
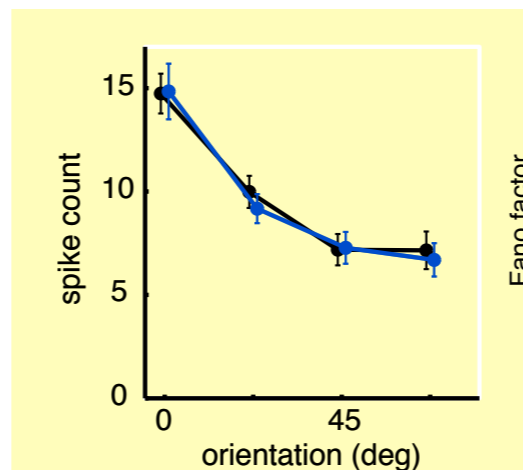
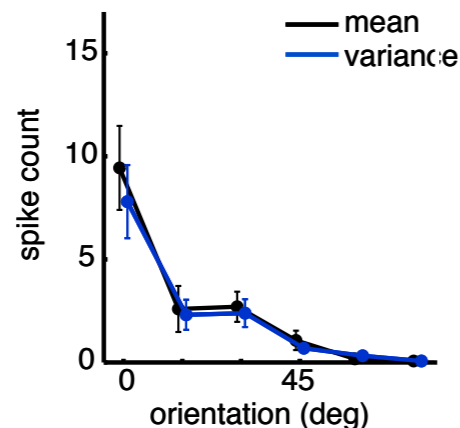


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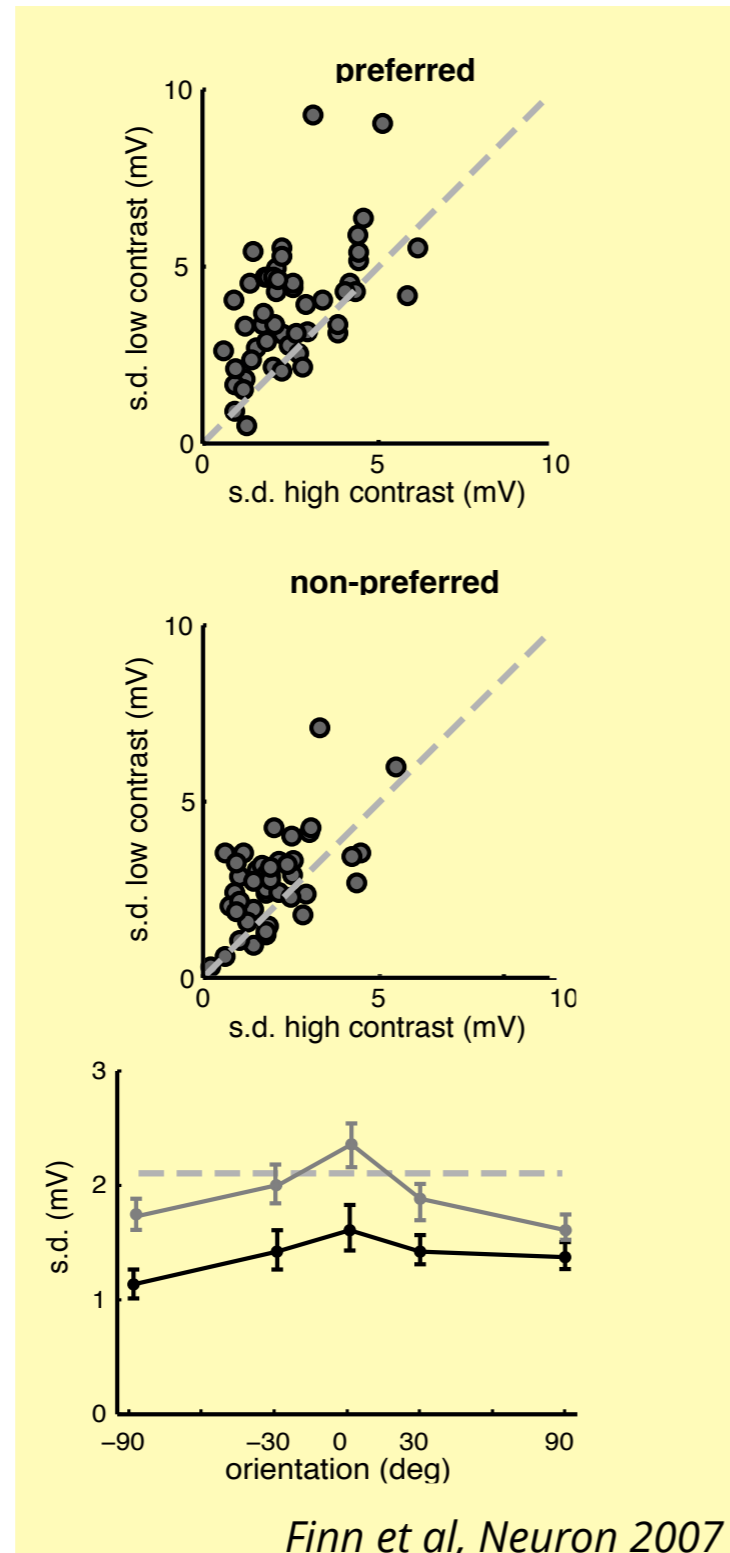
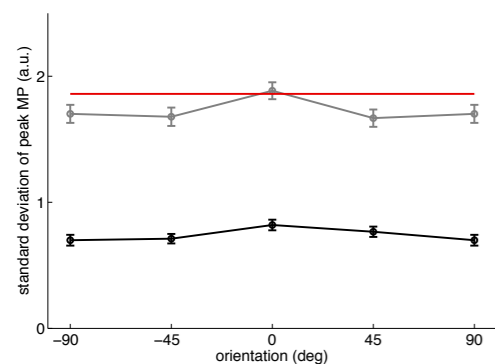
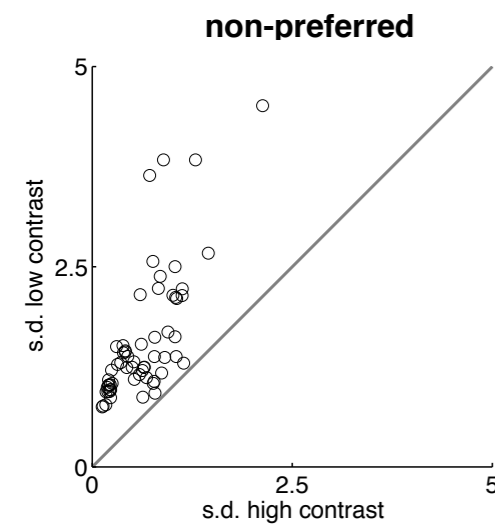
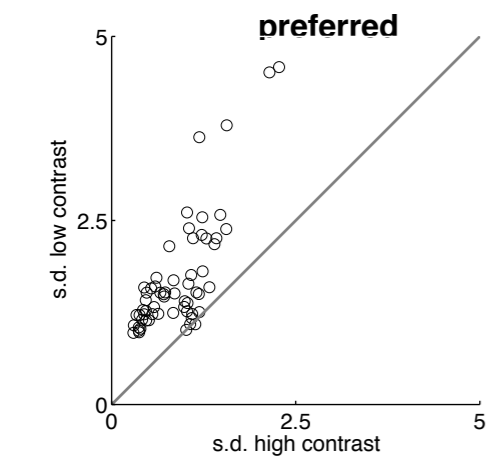
Contrast-dependence of response statistics

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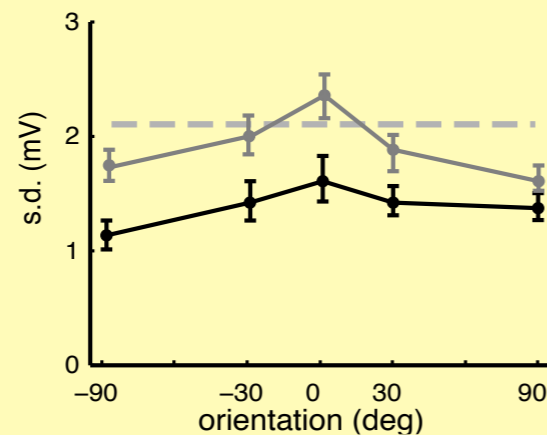
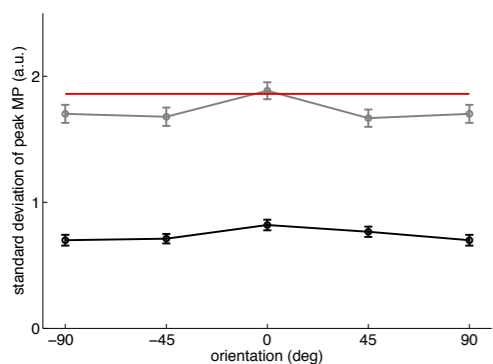
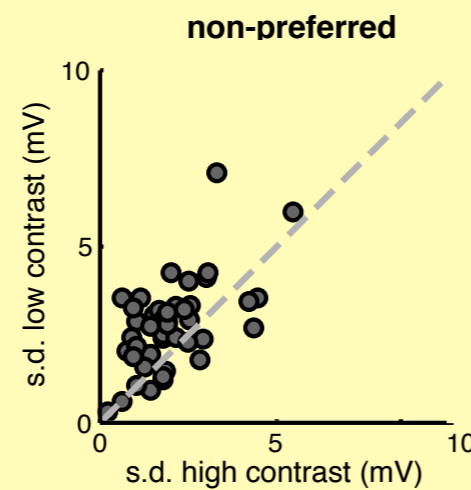
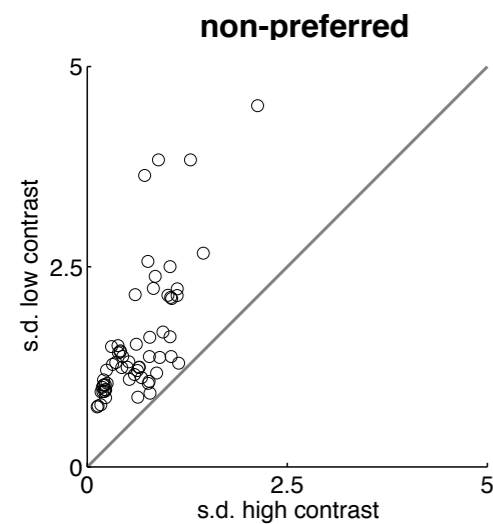
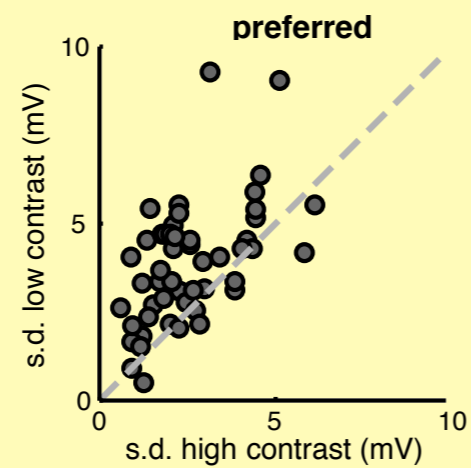
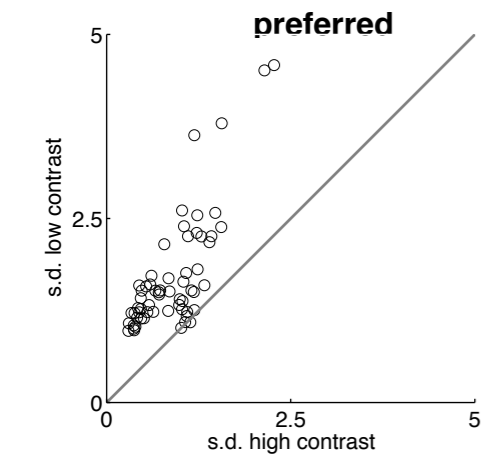
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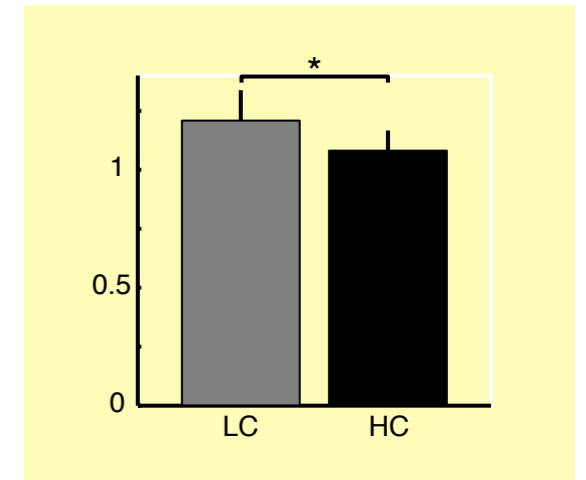
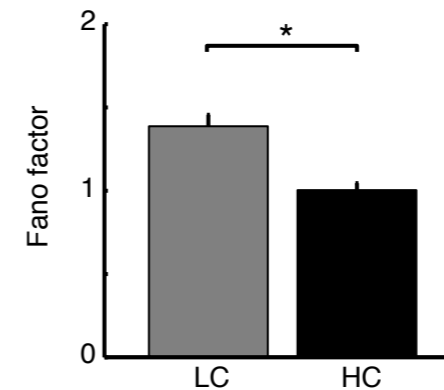


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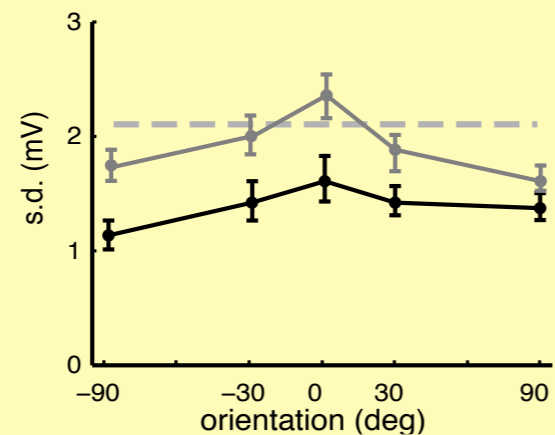
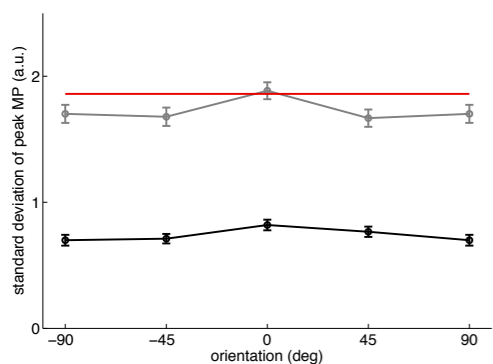
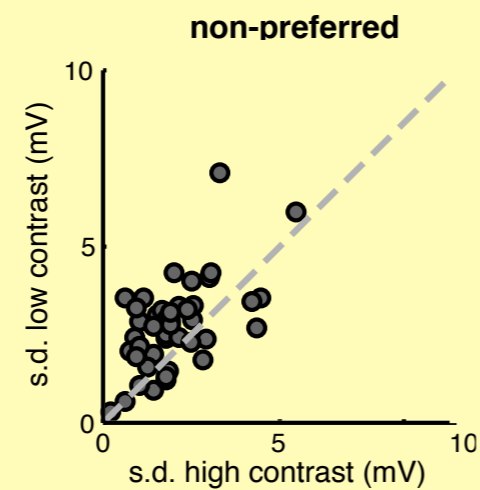
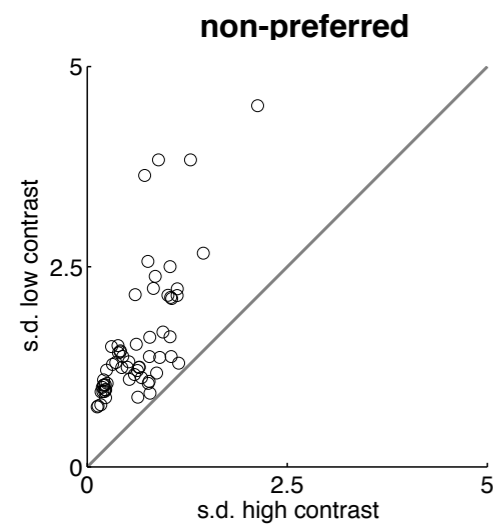
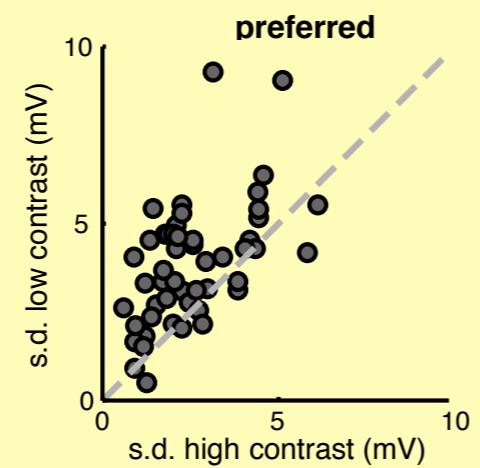
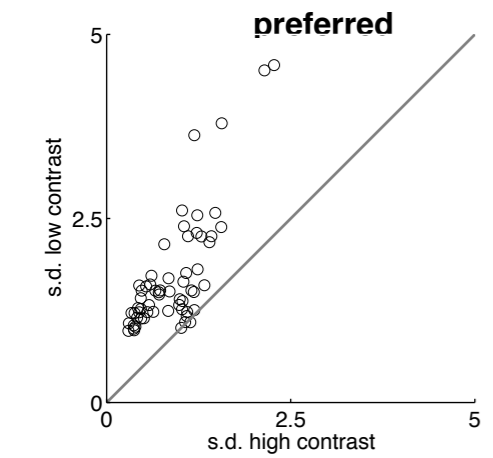


Finn et al, Neuron 2007

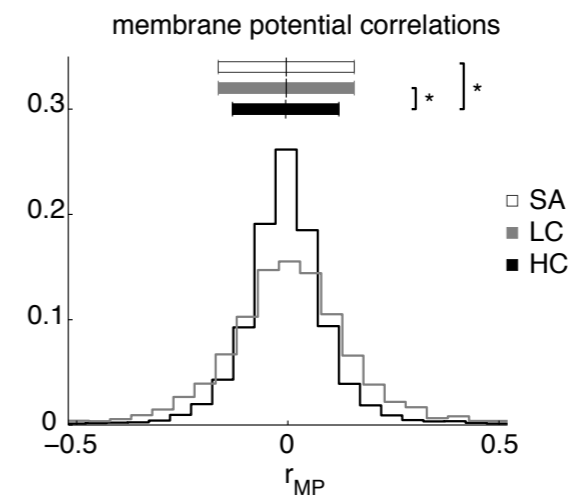
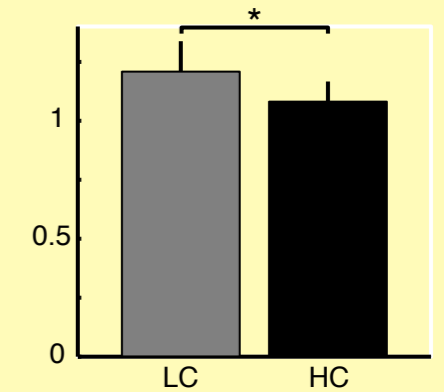
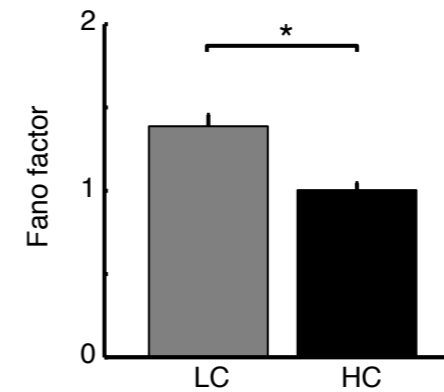


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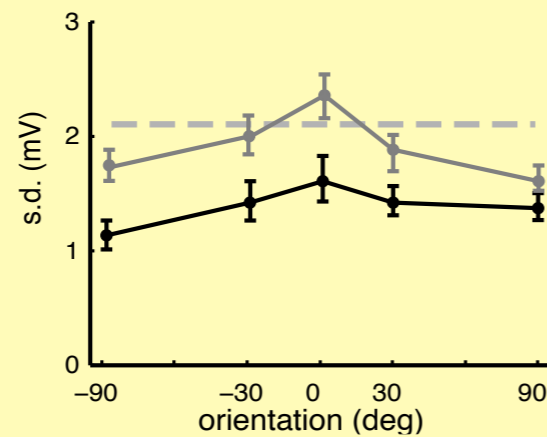
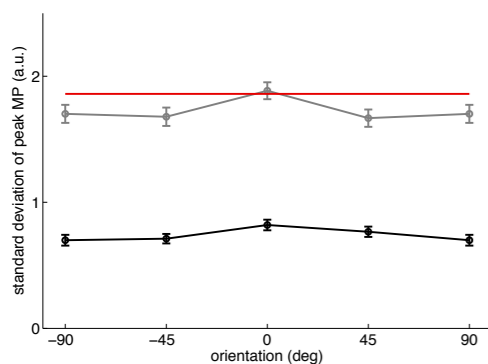
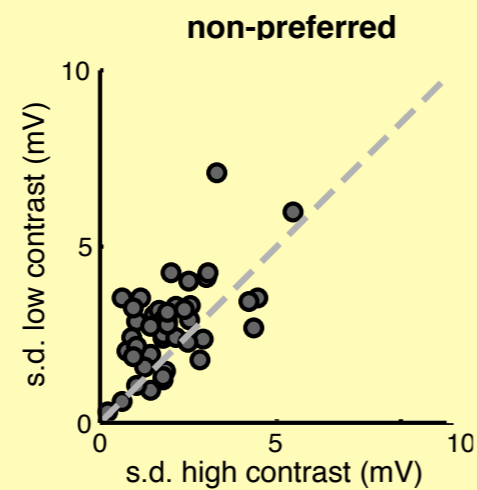
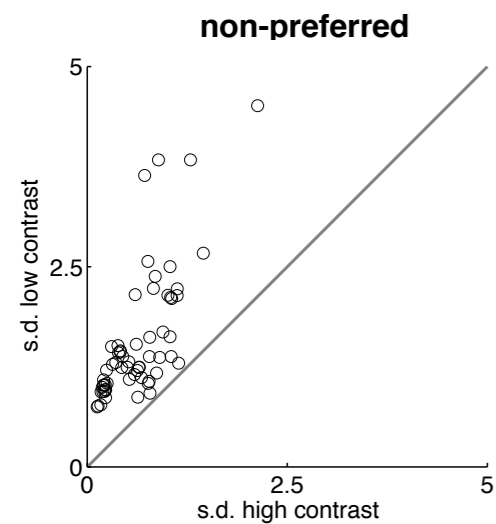
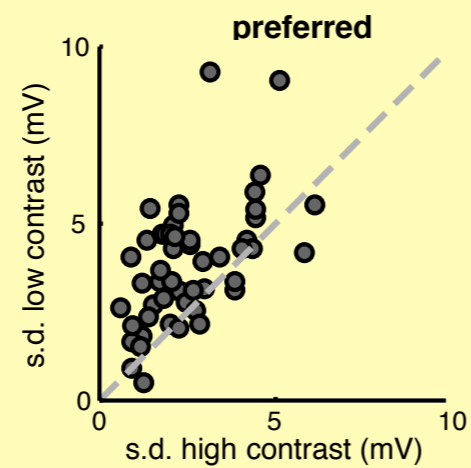
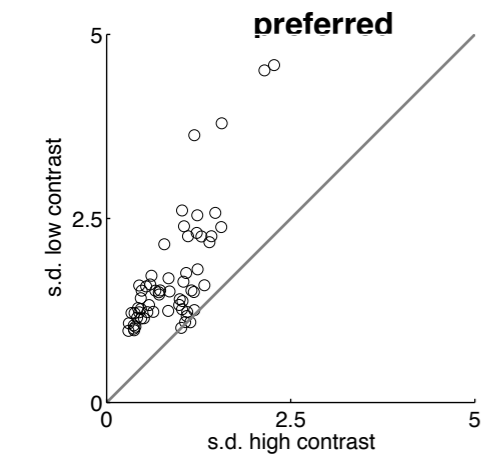


Finn et al, Neuron 2007

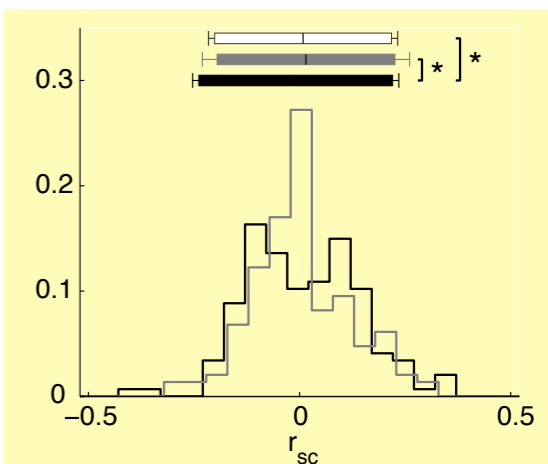
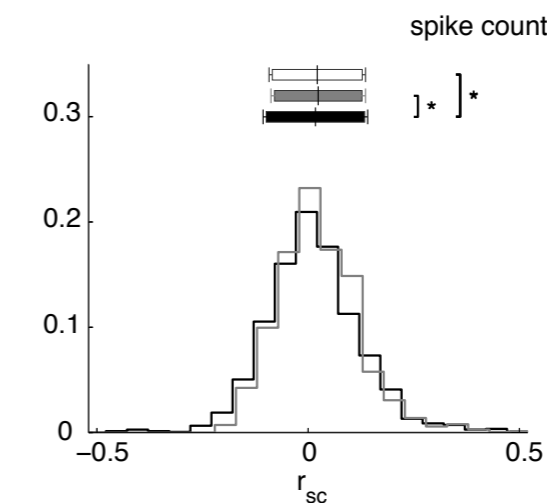
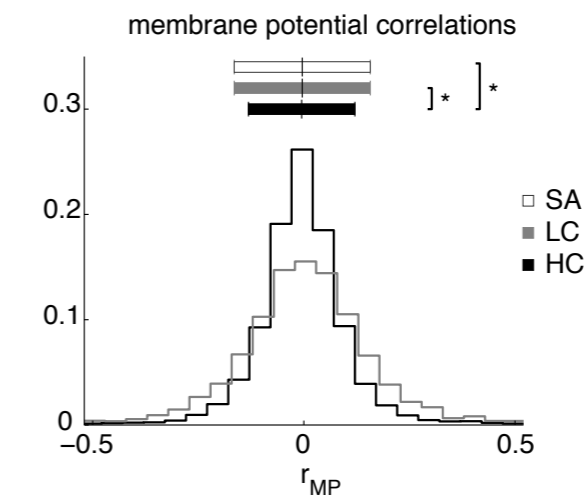
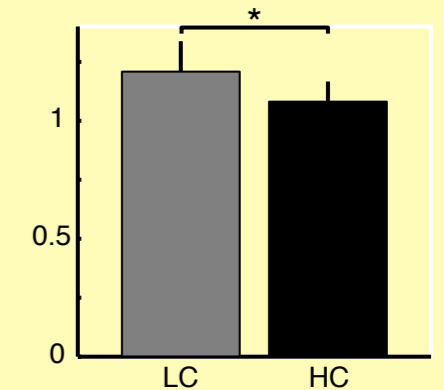
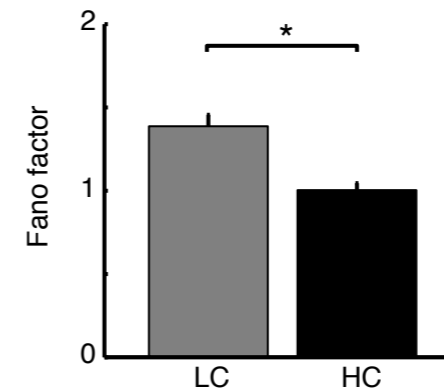


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Finn et al, Neuron 2007



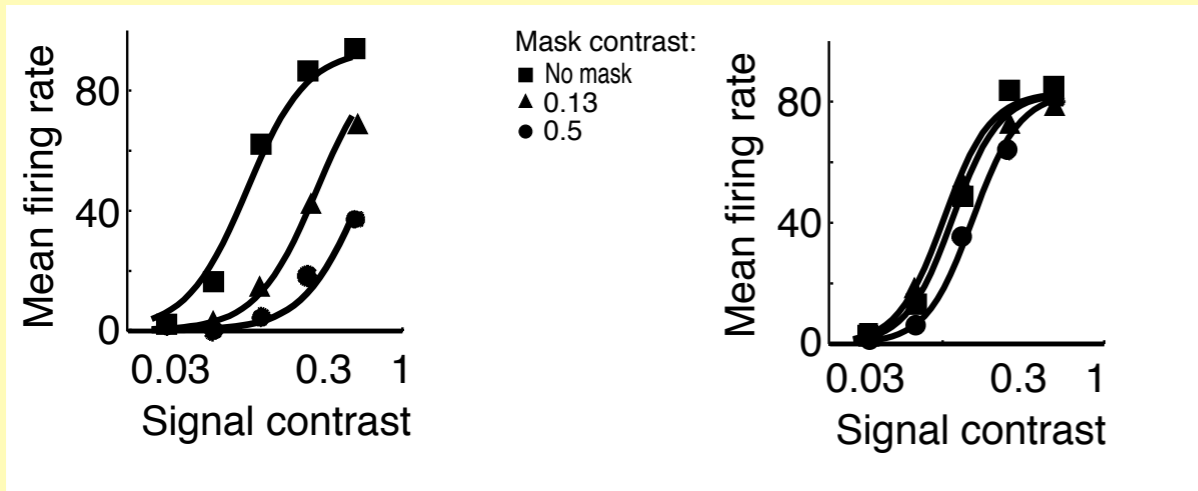
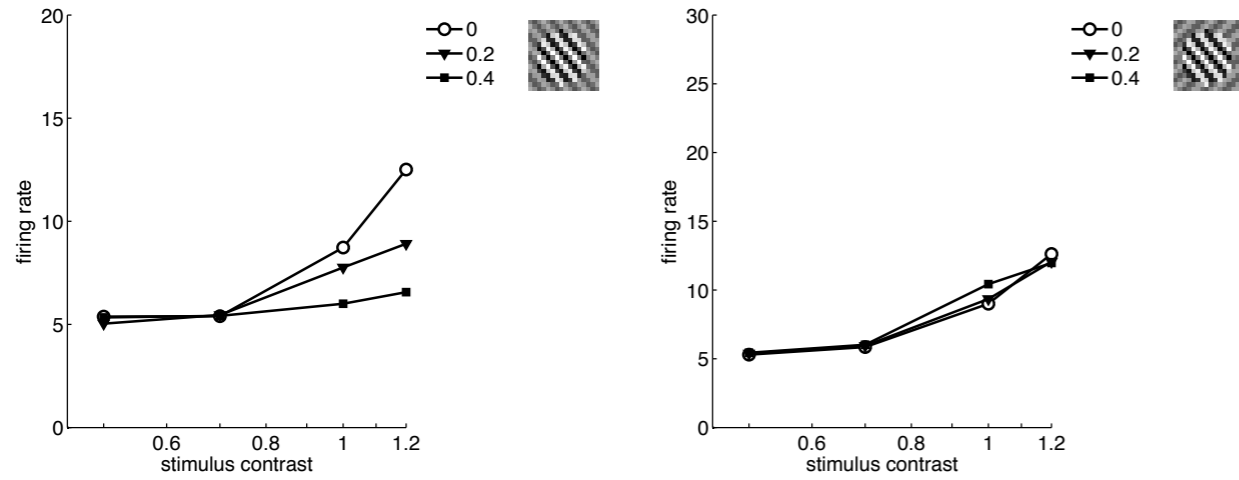
Non-classical RF dependence of response statistics

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- non-linear interaction between within-receptive field and extra-receptive field stimulation

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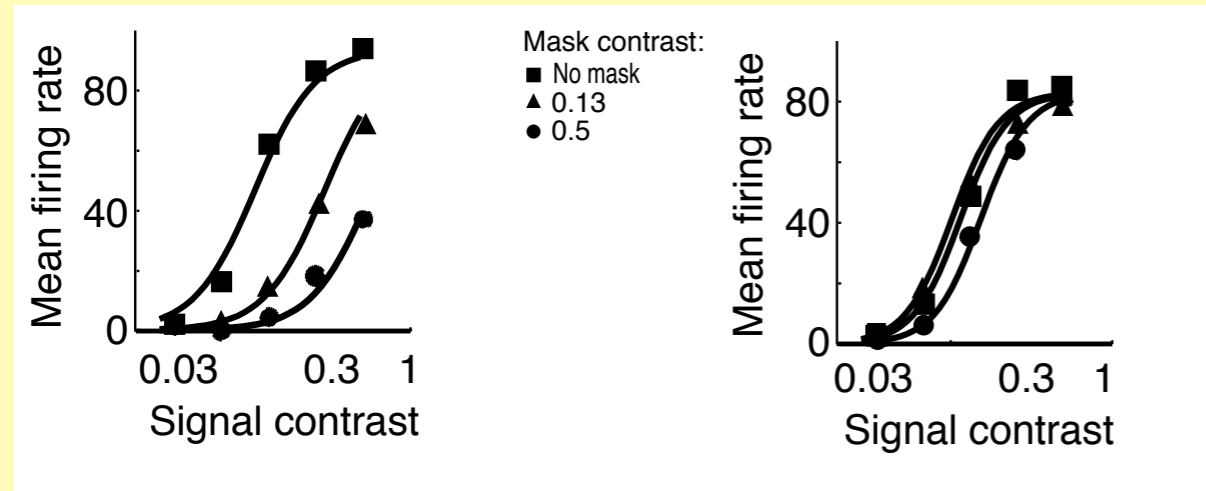
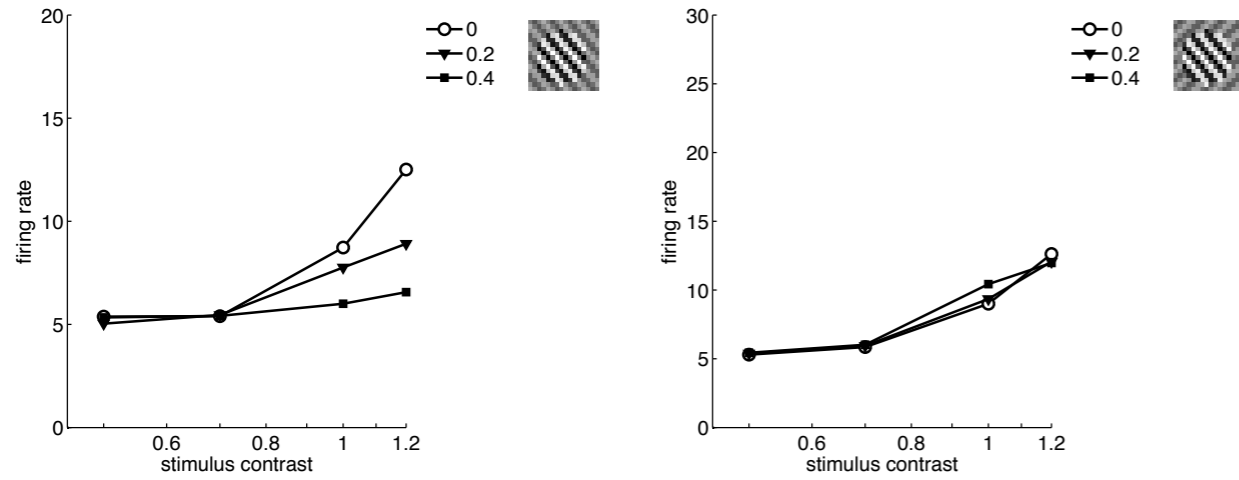
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Cavanagh, 2000

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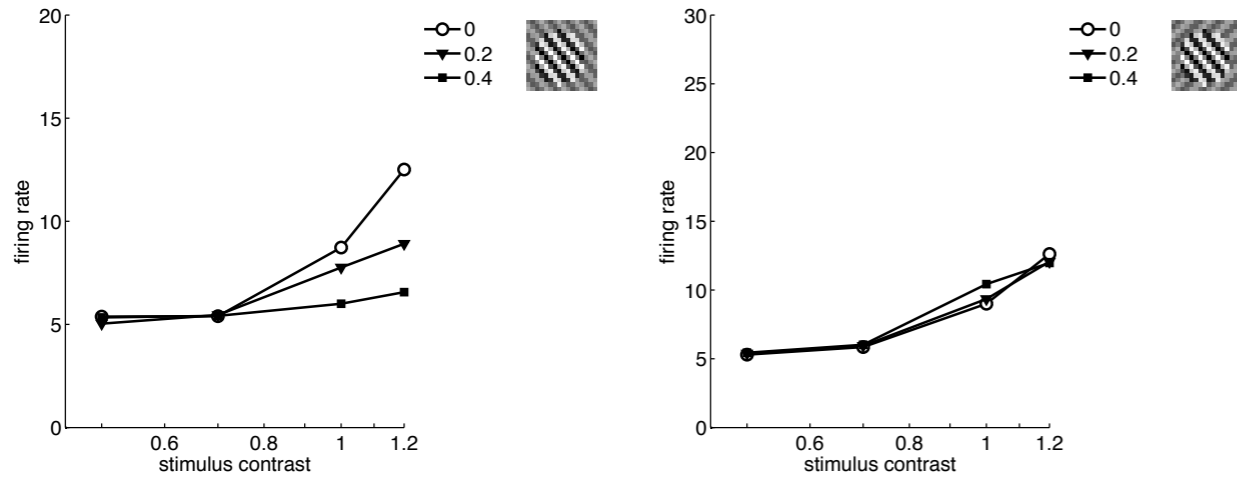
- non-linear interaction between with-receptive field and extra-receptive field stimulation
- uncertainty is affected by extra information



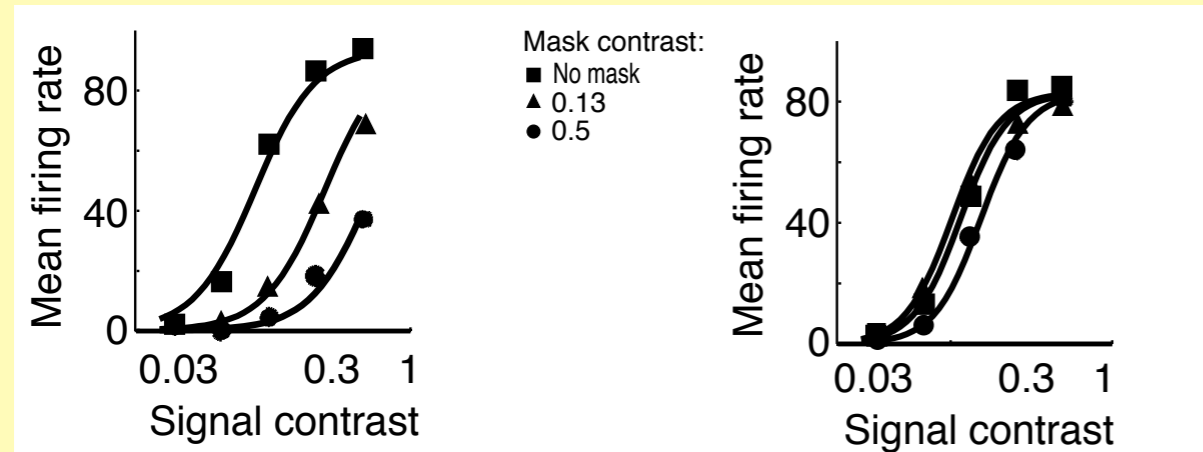
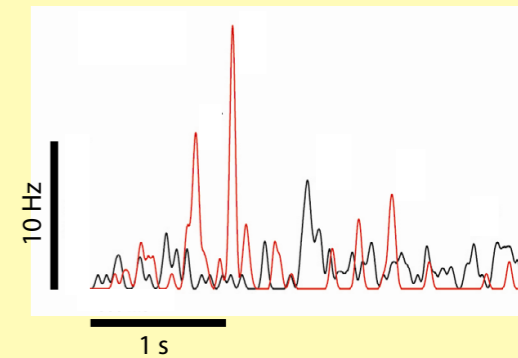
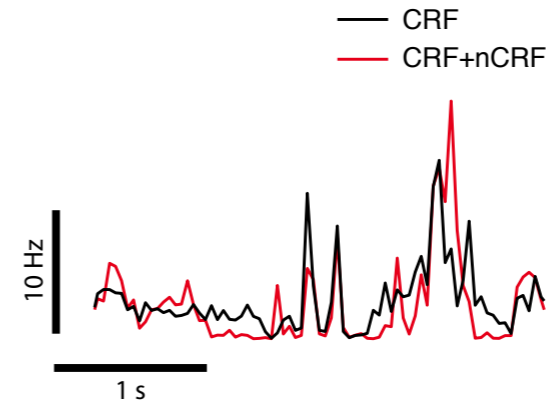
Cavanagh, 2000

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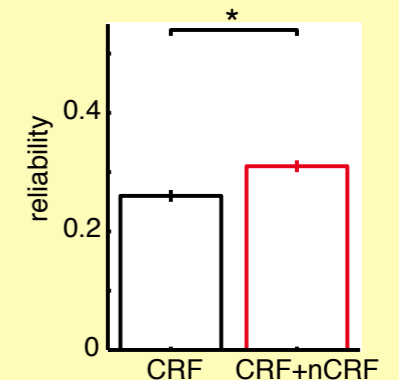
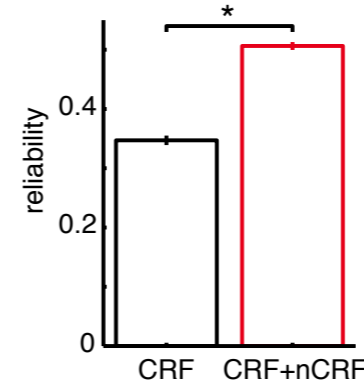
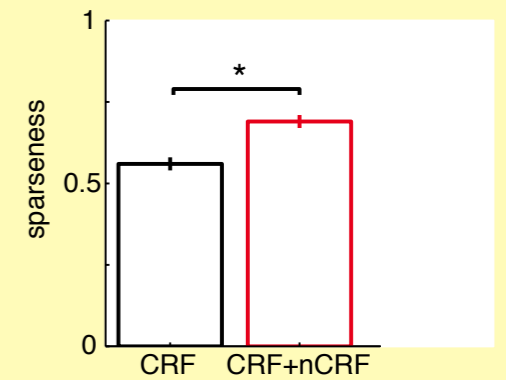
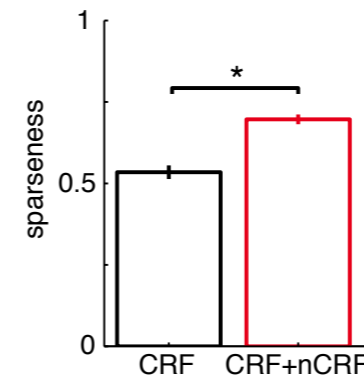
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Cavanagh, 2000



Haider et al, Neuron 2010

roadmap

- image model
- consequence of the representation of prior
- stimulus-dependence of variability
- stimulus dependence of covariability of multiple neurons

Learning and correlations structure

Learning and correlations structure

$$P(\text{responses} \mid \text{stimulus}) \propto P(\text{stimulus} \mid \text{responses}) \times P(\text{responses})$$

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prior
correlation

posterior
correlation

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Learning and correlations structure

$$P(\text{responses} \mid \text{stimulus}) \propto P(\text{stimulus} \mid \text{responses}) \times P(\text{responses})$$

The diagram illustrates the components of the matrix C^* . It features three yellow speech bubbles: 'prior correlation' pointing to a blue box containing C^* , 'posterior correlation' pointing to a green box containing $\sum_t \Sigma(t)$, and 'signal correlation' pointing to a red box containing $\sum_t \mu(t) \mu^T(t)$. The equation $C^* \approx \frac{1}{T} \left(\sum_t \Sigma(t) + \sum_t \mu(t) \mu^T(t) \right)$ is shown below the boxes.

$$C^* \approx \frac{1}{T} \left(\sum_t \Sigma(t) + \sum_t \mu(t) \mu^T(t) \right)$$

Learning and correlations structure

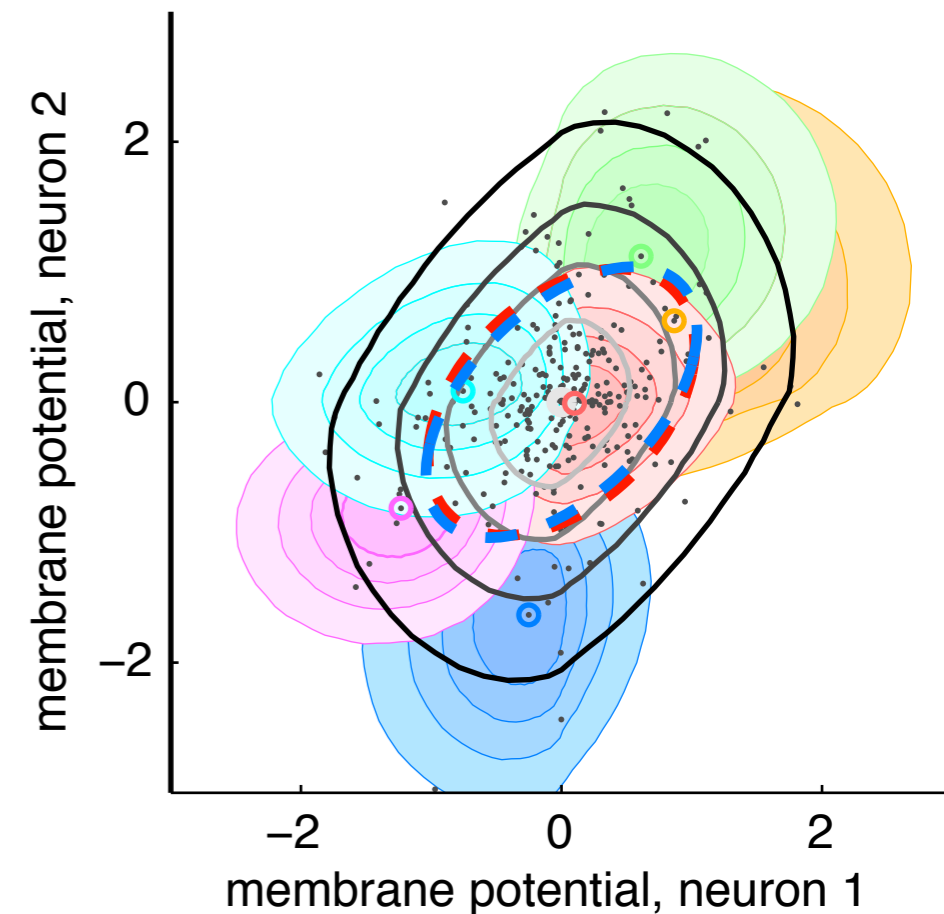
$$P(\text{responses} \mid \text{stimulus}) \propto P(\text{stimulus} \mid \text{responses}) \times P(\text{responses})$$

prior correlation

posterior correlation

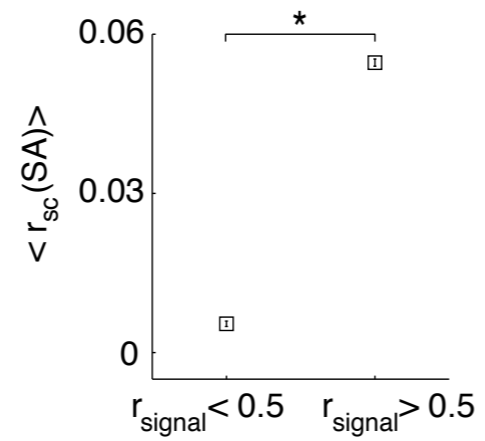
signal correlation

$$C^* \approx \frac{1}{T} \left(\sum_t \Sigma(t) + \sum_t \mu(t) \mu^T(t) \right)$$

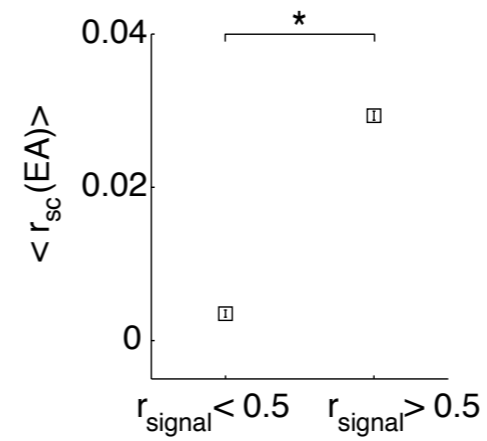


Relationship between various forms of correlations

signal vs. spontaneous correlation

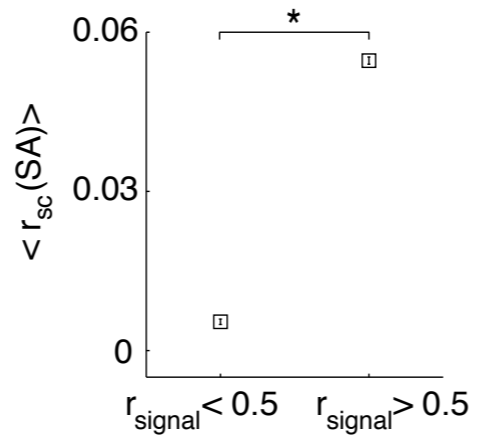


signal vs. noise correlation

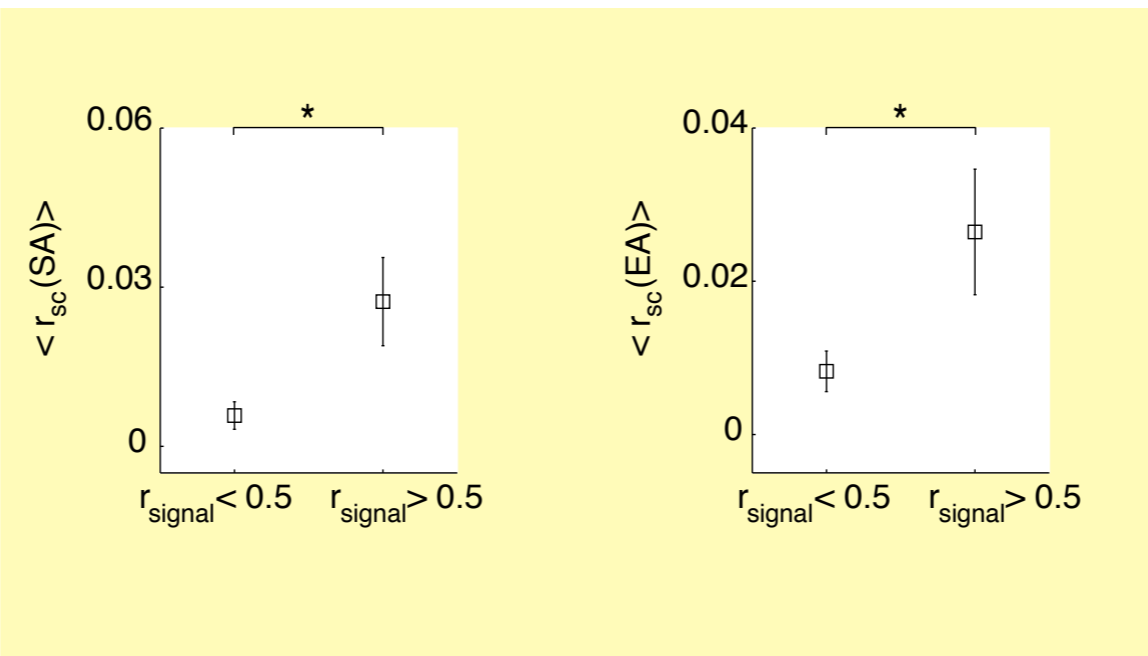
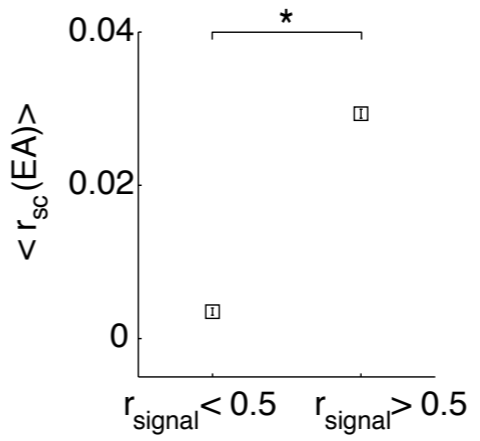


Relationship between various forms of correlations

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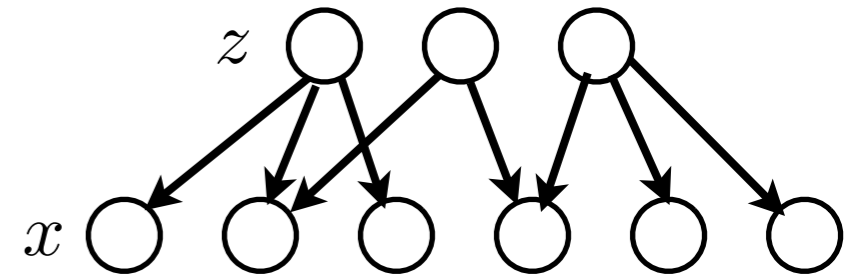


signal vs. noise correlation

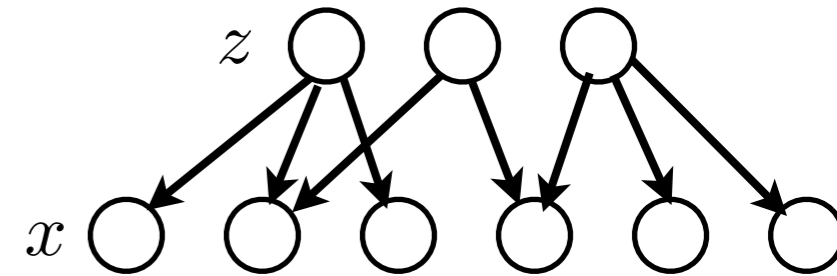
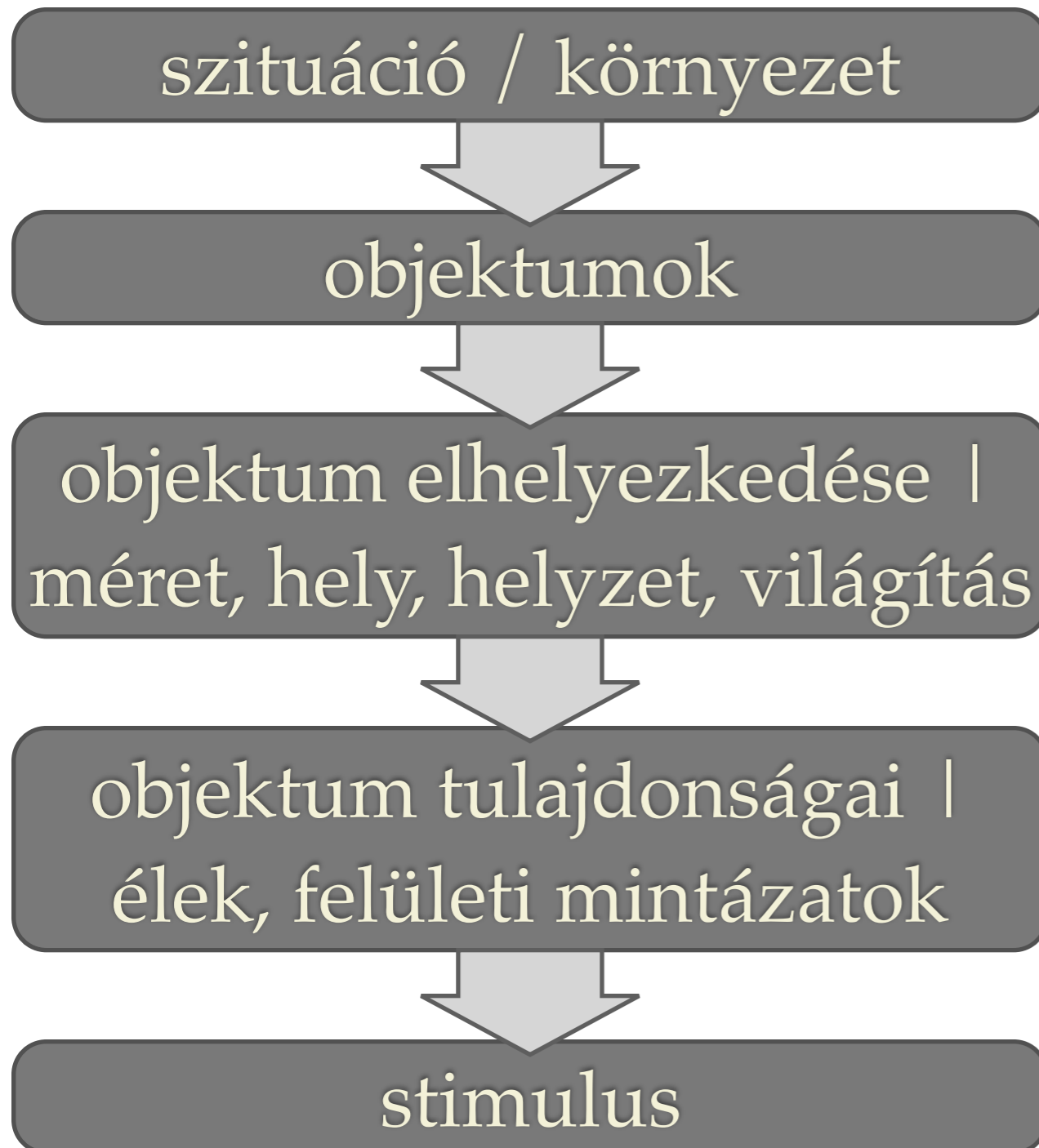


Bayes inferencia

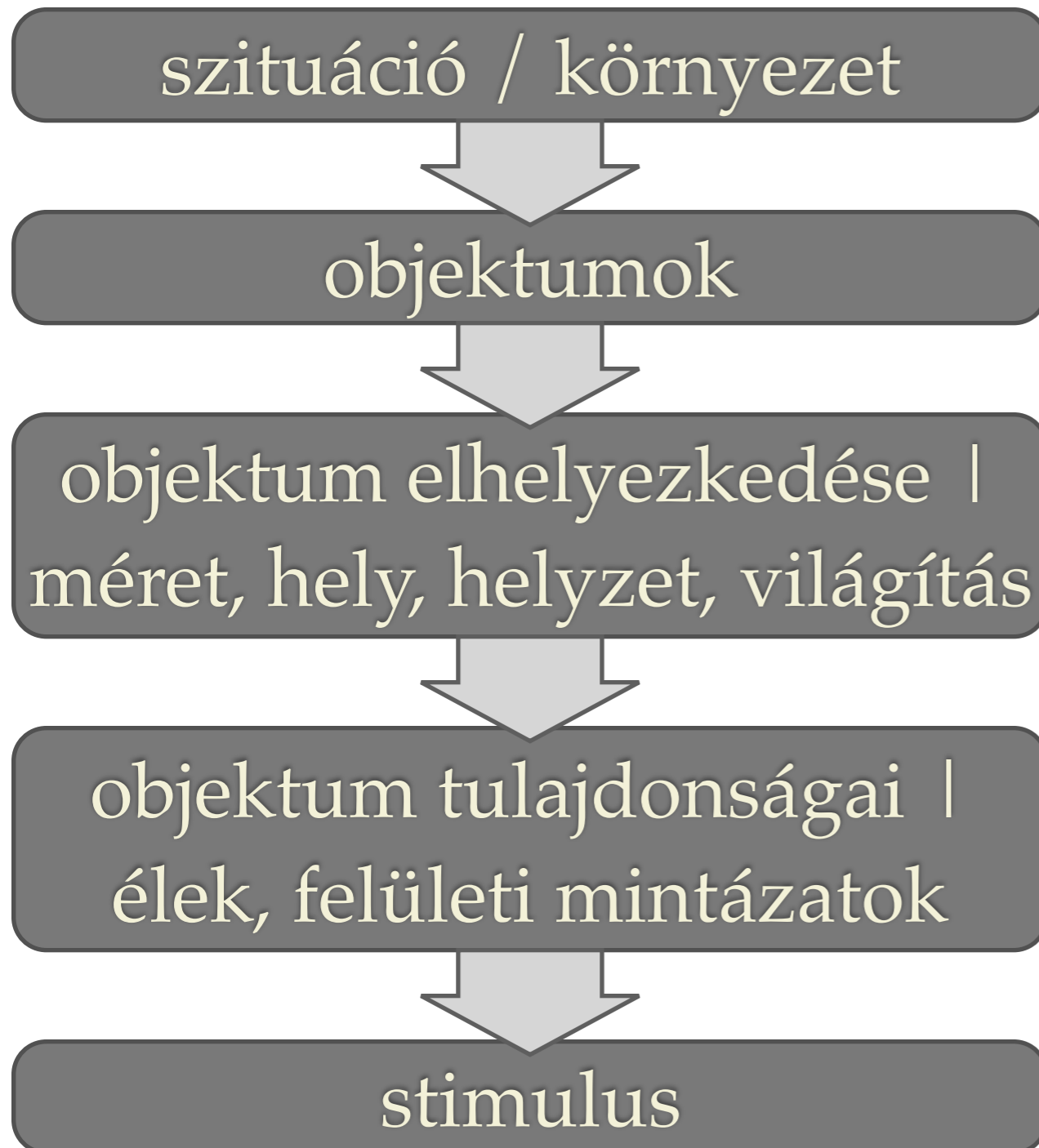
Bayes inferencia



Bayes inferencia

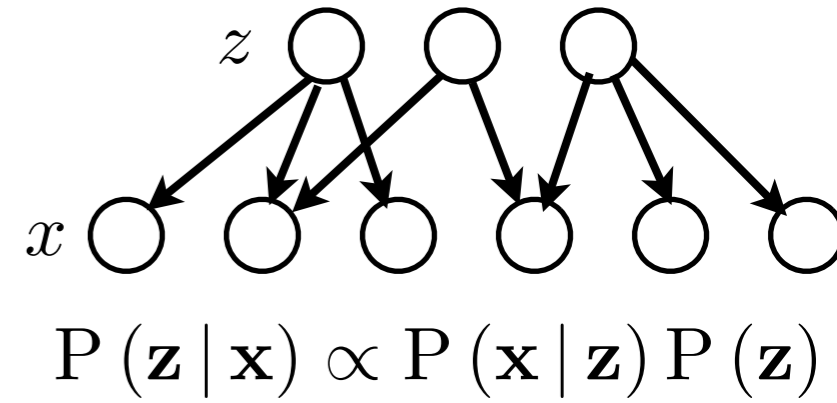


Bayes inferencia



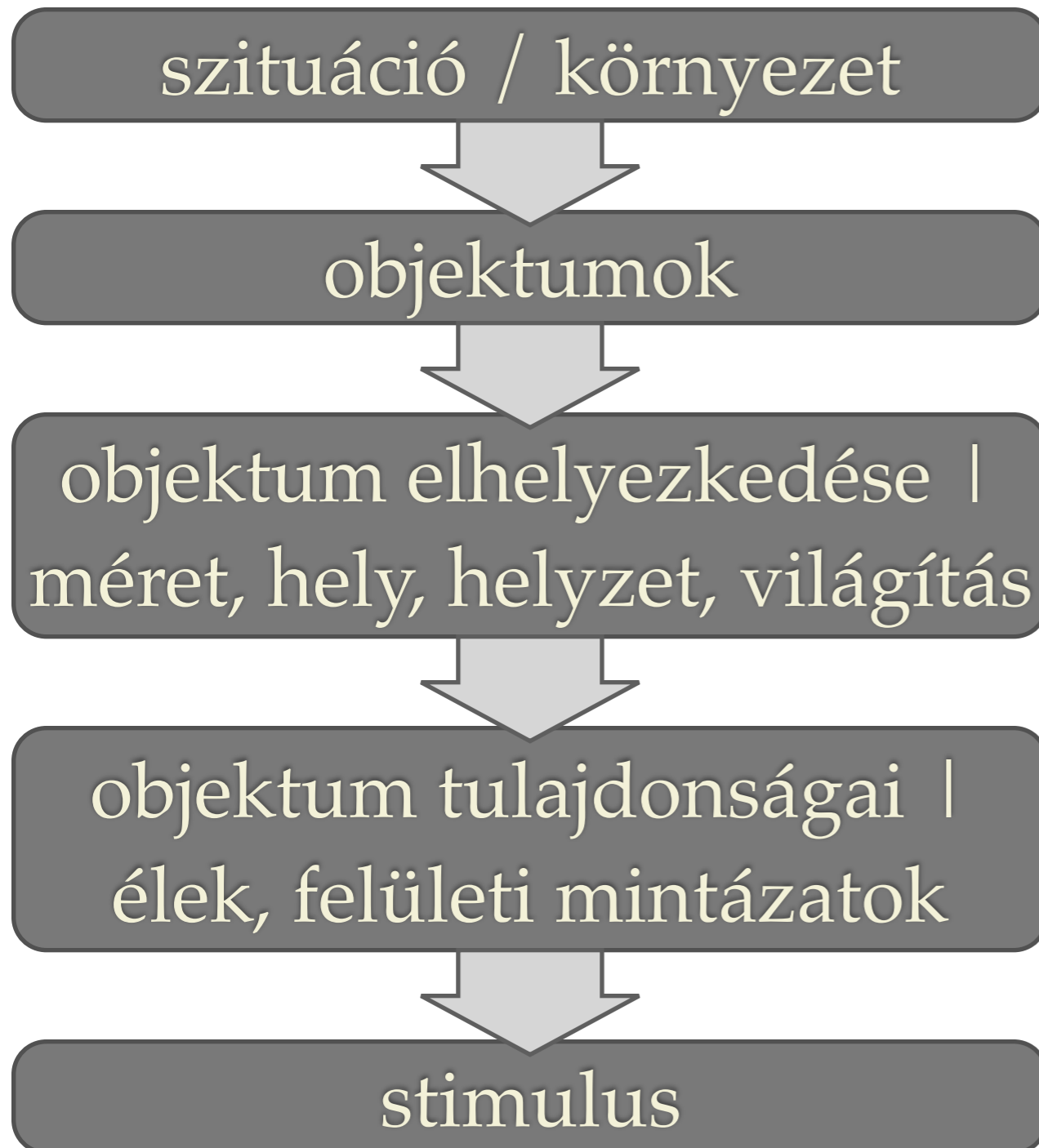
generatív modell

inferencia/felismerés



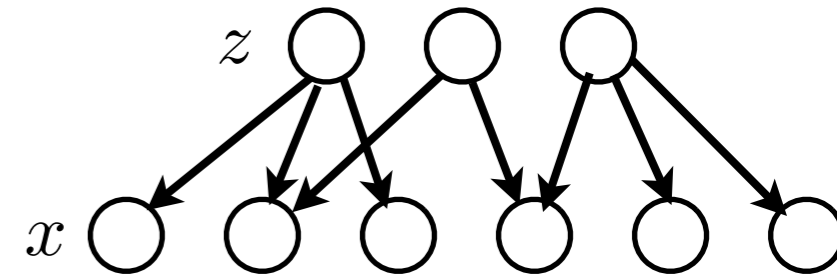
$$P(\mathbf{z} | \mathbf{x}) \propto P(\mathbf{x} | \mathbf{z}) P(\mathbf{z})$$

Bayes inferencia



generatív modell

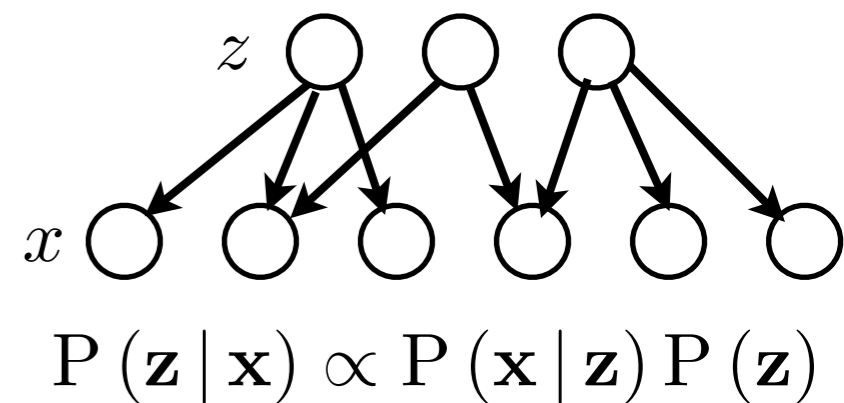
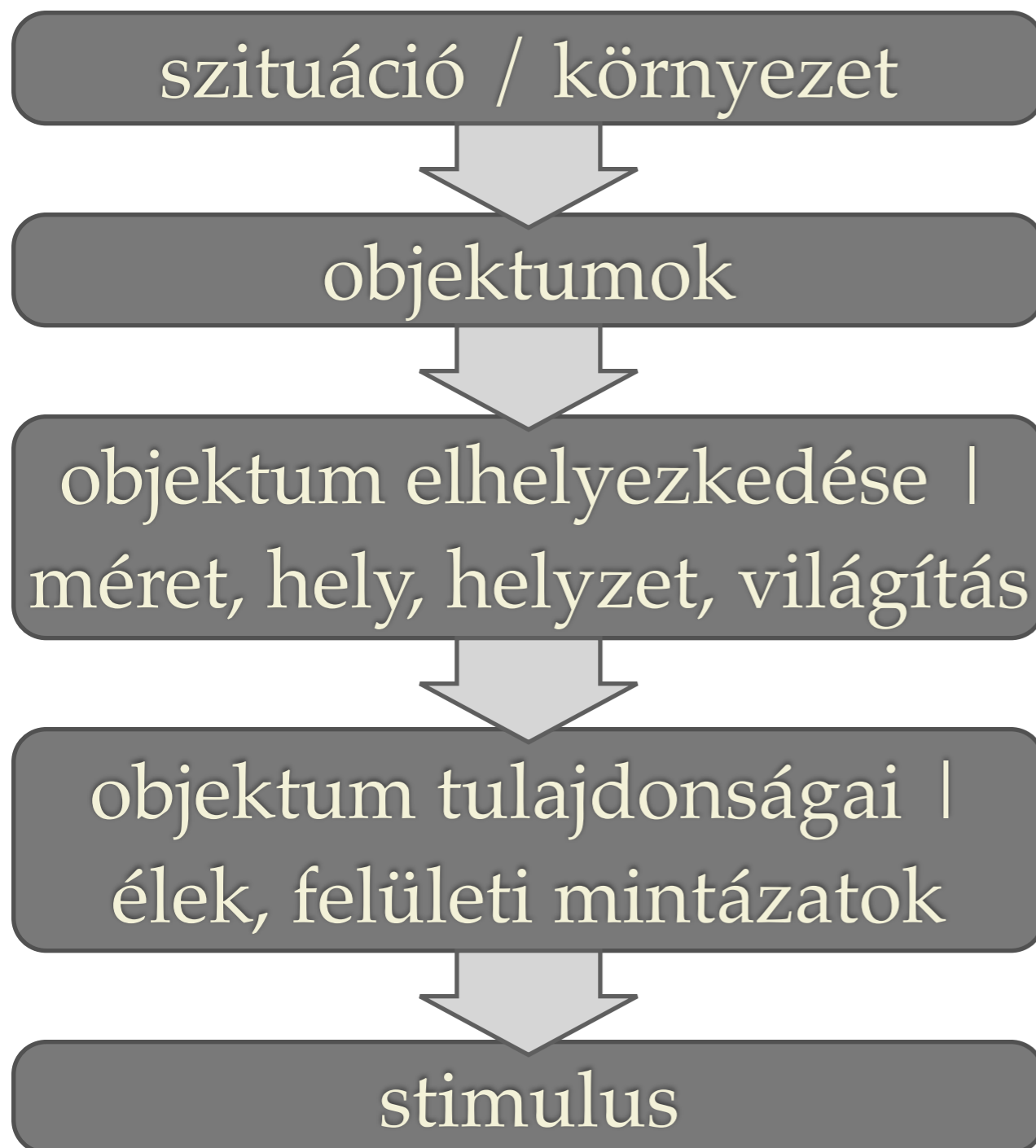
inferencia/felismerés



$$P(\mathbf{z} | \mathbf{x}) \propto P(\mathbf{x} | \mathbf{z}) P(\mathbf{z})$$

Eddig arra koncentráltunk,
hogy mi a legvalószínűbb
aktivitás

Bayes inferencia



Eddig arra koncentráltunk,
hogy mi a legvalószínűbb
aktivitás

Ez a maximum a posteriori
becslés (MAP)

inferencia/felismerés

Lineáris modellek

$$P(x | z) = \text{Normal}(x; z, \theta) = C \exp\left((x - Az)^T \Sigma^{-1} (x - Az)\right)$$

Lineáris modellek

$$P(x | z) = \text{Normal}(x; z, \theta) = C \exp\left(-\frac{1}{2}(x - Az)^T \Sigma^{-1} (x - Az)\right)$$

PCA

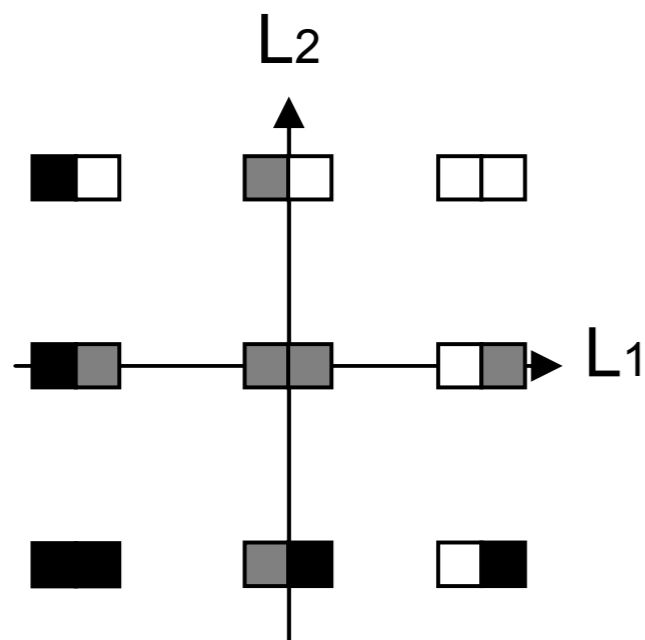
- A oszlopvektorai ortogonálisak
- $D(x) = D(z)$
- Izotróp zaj

Lineáris modellek

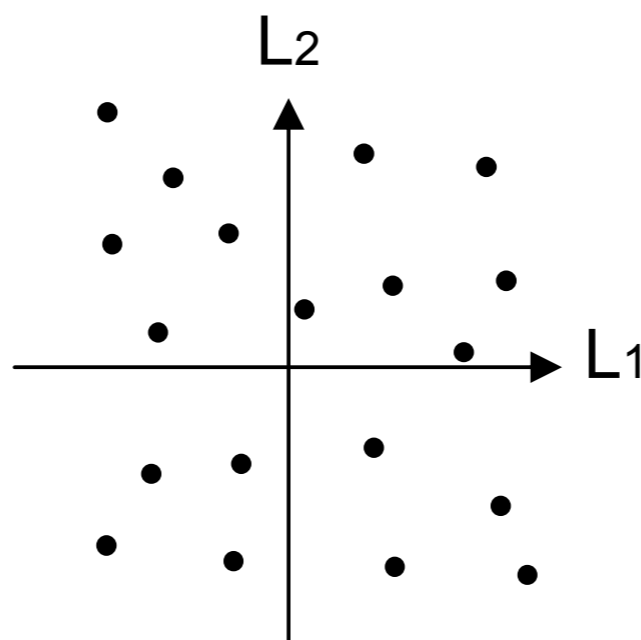
$$P(x | z) = \text{Normal}(x; z, \theta) = C \exp\left(-\frac{1}{2}(x - Az)^T \Sigma^{-1} (x - Az)\right)$$

PCA

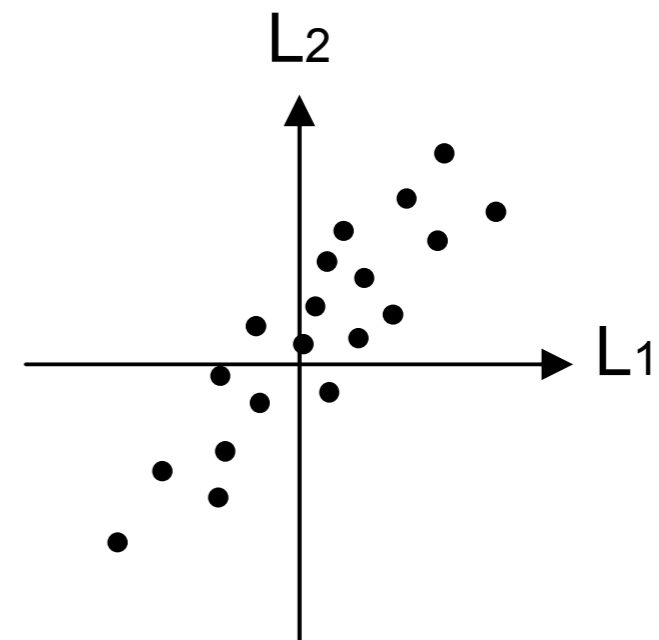
- A oszlopvektorai ortogonálisak
- $D(x) = D(z)$
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State space of two pixel images



Random images



Structured images

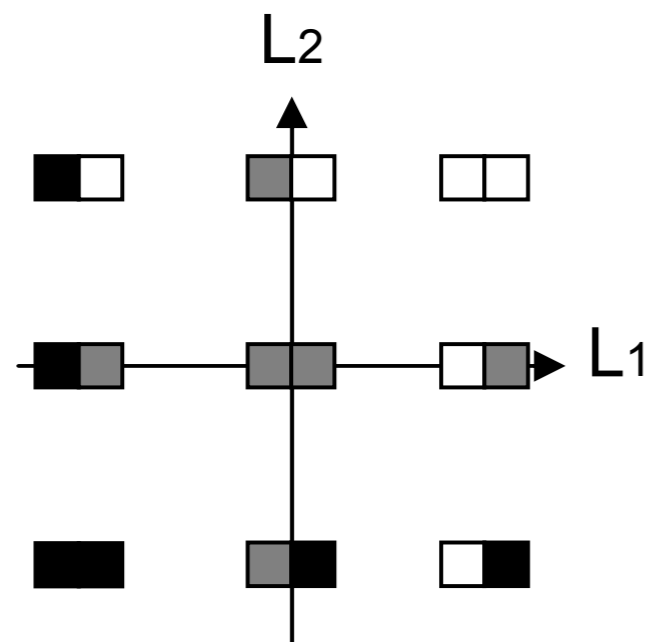
Lineáris modellek

$$P(x | z) = \text{Normal}(x; z, \theta) = C \exp\left(-\frac{1}{2}(x - Az)^T \Sigma^{-1} (x - Az)\right)$$

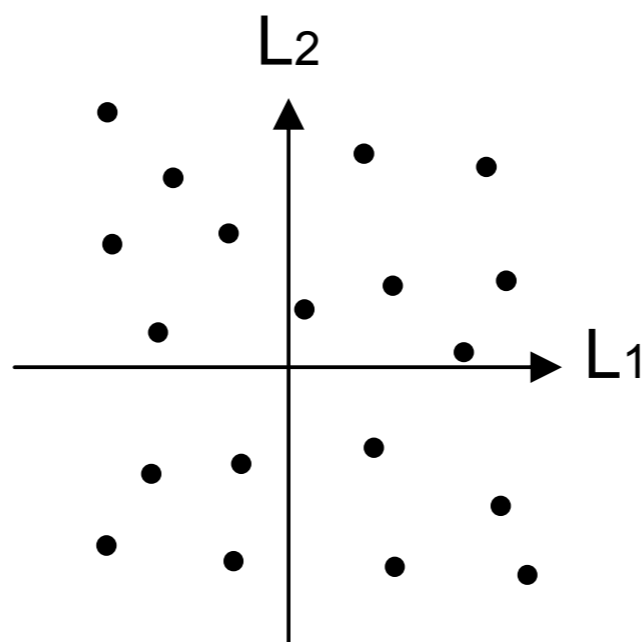
$$x = A \cdot z + \epsilon$$

PCA

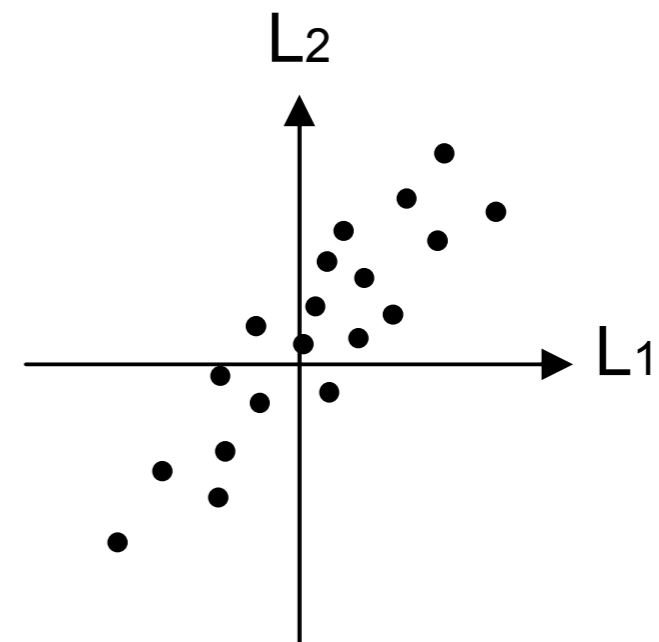
- A oszlopvektorai ortogonálisak
- $D(x) = D(z)$
- Izotróp zaj



State space of two pixel images



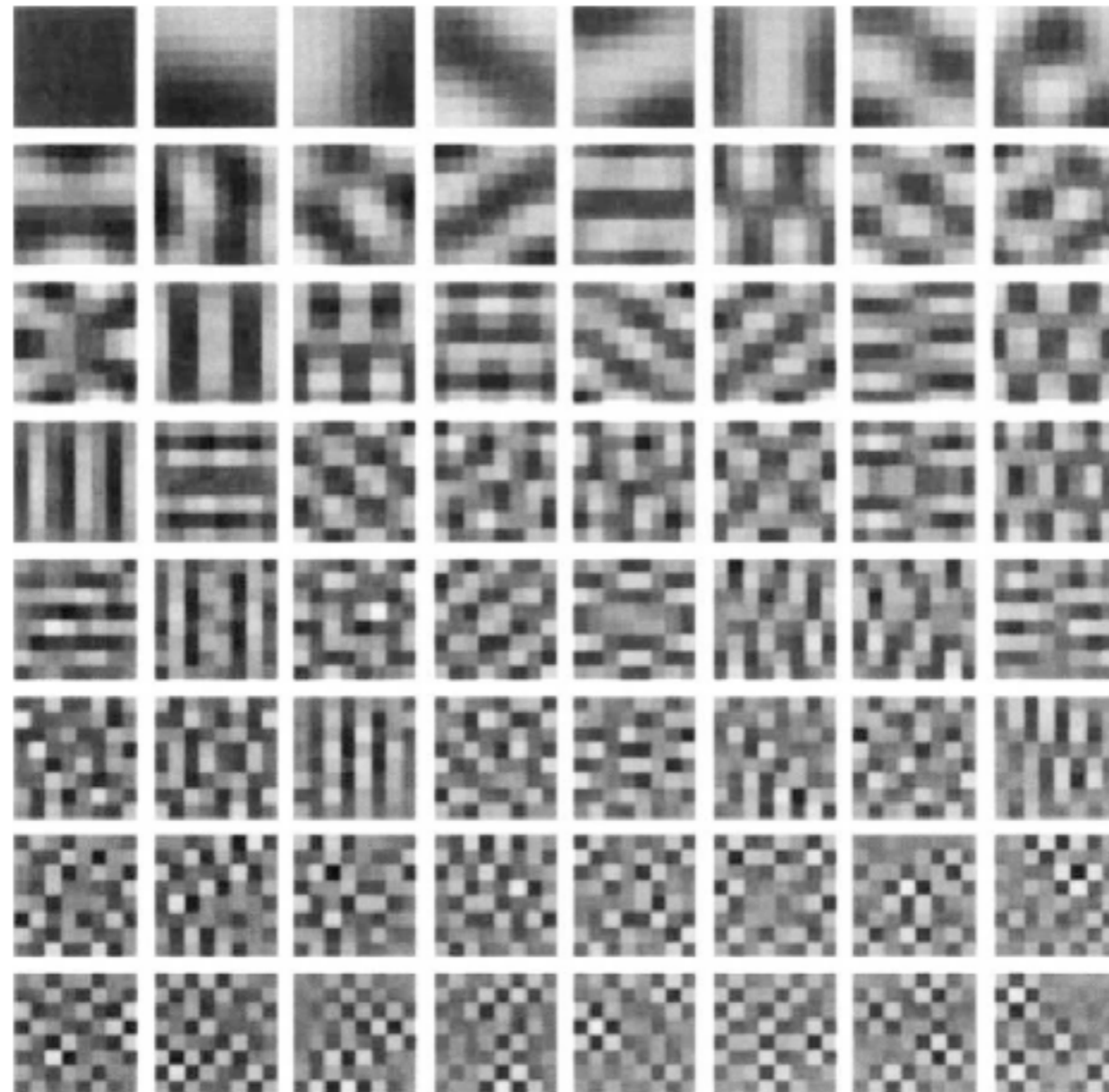
Random images



Structured images

PCA tulajdonságok

- Kompakt kódot eredményez
- Egy adatponért leírásáért általában a teljes hálózat felel



Sparse kódolás, ICA

- “z”-k függetlenek
- y priorja “ritka” ($P(z)$)

Sparse kódolás, ICA

$$x = A \cdot z + \epsilon$$

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Sparse kódolás, ICA

$$x = \mathbf{A} \cdot z + \epsilon$$

- “z”-k függetlenek
- y priorja “ritka” ($P(z)$)

Komputációs kritériumok:

- Hiteles rekonstrukció
költség egy adatpontra (képre):

$$\text{cost}_1 = \left(x - \sum_i A'_i \cdot z_i \right)^2$$

- Kis “energiafelhasználás (kevés szimultán aktiv neuron)
további költség a kód “ritkasága”:

$$\text{cost}_2 = - \sum_i S \left(\frac{z_i}{\sigma} \right)$$

S a Gauss-nál nagyobb kurtózissal bíró eloszlás

- teljes költség (~energia):

$$E = -\text{cost}_1 - \lambda \text{cost}_2$$

Sparse kód tanulása: E-M

Algoritmus:

- Itáráció EM lépésekkel
- Random kezdeti feltételek
- Adott konnektivitási mátrixnál az aktiviások segítségével a költség minimalizálása
- Adott aktivitásokkal a költség minimalizálása a súlyok adaptálásával

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Adott konnektivitási mátrix esetén a legjobb aktivitások megtalálása:

$$\dot{z}_i = \mathbf{A}_i x_t - \sum_j \mathbf{A}'_i \mathbf{A}_j z_j - \frac{\lambda}{\sigma} S' \left(\frac{z_i}{\sigma} \right)$$

Sparse kód tanulása: E-M

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Adott konnektivitási mátrix esetén a legjobb aktivitások megtalálása:

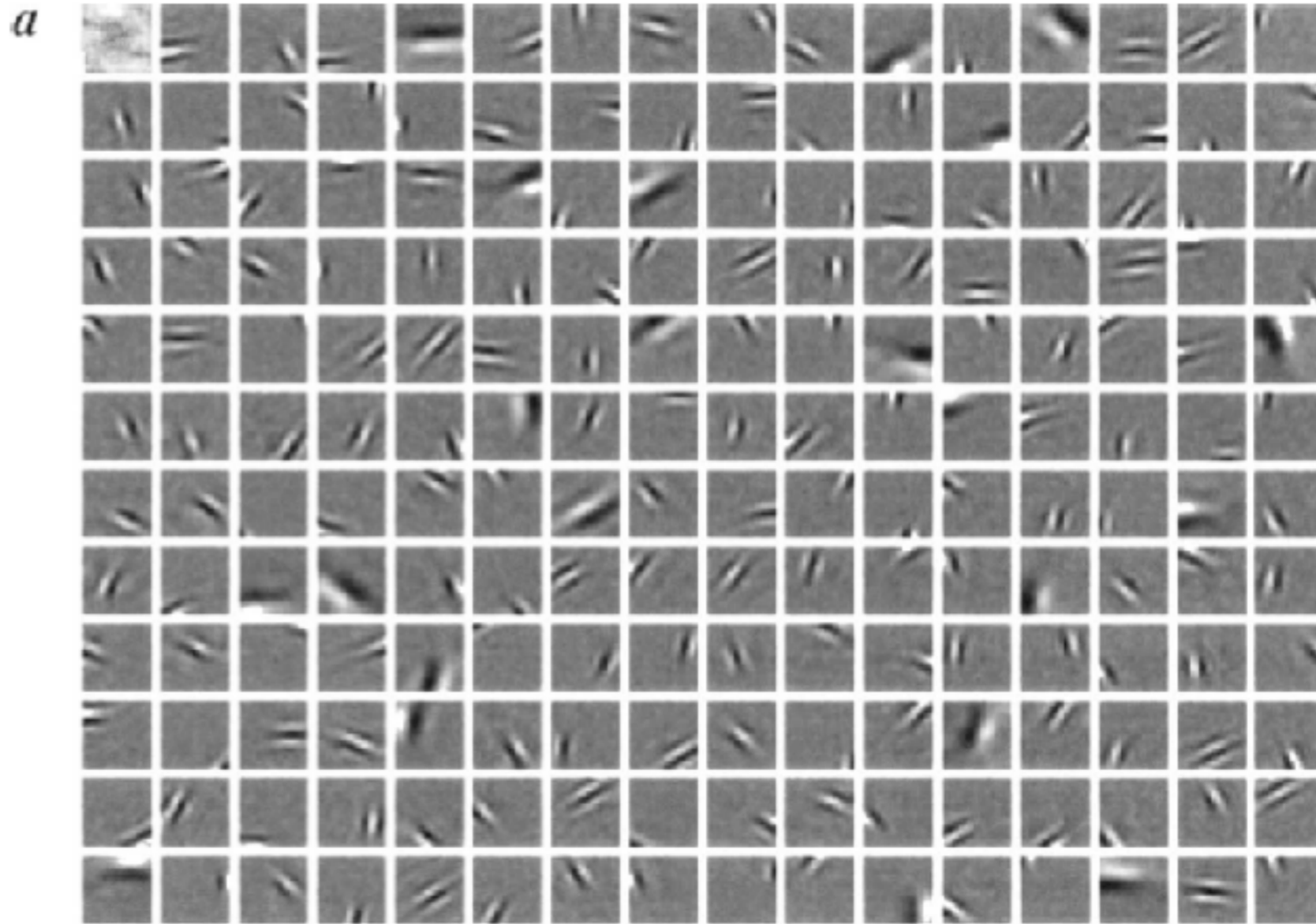
$$\dot{z}_i = \mathbf{A}_i x_t - \sum_j \mathbf{A}'_i \mathbf{A}_j z_j - \frac{\lambda}{\sigma} S' \left(\frac{z_i}{\sigma} \right)$$

Adott konnektivitási aktivációk esetén a legjobb súlyok megtalálása:

$$\Delta A_i = \eta \langle a_i [x - \hat{x}] \rangle_t$$

Sparse kódolás: eredmény

tréningezés természetes képekkel

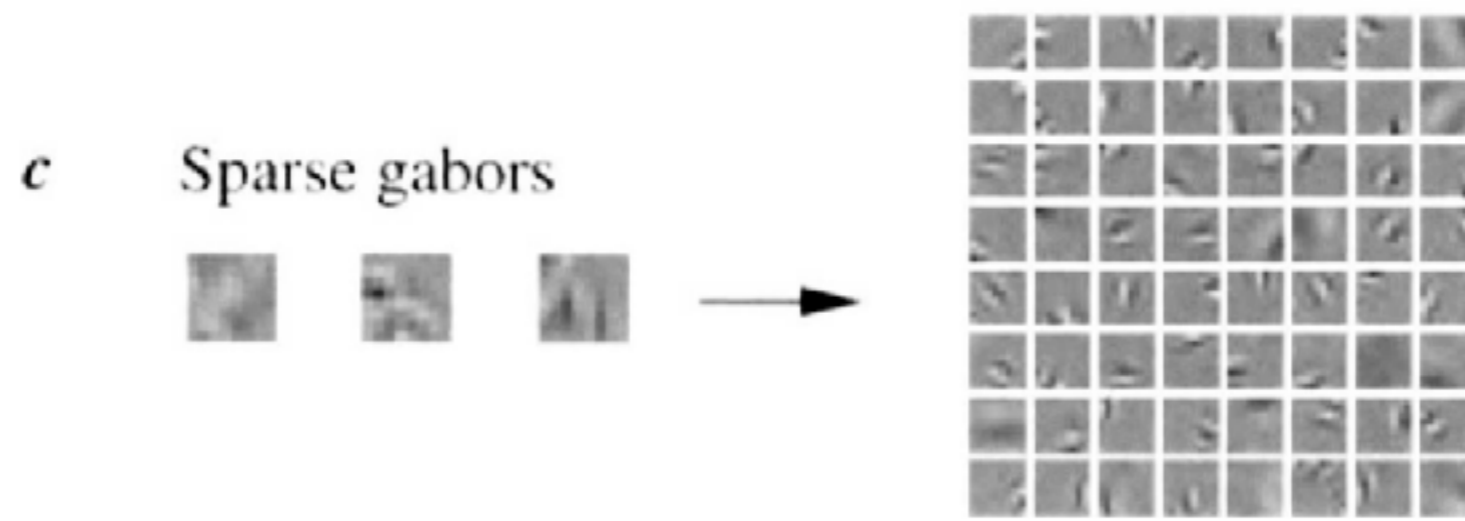


Olshausen & Field '96

A kialakult bázis:

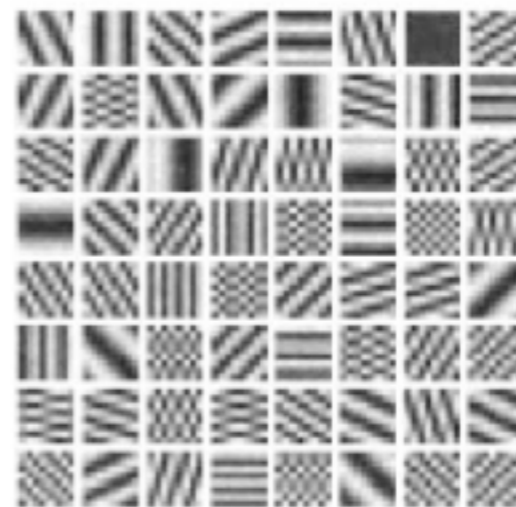
- irányított
- térbeli sávszűrést valósít meg
- lokalizált

Tanulás és stimulus statisztika

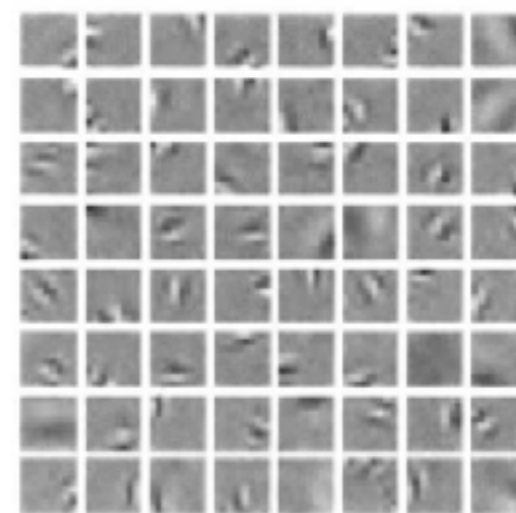


Tanulás és stimulus statisztika

b Sparse gratings

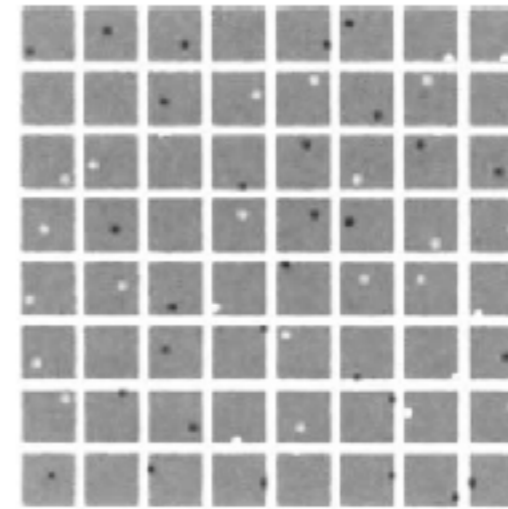
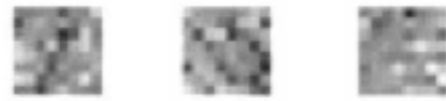


c Sparse gabors

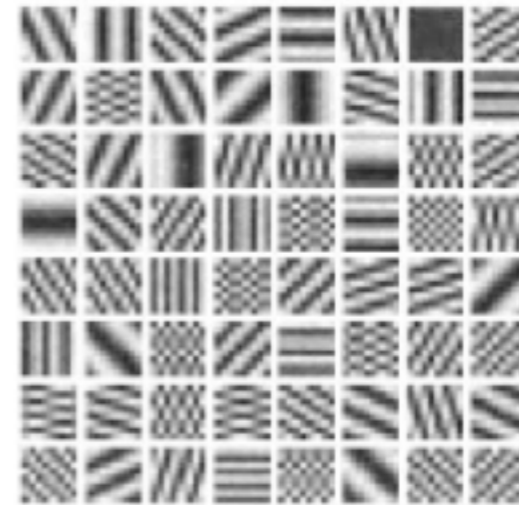


Tanulás és stimulus statisztika

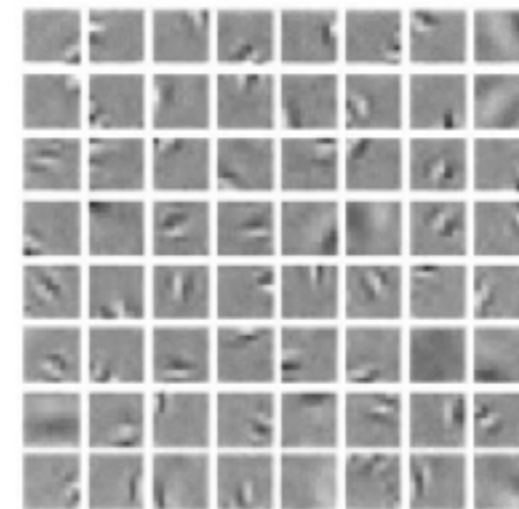
a Sparse pixels



b Sparse gratings



c Sparse gabors

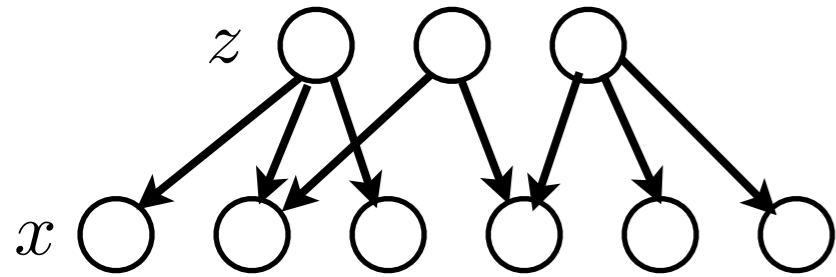


Generatív/rekogníciós modell

$$P(\mathbf{x}) = P(\mathbf{x} | \mathbf{z}) P(\mathbf{z})$$

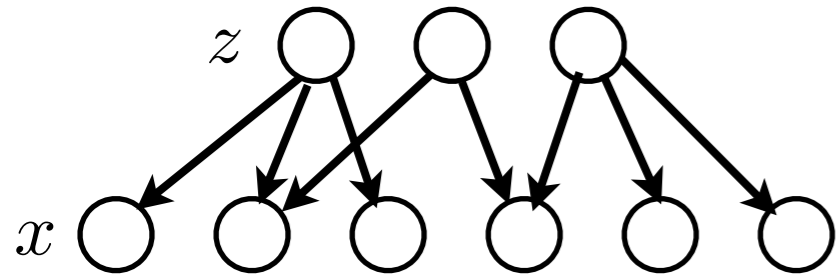
Generatív/rekogníciós modell

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Generatív/rekogníciós modell

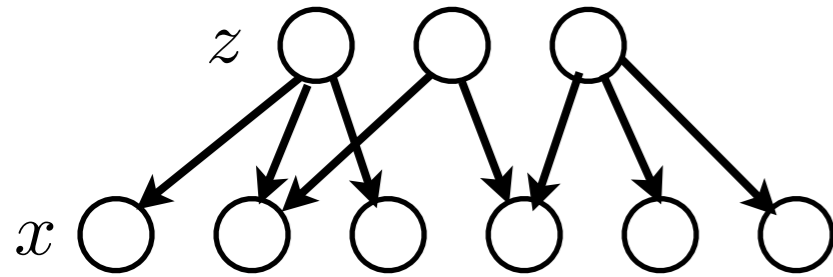
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szituáció / környezet

Generatív/rekogníciós modell

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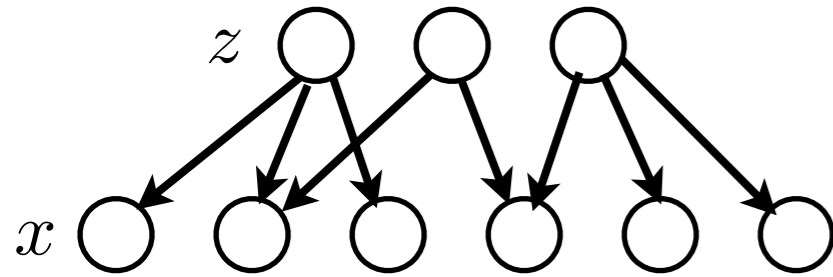


szituáció / környezet

objektumok

Generatív/rekogníciós modell

$$P(\mathbf{x}) = P(\mathbf{x} | \mathbf{z}) P(\mathbf{z})$$



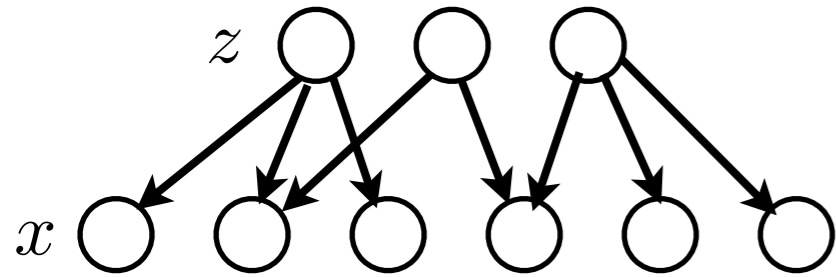
szituáció / környezet

objektumok

objektum elhelyezkedése |
méret, hely, helyzet, világítás

Generatív/rekogníciós modell

$$P(\mathbf{x}) = P(\mathbf{x} | \mathbf{z}) P(\mathbf{z})$$



szituáció / környezet

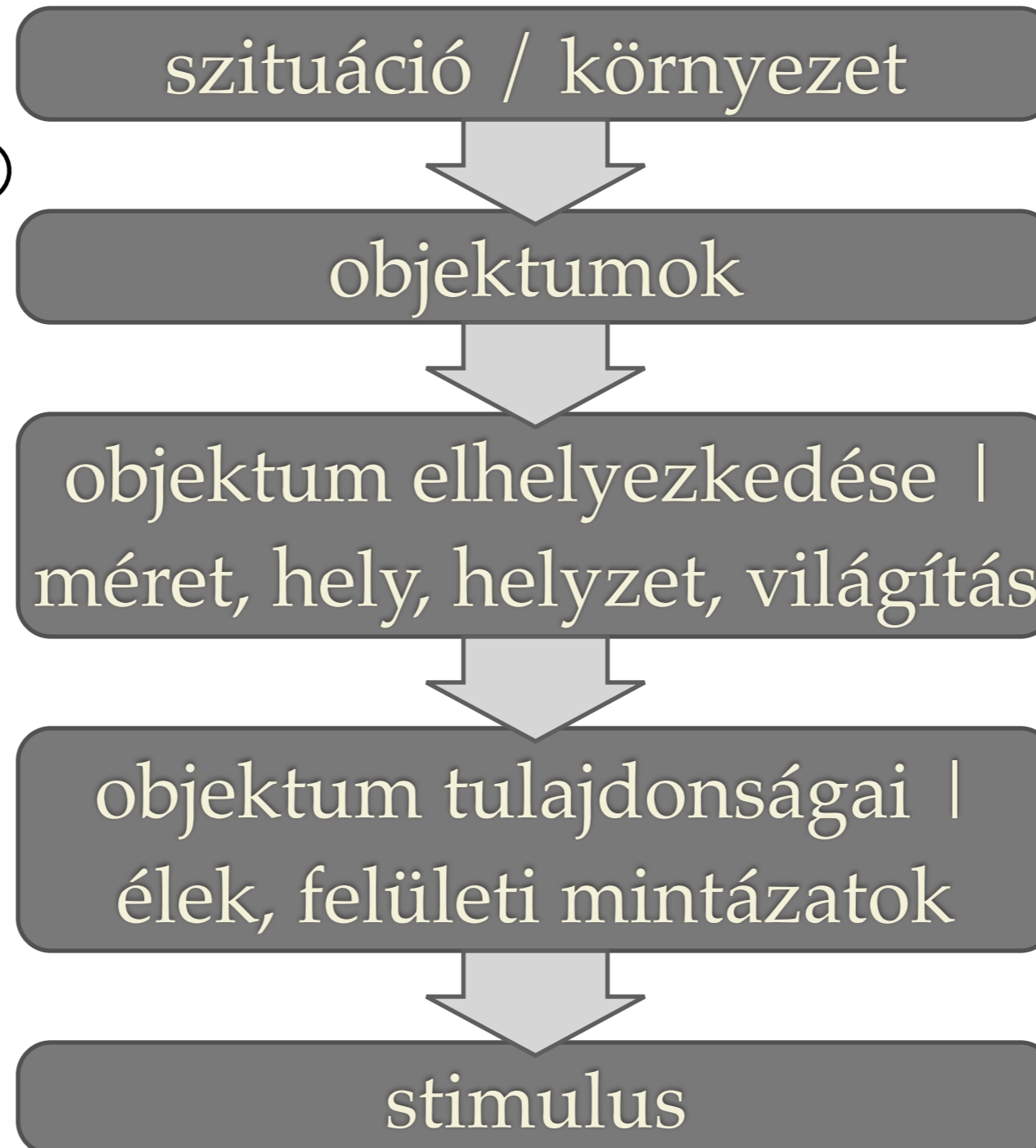
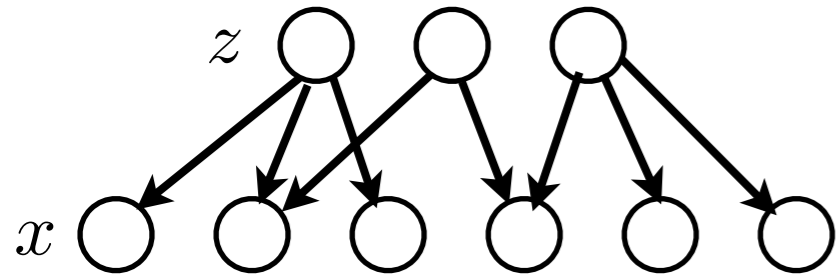
objektumok

objektum elhelyezkedése |
méret, hely, helyzet, világítás

objektum tulajdonságai |
élek, felületi mintázatok

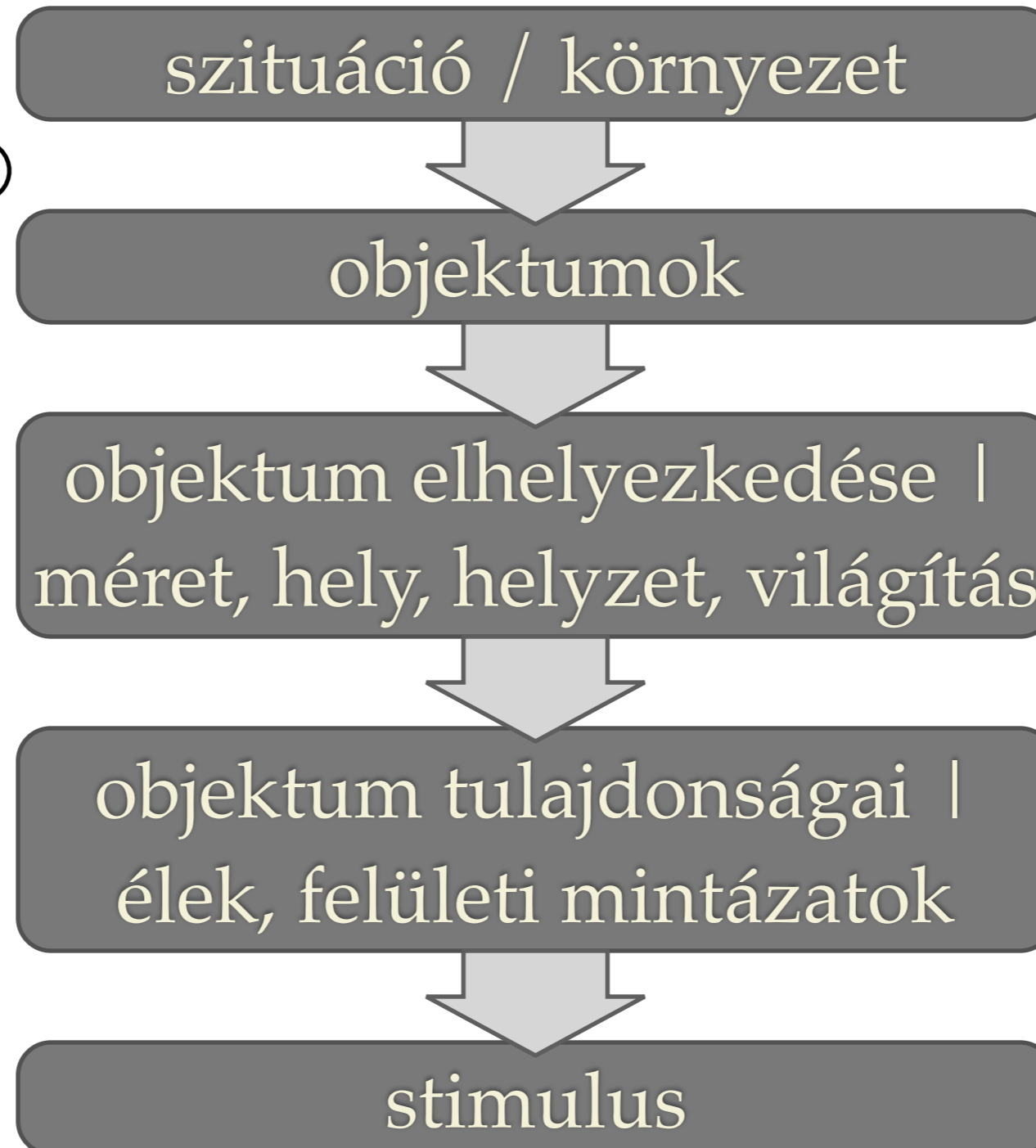
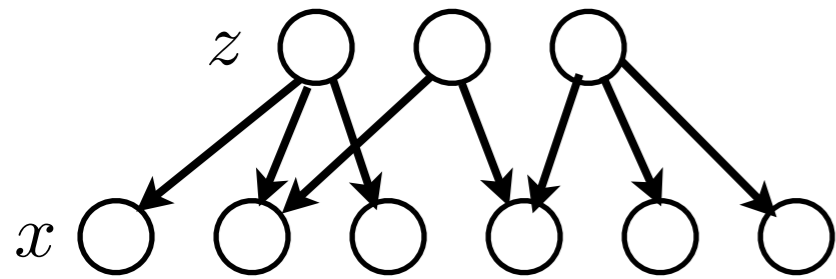
Generatív/rekogníciós modell

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Generatív/rekogníciós modell

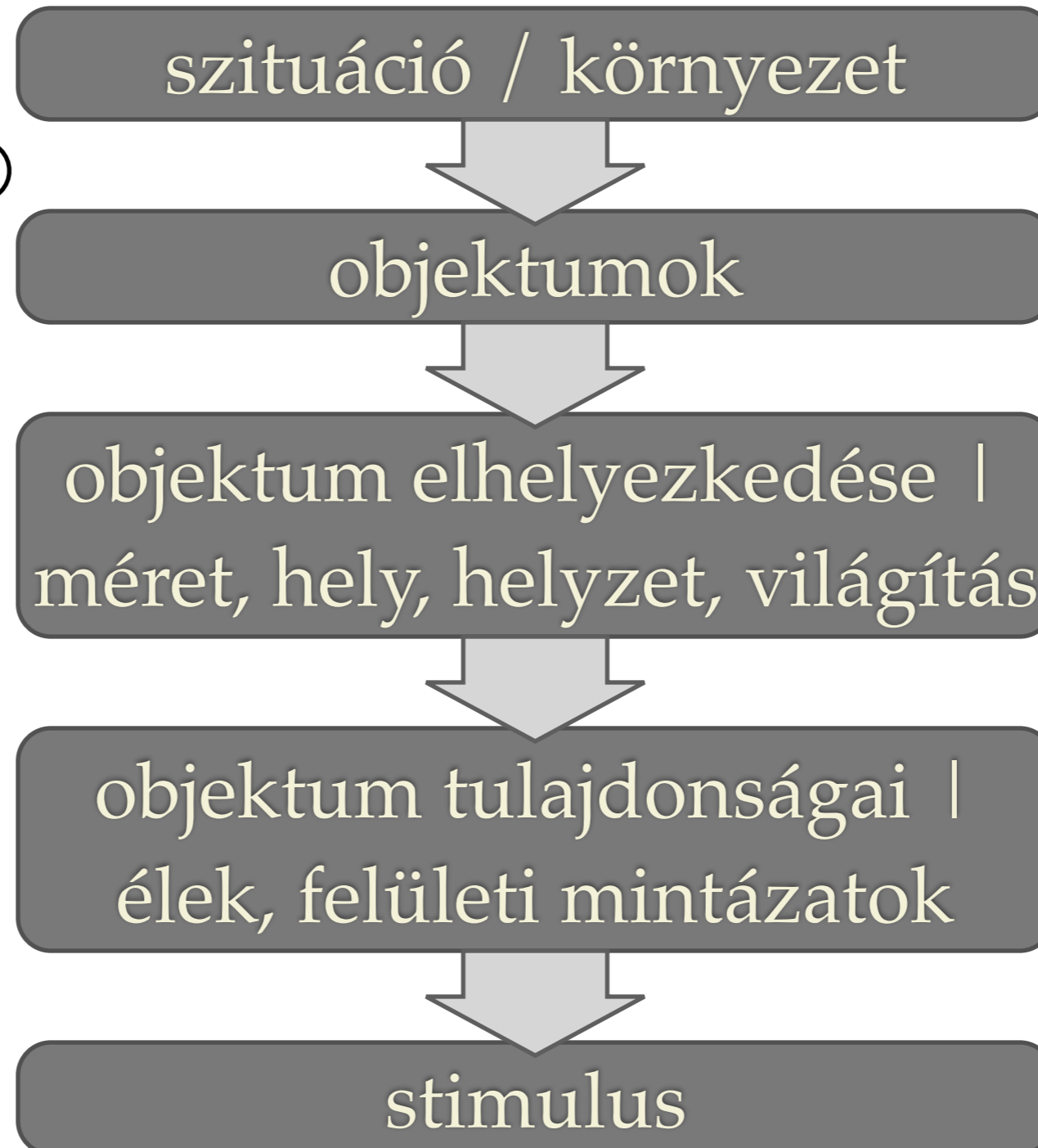
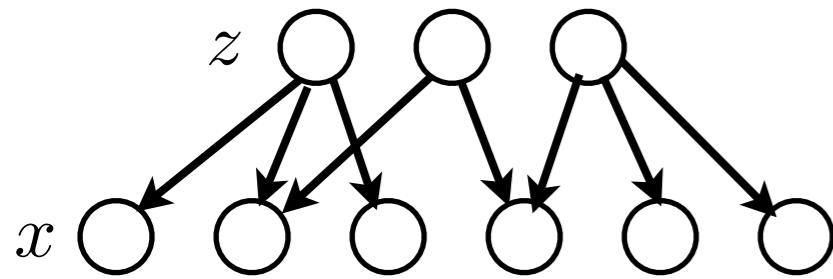
$$P(\mathbf{x}) = P(\mathbf{x} | \mathbf{z}) P(\mathbf{z})$$



Generatív modell

Generatív/rekogníciós modell

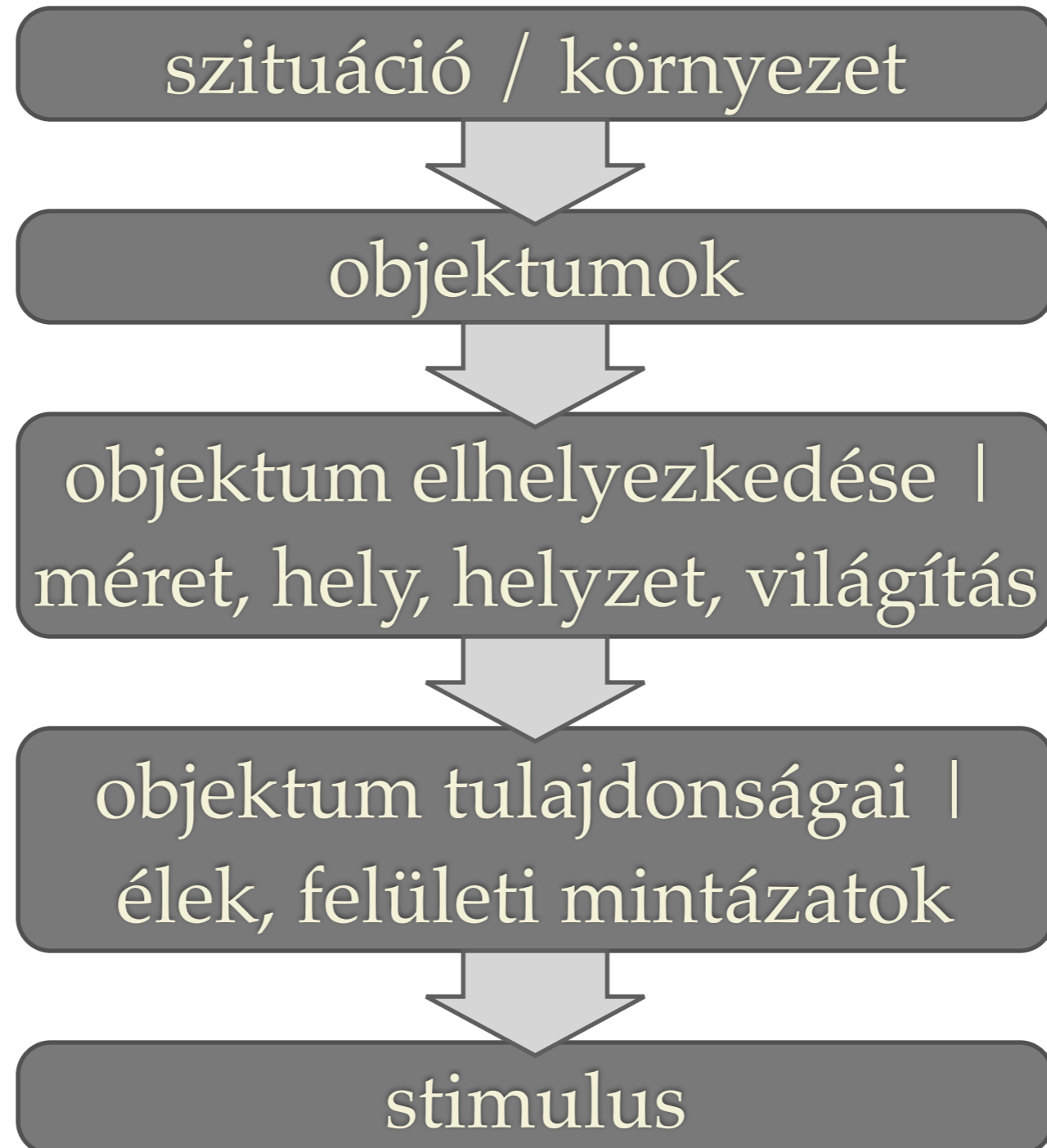
$$P(\mathbf{x}) = P(\mathbf{x} | \mathbf{z}) P(\mathbf{z})$$



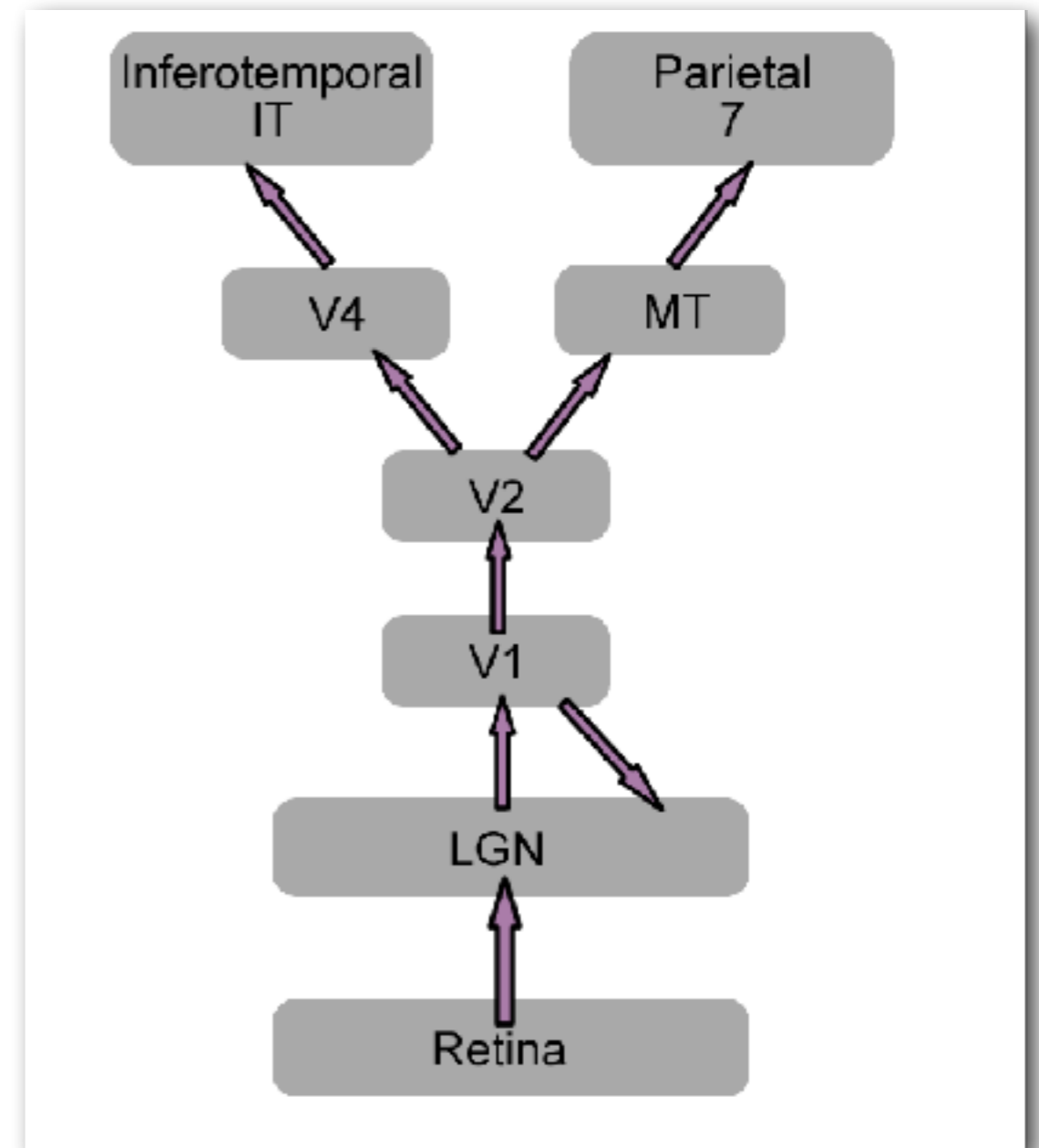
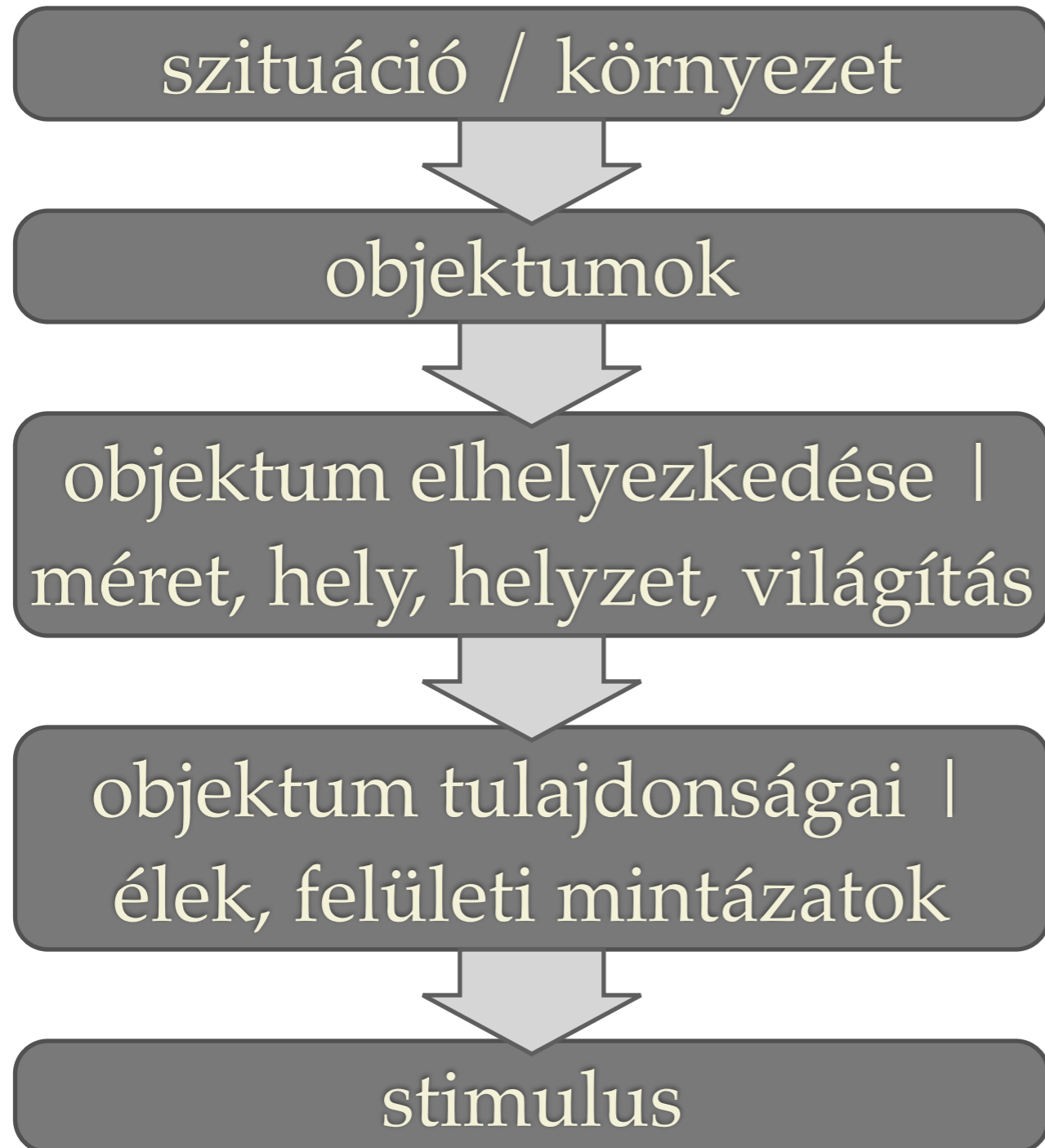
generatív modell

inferencia/felismerés

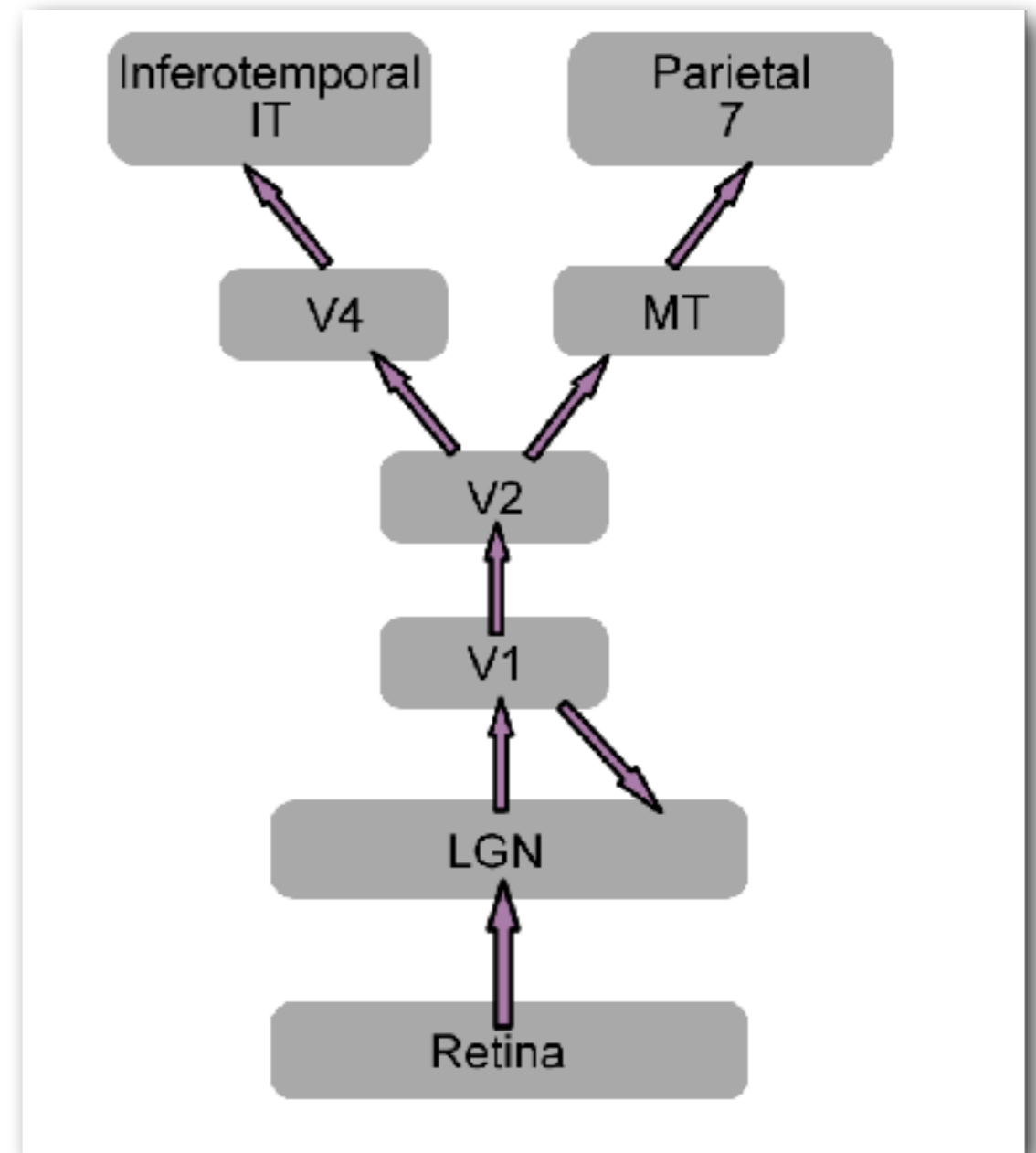
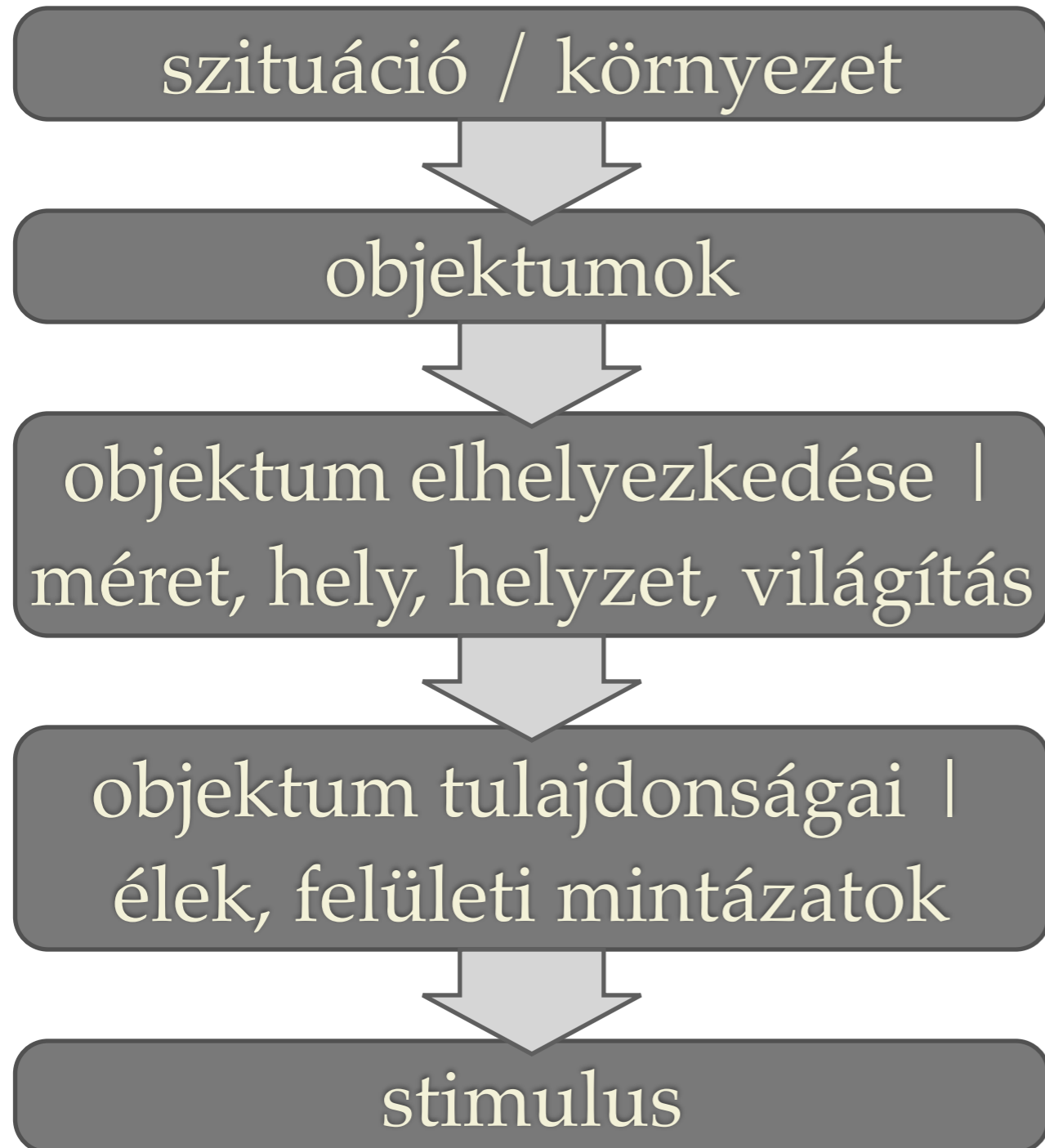
Generatív/rekogníciós modell



Generatív/rekogníciós modell



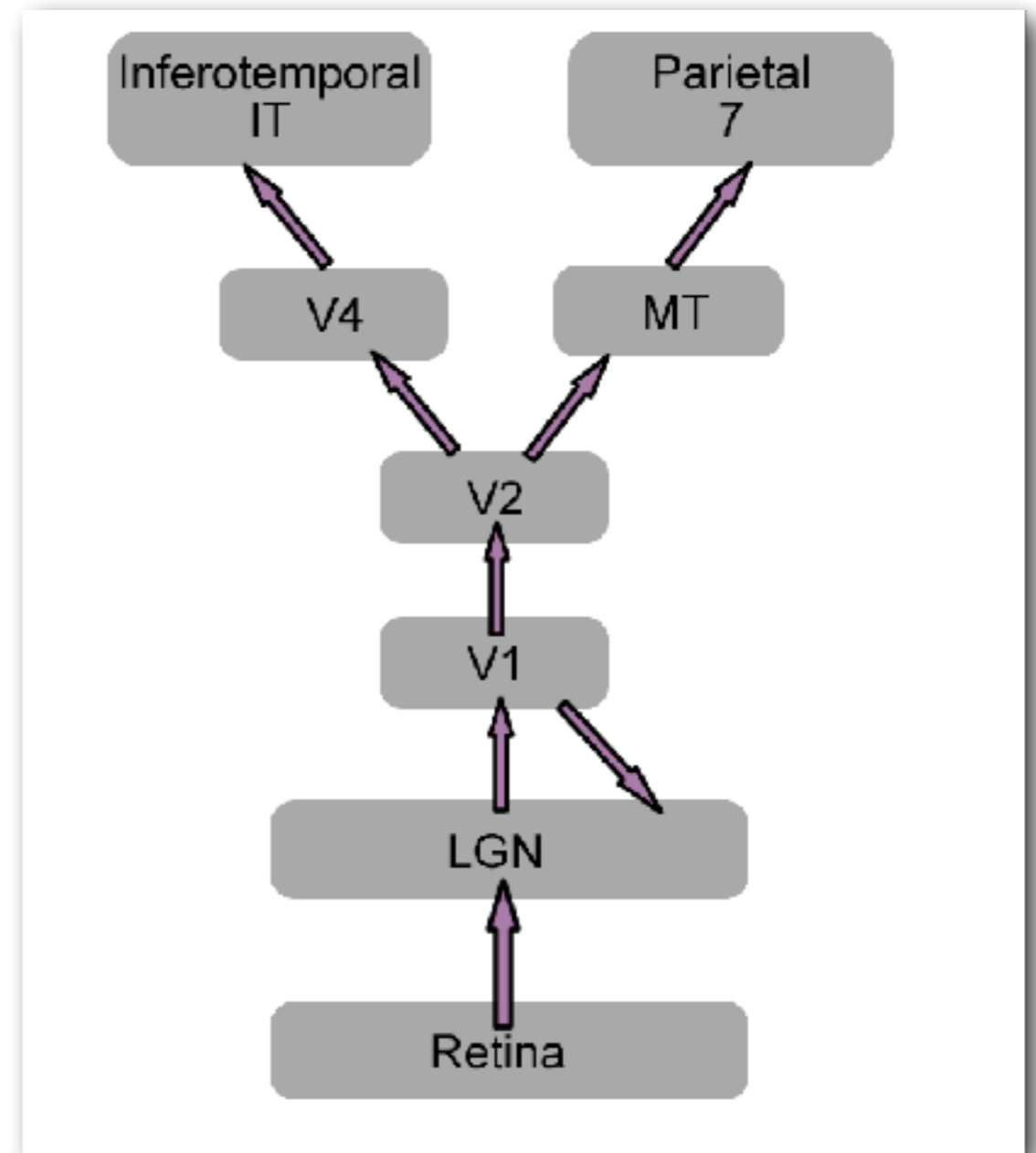
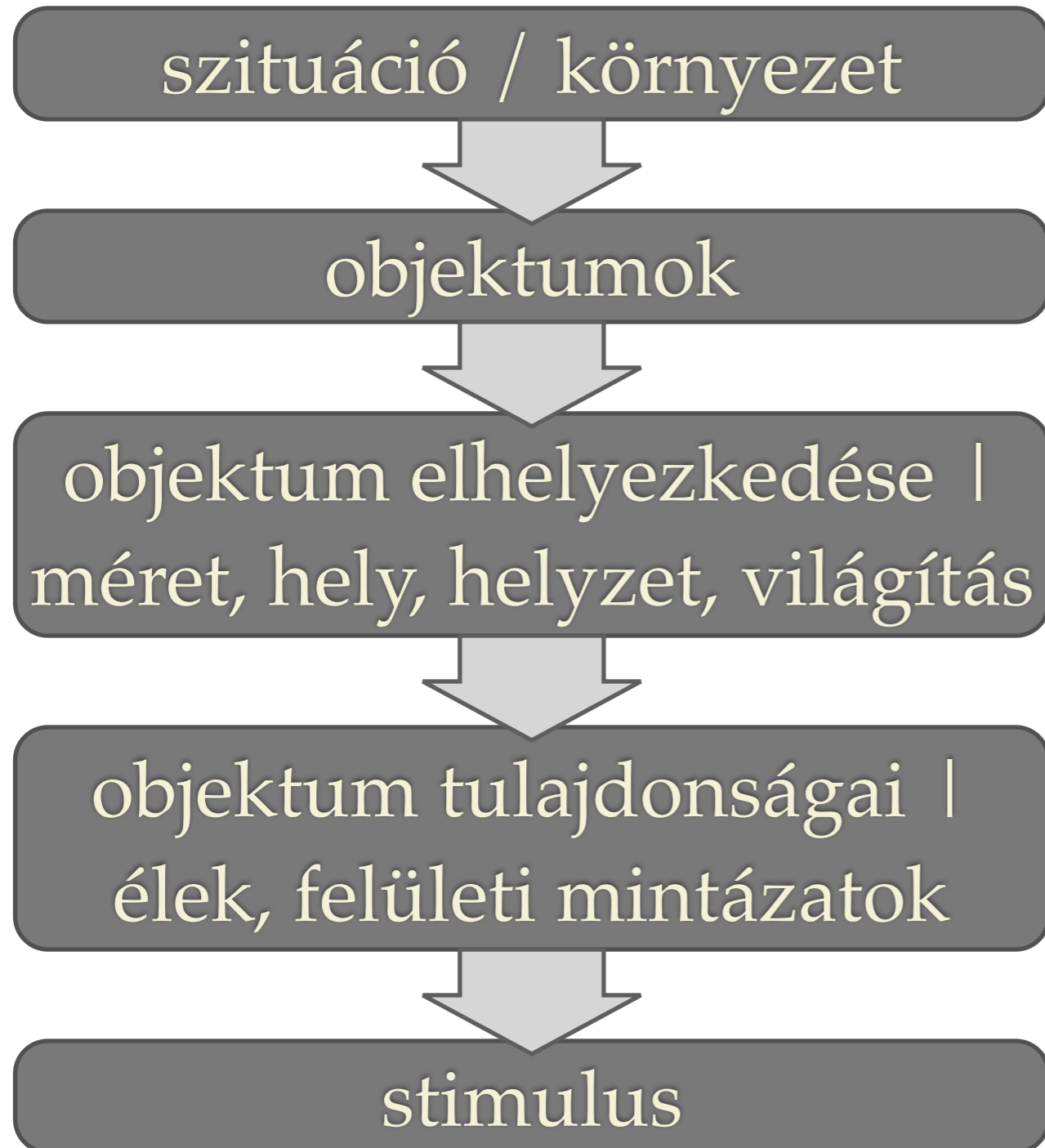
Generatív/rekogníciós modell



Modell definíció -> rekogníció:

$$P(x|z)$$

Generatív/rekogníciós modell

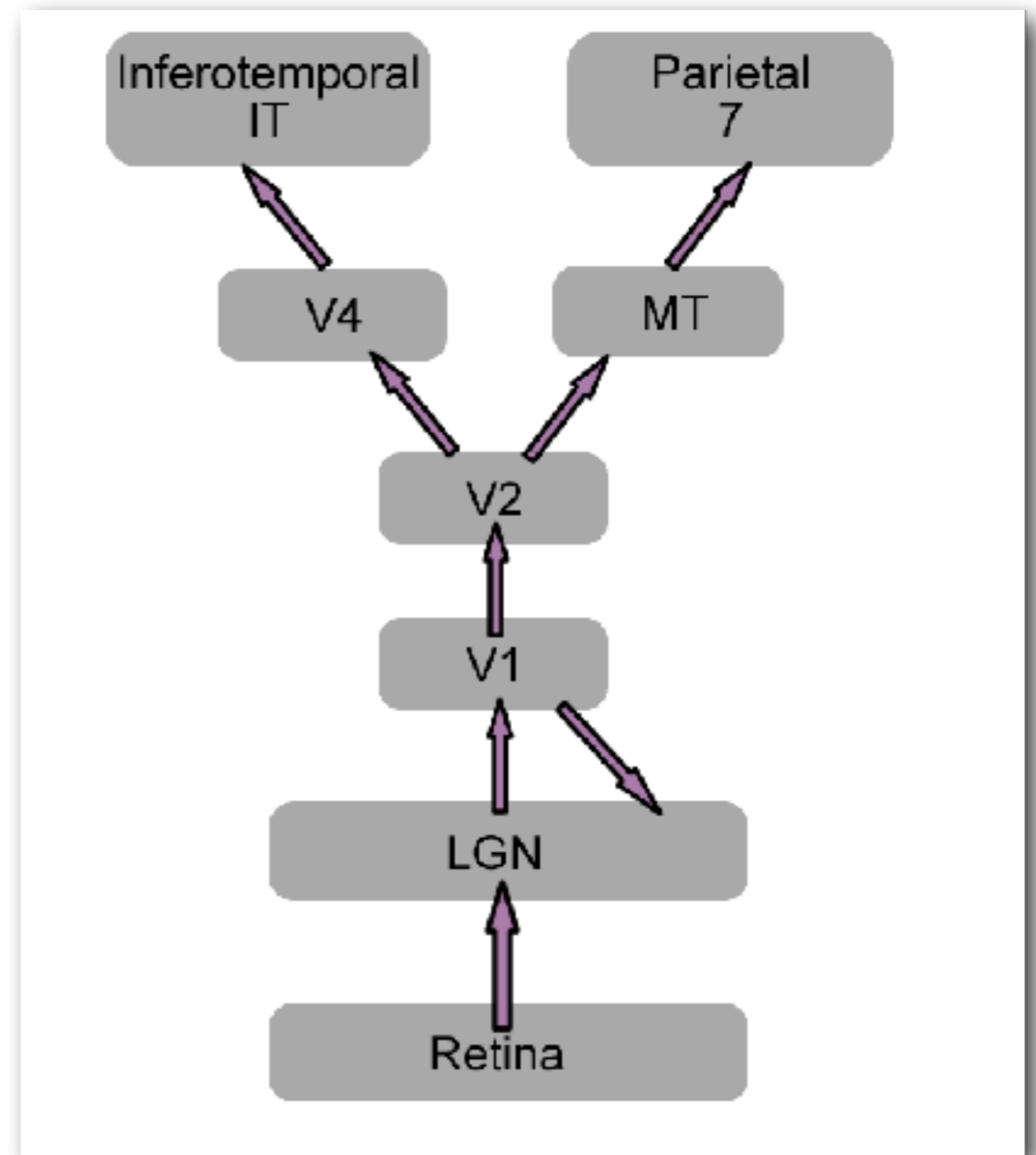
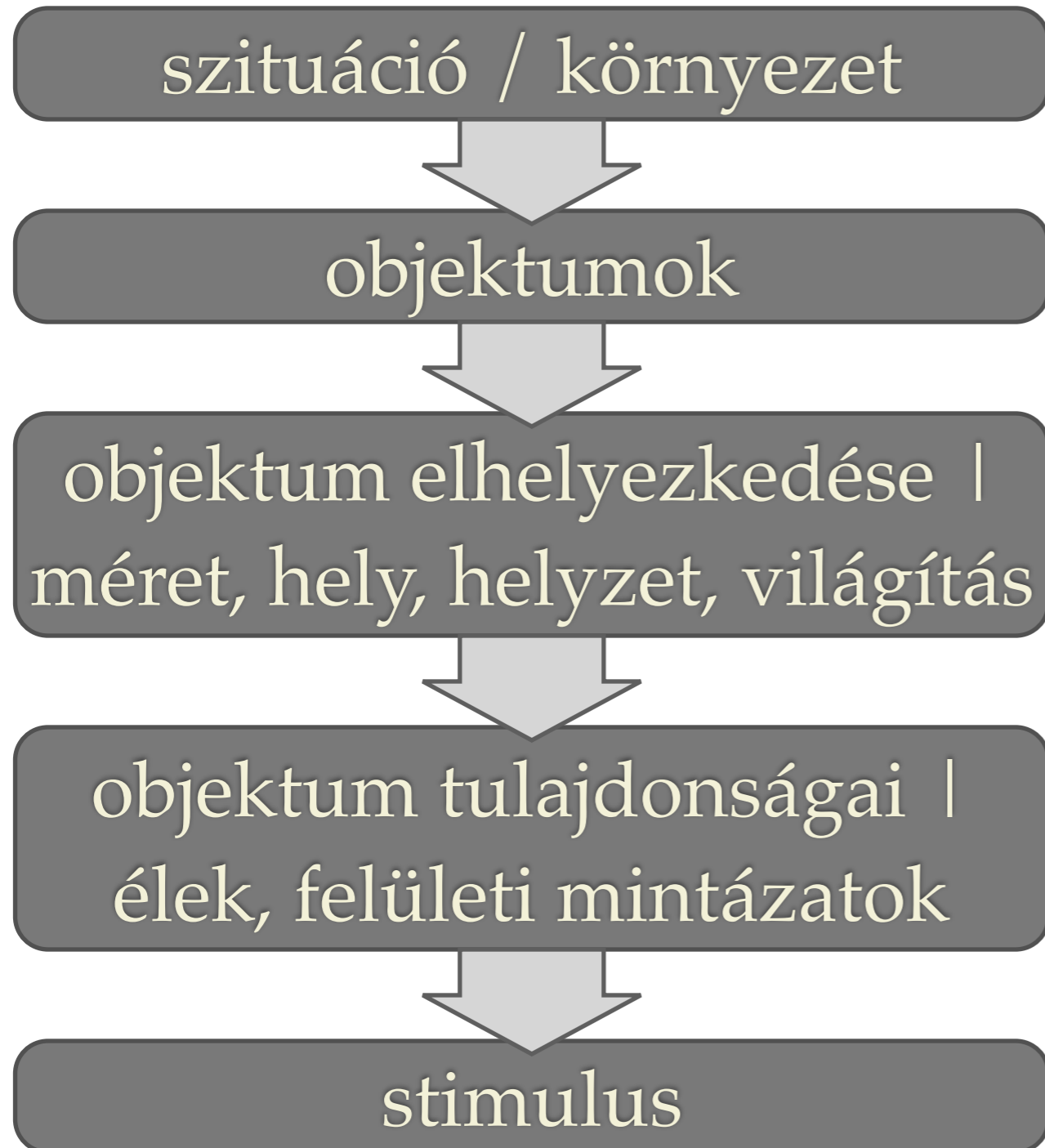


Modell definíció -> rekogníció:

$$P(x|z)$$

$$P(z|x)$$

Generatív/rekogníciós modell



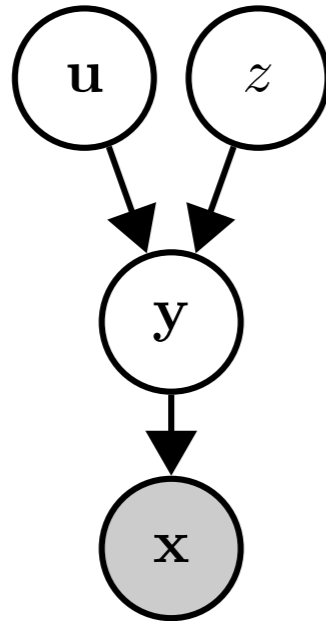
Modell definíció -> rekogníció:

$$P(x|z)$$

Inferencia igénye -> rekogníció:

$$P(z|x)$$

Gaussian Scale Mixtures



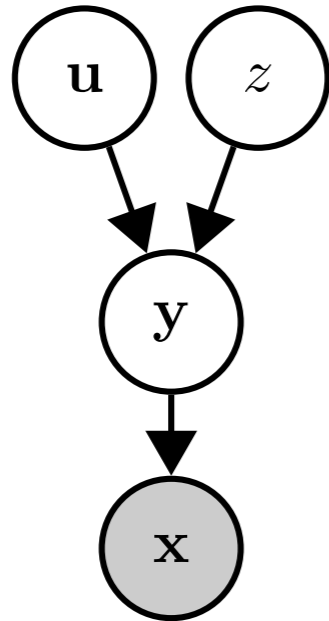
$$P(\mathbf{x}|\mathbf{y}) = \mathcal{N}(\mathbf{x}; \mathbf{A}\mathbf{y}, \sigma_x^2 \mathbf{I})$$

$$\mathbf{y} = z \mathbf{u}$$

$$P(\mathbf{u}) = \mathcal{N}(\mathbf{u}; \mathbf{0}, \mathbf{C})$$

$$P(z) = \text{Gamma}(z; k, \theta)$$

Gaussian Scale Mixtures



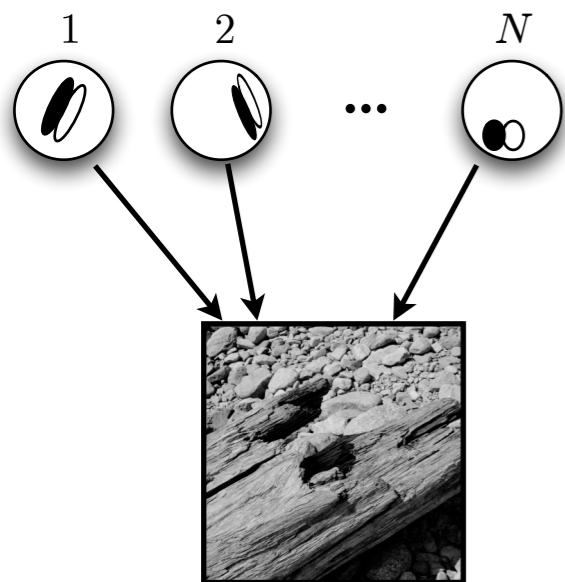
$$P(\mathbf{x}|\mathbf{y}) = \mathcal{N}(\mathbf{x}; \mathbf{A}\mathbf{y}, \sigma_x^2 \mathbf{I})$$

$$\mathbf{y} = z \mathbf{u}$$

$$P(\mathbf{u}) = \mathcal{N}(\mathbf{u}; \mathbf{0}, \mathbf{C})$$

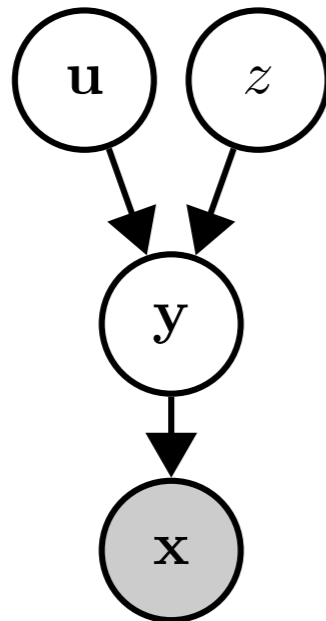
$$P(z) = \text{Gamma}(z; k, \theta)$$

linear features



image

Gaussian Scale Mixtures



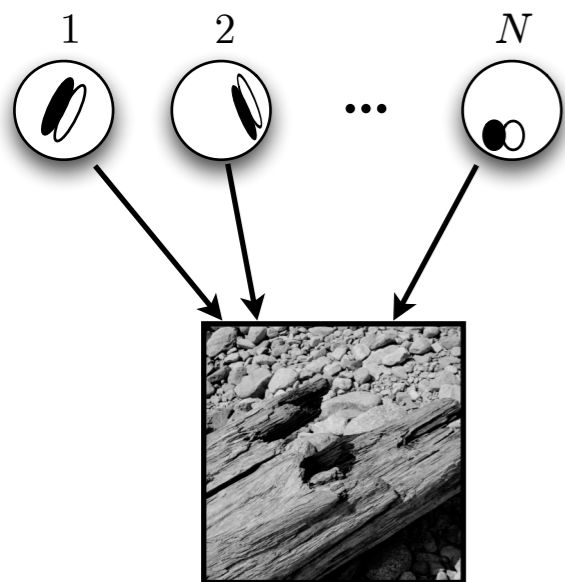
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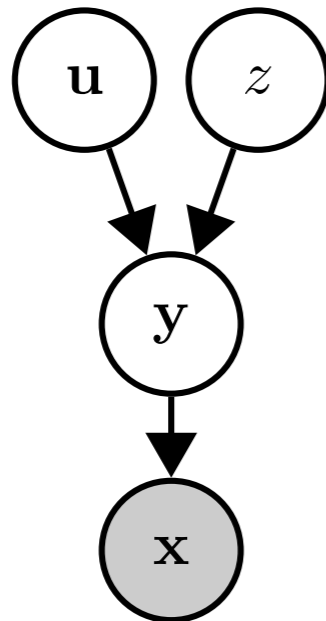
linear features



$$\text{image} = a_1 \text{feature}_1 + a_2 \text{feature}_2 + \dots + a_N \text{feature}_N + \text{noise}$$

image

Gaussian Scale Mixtures



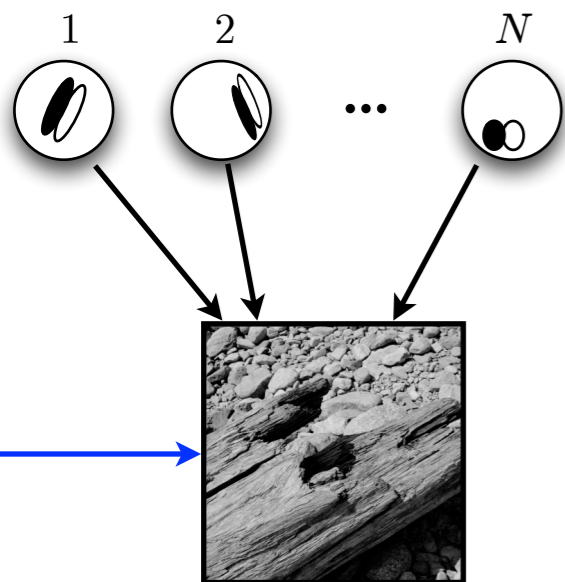
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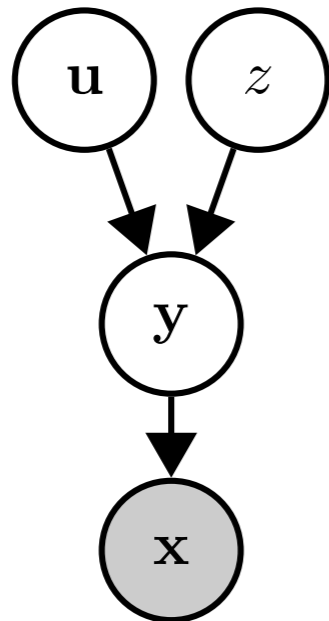
linear features



image

$$\text{image} = \text{contrast} \times (a_1 \text{feature}_1 + a_2 \text{feature}_2 + \dots + a_N \text{feature}_N + \text{noise})$$

Gaussian Scale Mixtures



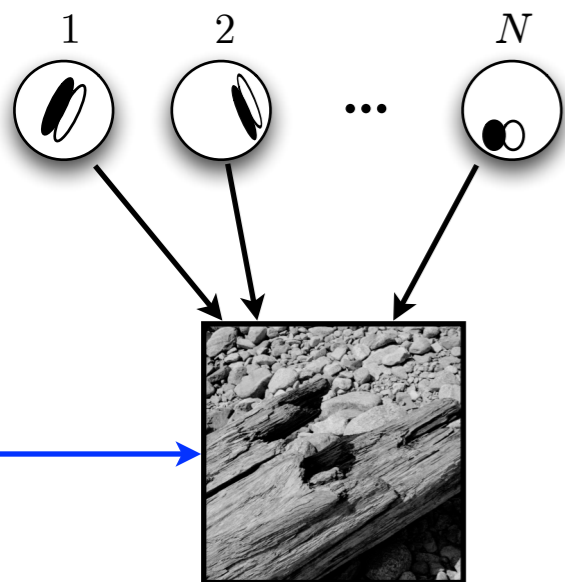
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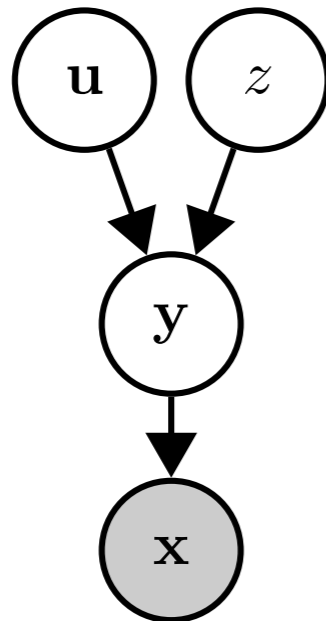
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$$\text{var}(L_1|L_2) = wL_2^2 + \sigma^2$$

$$R_1 = \frac{L_1^2}{wL_2^2 + \sigma^2}$$

image

Gaussian Scale Mixtures



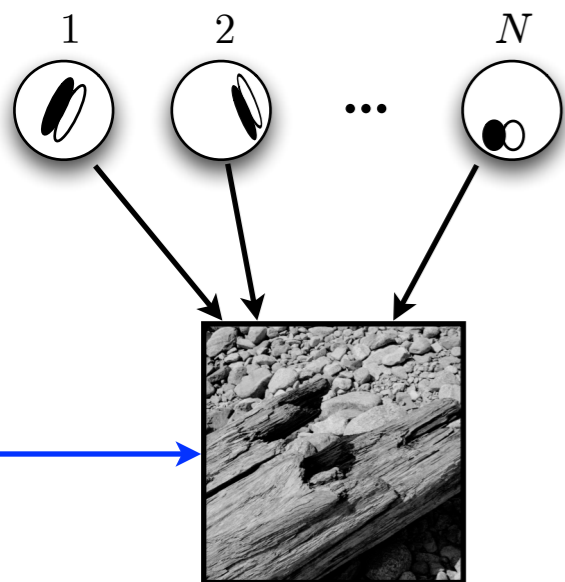
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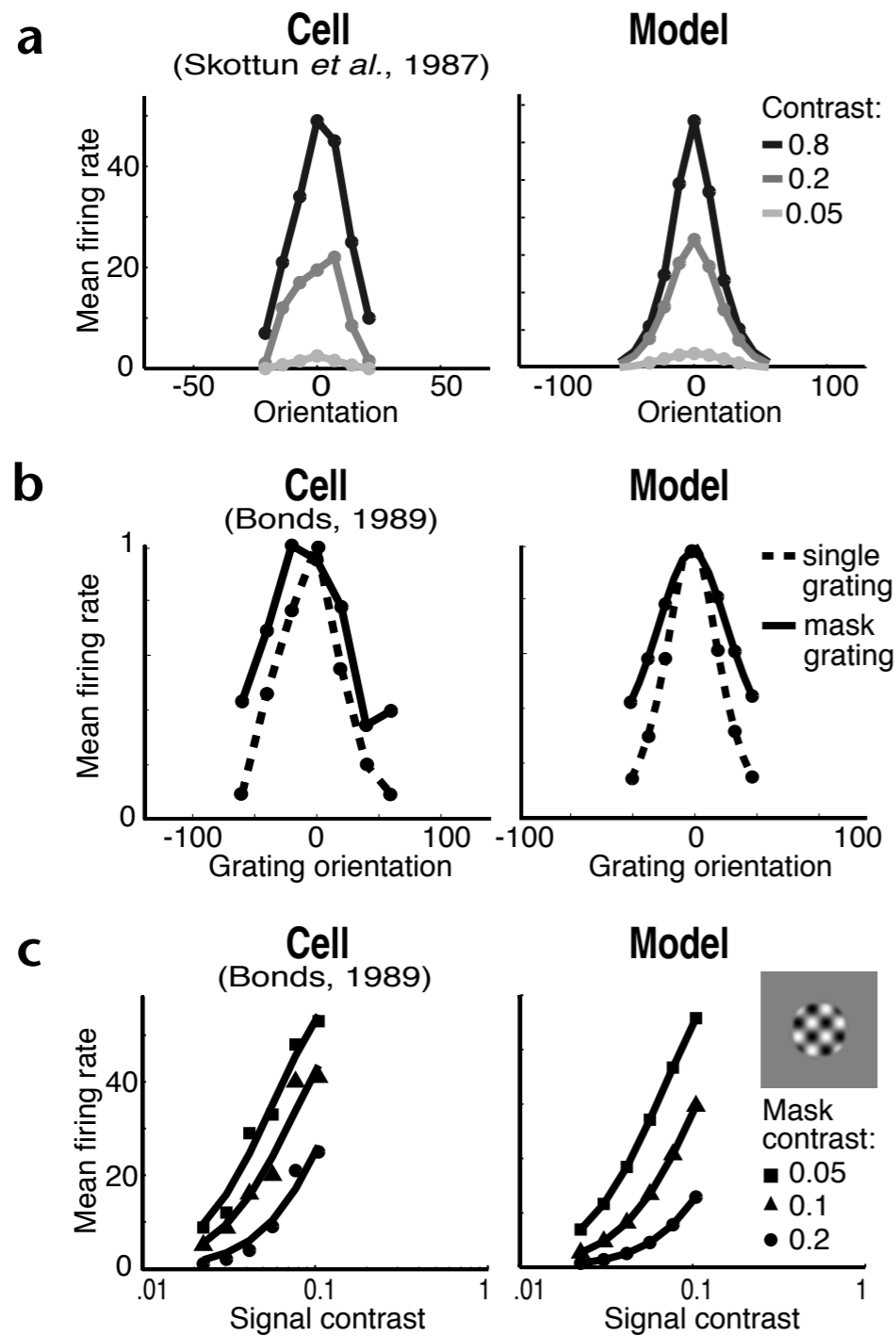
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$$\text{var}(L_i|\{L_j, j \in N_i\}) = \sum w_{ji} L_j^2 + \sigma^2$$

$$R_i = \frac{L_i^2}{\sum_j w_{ji} L_j^2 + \sigma^2}$$

Neurális adatok és GSM



Schwartz & Simoncelli, 2001

